



Variable Gain Super Twisting Sliding Mode Control (VGSTW) application in longitudinal plane of Autonomous Underwater Glider (AUG)

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This paper presents a robust control design using variable gain super twisting sliding mode control application in Autonomous Underwater Glider (AUG). AUGs are known as underactuated systems and very nonlinear in nature make difficult to control. The controller is designed for longitudinal plane of an AUG that tracks the pitching angle and net buoyancy of a ballast pump for nominal system, system in existence of external disturbance and parameter variations in hydrodynamic coefficients. The Lyapunov stability theorem has proved that the proposed control law is satisfied the stability sufficient condition. The simulation results have shown that the proposed controller has improved the transient response, reduced steady error and chattering effects in control input and sliding surface in all cases.

[**Keywords:** Autonomous Underwater Glider (AUG), Chattering reduction, Robustness, Variable Gain Super Twisting Sliding Mode Control (VGSTW)]

Introduction

The idea of glider concept was proposed by Henry Stommel in 1989^(ref. 1). Later in 1990 Henry together with Webb were awarded a grant from naval office and they successfully developed battery-powered glider prototype and completed a total of 71 dives. With this success, promotes many researchers to embark in this field of research where later three operational gliders classified as a special class of Autonomous Underwater Vehicle (AUV) were successfully developed name as Slocum glider, Spray glider and Seaglider. Many agencies and research laboratories using these three AUGs to collect oceanography data². There are many lab-scale AUGs have been developed by many universities and institutes such as robotic gliding fish (Michigan State University)³, USM glider⁴ (University Science Malaysia), a light-weight underwater glider (Shanghai Jiao Tong University)⁵, RoBuoy underwater glider (Indian Institute of Technology Madras, India)⁶ and Sepiida glider (University of Cambridge)⁷. Autonomous Underwater Glider (AUG) glides through the water column by shifting its internal movable masses translationally or rotationally and pumping the ballast. Many control techniques were

proposed to control the motion of AUG. AUG save energy usage since the design of AUG with the absence of external propellers that contribute to high power usage.

Gliders are known as Multi-Input-Multi-Output (MIMO) nonlinear under-actuated systems. Therefore, AUG is considered as highly nonlinear, uncertainties in hydrodynamic coefficients, time-varying dynamic in nature and also ocean disturbances such as ocean currents and waves⁸. Numerous control methods have been proposed for AUG. Review on control techniques implementations in AUG has been done by Ullah *et al.*⁹. The control techniques reviewed in the paper include classical Proportional-Integral-Derivative (PID), optimal control of Linear Quadratic Regulator (LQR), robust nonlinear control up to intelligent control such as neural network and fuzzy logic. Review on sliding mode control implementation in AUV was done by Mat-Noh¹⁰ where the review is limited to all of SMC families implemented in AUV including AUG. As proposed in Garcia *et al.*⁷, Latifah *et al.*¹¹, Vidya *et al.*¹², classical PID control has a basic architecture with fewer tuning parameters, making it easier to implement. Optimal control is another linear control method¹³⁻¹⁶, which

has a simple architecture and requirements. In order to get the desired performance, only two tuning parameters had to be changed. Both PID and optimal provide good performance. However, both effective only in a small neighbourhood of the equilibrium since the model is linearized about the equilibrium point.

The Model Predictive Control (MPC) had been proposed in by Shan & Yan¹⁷, Abraham & Yi¹⁸ and Liu *et al.*¹⁹. Shan & Zheng¹⁷ designed the MPC using one-layer recurrent neural network to improve the computational problem in MPC to control the longitudinal plane of AUG. The MPC was used by Abraham & Yi¹⁸ in conjunction with the path-following technique for online tuning of the desired vehicle velocity along with the trajectory and thus validated the 3D motion dynamics of the Slocum glider. Liu *et al.*¹⁹ used adaptive MPC for steering control of AUG where online estimator was designed to update the dynamic model of yaw system.

The intelligent control had been proposed in by Isa²⁰ & Liu²¹. The intelligent control does not need a precise mathematical model of the plant however they will suffer from high computational time and need high tuning effort in order to attain a good performance. The Sliding Mode Control (SMC) is the another technique used²²⁻²⁶. The boundary layer SMC was proposed by Mat-Noh *et al.*^{22,24} for 1 Degree of Freedom (DOF) and 2 DOF internal movable sliding mass, respectively. The Taylor's series expansion method is used in obtaining the linearized model of AUG. Yang & Ma^{25,26} proposed the SMC for nonlinear system of longitudinal plane of AUG. The reaching law is designed based on rapid-smooth reaching law. In Yang & Ma²⁶ the performance of Yang & Ma²⁵ is improved using inverse system method where the output equations are differentiated repeatedly until the input is appeared in the equations, then the control law is designed based on that equations. Mat-Noh *et al.*²³ proposed SMC to control the pitching and the net buoyancy of the longitudinal plane system. The control law is designed based on Super Twisting Sliding Mode Control (STSMC). The standard STSMC composed only discontinuous part, however in Mat-Noh *et al.*²³ the control law consists of equivalent and discontinuous parts.

This paper proposes the application of Variable Gain Super Twisting Siding (VGSTW) control in longitudinal plane of an AUG. The performance of the proposed controller is compared to the performance of original super twisting SMC (STW) to evaluate robustness of the proposed controller against

external disturbance and parameter variations. Since VGSTW has never been implemented in any of AUV or AUG systems, therefore it becomes contribution for this paper.

Approach and Methods

Autonomous Underwater Glider (AUG) model

Detail derivation of the motion equation of longitudinal plane can be found in Graver². The dynamics of the longitudinal plane is controlled via internal movable mass and variable ballast mass and uses fixed rudder to stabilize the glider straight motion in longitudinal plane where lateral dynamics can be ignored. Therefore, all the lateral parameters are equal to zero except pitching related parameter. Figure 1 and Table 1 show the reference frame and all related parameters of the glider respectively.

The desired angle glide path, ξ_d and speed V_d determines the glide path.

$$\xi = \theta - \alpha \quad \dots (1)$$

Where, θ = angle of pitching, α = angle of attack

$$V = \sqrt{v_1^2 + v_3^2} \quad \dots (2)$$

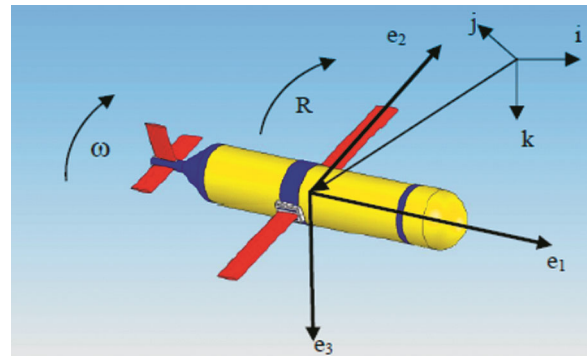


Fig. 1 — The reference frame of the glider

Table 1 — The notation used for the glider

No.	Motion axis	Linear and angular velocity	Position and orientation
1.	Motion in the x-direction (surge)	v_1 (m/s)	x
2.	Motion in the y-direction (sway)	v_2 (m/s)	y
3.	Motion in the z-direction (heave)	v_3 (m/s)	z
4.	Rotation about the x-axis (roll)	ω_1 (rad/s)	ϕ
5.	Rotation about the y-axis (pitching)	ω_2 (rad/s)	θ
6.	Rotation about the z-axis (yaw)	ω_3 (rad/s)	ψ

The initial coordinates (x', z') where x' is the position along the desired path and is defined as

$$\begin{pmatrix} x' \\ z' \end{pmatrix} = \begin{pmatrix} \cos \xi_d & -\sin \xi_d \\ \sin \xi_d & \cos \xi_d \end{pmatrix} \begin{pmatrix} x \\ z \end{pmatrix} \quad \dots (3)$$

Where, z' is the vehicle's position in the direction perpendicular to the desired path. The dynamics of the z' is written in Eq. (4)

$$\dot{z}' = \sin \xi_d \dot{x} + \cos \xi_d \dot{z} \quad \dots (4)$$

Practically, most internal movable mass of the AUG only moves along x-axis² and ballast pumping rate makes the glider glides through in water column. Thus, 1 Degree of Freedom (DOF) internal movable mass is considered in paper as proposed in Graver².

The motion equations in Leonard & Graver²⁷ is rewritten; where, r_{p3} is fixed at one position. The motion equations for 1 DOF internal movable mass are written in Eqs. (5 – 11)

$$\dot{\theta} = \omega_2 \quad \dots (5)$$

$$\dot{\omega}_2 = \frac{1}{a} \{ (m_p + m_1)(m_p + m_3)Y - m_p m_3 (m_p + m_1) r_{p1} \dot{r}_{p1} \omega_2 - m_p (m_p + m_3) r_{p3} X_1 + m_p (m_p + m_1) r_{p1} X_3 - m_p m_1 (m_p + m_3) r_{p3} u_1 \} \quad \dots (6)$$

$$\dot{v}_1 = \frac{1}{a} \{ -m_p (m_p + m_3) r_{p3} Y - m_p^2 m_3 r_{p1} r_{p3} \dot{r}_{p1} \omega_2 + m_p [J_2 (m_p + m_3) + m_p m_3 r_{p1}^2 + m_p (m_p + m_3) r_{p3}^2] X_1 - m_p^2 r_{p1} r_{p3} X_3 - m_p [J_2 (m_p + m_3) + m_p m_3 r_{p1}^2] u_1 \} \quad \dots (7)$$

$$\dot{v}_3 = \frac{1}{a} \{ m_p [J_2 (m_p + m_1) + m_p m_1 r_{p3}^2] r_{p1} \omega_2 - m_p^2 r_{p1} r_{p3} X_1 [J_2 (m_p + m_1) r_{p1}^2 + m_p m_1 r_{p3}^2] X_3 - m_p^2 r_{p1} r_{p3} X_3 - m_p^2 m_1 r_{p1} r_{p3} u_1 \} \quad \dots (8)$$

$$\dot{r}_{p1} = \dot{r}_{p1} \quad \dots (9)$$

$$\dot{r}_{p1} = u_1 \quad \dots (10)$$

$$\dot{m}_b = u_b \quad \dots (11)$$

Where,

$$a = J_2 (m_p + m_1)(m_p + m_3) + m_p m_3 (m_p + m_1) r_{p1}^2 + m_p m_1 (m_p + m_3) r_{p3} \quad \dots (12)$$

$$X_1 = -m_3 v_3 \omega_2 - P_{p3} \omega_2 - m_{em} g \sin \theta + L \sin \alpha - D \cos \alpha \quad \dots (13)$$

$$X_3 = m_1 v_1 \omega_2 + P_{p1} \omega_2 + m_{em} g \cos \theta - L \cos \alpha - D \sin \alpha \quad \dots (14)$$

$$Y = (m_{f3} - m_{f1}) v_1 v_3 - [r_{p1} P_{p1} + r_{p3} m_p (v_3 - r_{p1} \omega_2) \omega_2] - m_p g (r_{p1} \cos \theta + r_{p3} \sin \theta) + M_{DL2} \quad \dots (15)$$

$$P_{p1} = m_p (v_3 - r_{p3} \omega_2) \quad \dots (16)$$

$$P_{p3} = m_p (v_3 - r_{p1} \omega_2) \quad \dots (17)$$

Where, m_{em} is the net buoyancy, m_{f1} , and m_{f3} denote the added masses, D , L , and M_{DL2} denote the drag, lift and viscous moment of the hydrodynamic force and moment as defined in Graver² as

$$m_{em} = m_h + m_p + m_b - m_{df} \quad \dots (18)$$

$$L = (K_{LO} + K_L \alpha)(v_1^2 + v_3^2) \quad \dots (19)$$

$$D = (K_{DO} + K_D \alpha^2)(v_1^2 + v_3^2) \quad \dots (20)$$

$$M_{DL2} = (K_{MO} + K_M \alpha)(v_1^2 + v_3^2) + K_{\omega_2^1} \omega_2 + K_{\omega_2^2} \omega_2^2 \quad \dots (21)$$

Where, m_h , m_p , and m_{df} are the mass of the hull, internal movable mass, and displaced fluid. K_L , K_{LO} , K_D , K_{DO} , K_M , and K_{MO} are the hydrodynamic lift, drag and pitching moment coefficients. $K_{\omega_2^1}$, and $K_{\omega_2^2}$ are the linear and quadratic damping constant coefficients.

From Eqs. 5 – 11, it can be observed that the system is under-actuated system with two inputs and six outputs. The state and input vectors are written in Eqs. 22 and 23, respectively.

$$x = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7]^T = [\theta \ \omega_2 \ v_1 \ v_3 \ r_{p1} \ \dot{r}_{p1} \ m_b]^T \quad \dots (22)$$

$$u = [u_1 \ u_b]^T \quad \dots (23)$$

However, in this study only two parameters are considered that are pitching angle, θ and net buoyancy, m_{em} as written in Eqs. 24 and 25. The net buoyancy is obtained through the ballast mass, m_b .

$$y_1 = x_1 = \theta \quad \dots (24)$$

$$y_2 = m_{em} = m_h + m_p + m_b - m_{df} \quad \dots (25)$$

Controller design

This section discusses the design methodology of the controller. The proposed controller is a Variable Gain Super Twisting SMC (VGSTW). Therefore, here two controllers will be designed that are the proposed controller and super twisting Sliding Mode Control (STW).

Before designing the controller, the motion equations in Eqs. 5 – 11 are rewritten in the general form of nonlinear equation as given in Eq. 26.

$$\dot{x}_k = f_k(x, t) + g_k(x, t)u_i + g_k \delta_k(x, t) \quad \dots (26)$$

Where, $x \in R^n$, and $u \in R^m$ are defined as state, and input vectors, $\delta_k(x, t)$ represents the bounded matched perturbations, $k = 1, 2, \dots, n$ and $i = 1, 2, \dots, m$

$\delta_k(x, t)$ is bounded with a known norm upper bound,
 $|\delta_k(x, t)| \leq |\rho_k(x, t)|$... (27)

Where, $\rho_k \geq 0$ for $k = 1, 2, \dots, n$.

Eq. 8 and 11 are written in the form of Eq. 26 as given in Eqs. 28 – 30.

$$\dot{x}_1 = x_2 \quad \dots (28)$$

$$\dot{x}_2 = f_1(x, t) - g_1(x, t)u_1 - g_1(x, t)\delta_1(x, t) \quad \dots (29)$$

$$\dot{x}_7 = u_2 + \delta_2(x, t) \quad \dots (30)$$

Where, δ_1 and δ_2 are the external disturbances. The controllers are designed for the tracking problems. The errors of the outputs are defined in Eqs. 31 and 32.

$$e_1 = x_1 - x_{1d} \quad \dots (31)$$

$$e_2 = x_7 - x_{7d} \quad \dots (32)$$

A. Super Twisting sliding mode control (STW)

Super twisting SMC (STW) is categorised as second order SMC which was introduced in 1993 by Levant²⁸. STW is an alternative to the conventional first order SMC for the systems with relative degree one which ensures finite time stability^{29,30}.

Super twisting algorithm also known as model free SMC because its control law consists only the discontinuous control part which free from the system parameters. However, in this study the STW the control consists of equivalent control that derived using conventional SMC and the discontinuous control law is designed based on super-twisting algorithm. Therefore, the control law for tracking the pitching angle and the net buoyancy is written in Eqs. 33 and 34, respectively.

$$u_{1stw} = u_{1stw_eq} + u_{1stw_dis} \quad \dots (33)$$

$$u_{2stw} = u_{2stw_eq} + u_{2stw_dis} \quad \dots (34)$$

The sliding surfaces and their derivatives are defined in Eqs. 35, 36, 37 and 38 for pitching angle and net buoyancy, respectively.

$$s_{1stw} = c_1 e_1 + \dot{e}_1 \quad \dots (35)$$

$$s_{2stw} = e_2 \quad \dots (36)$$

and

$$\dot{s}_{1stw} = c_1 \dot{e}_1 + \ddot{e}_1 \quad \dots (37)$$

$$\dot{s}_{2stw} = \dot{e}_2 \quad \dots (38)$$

The equivalent control laws are defined as $\dot{s}_1 = 0$ and $\dot{s}_2 = 0$

$$u_{1stw_eq} = \frac{1}{g_1} (f_1 - g_1 \delta_1 + c_1 \dot{e}_1 - \ddot{x}_{1d}) \quad \dots (39)$$

$$u_{2stw_eq} = -\delta_2(x, t) \quad \dots (40)$$

The reachability conditions are chosen as super-twisting SMC given as in Eqs. 41 and 42.

$$u_{1stw_dis} = -\beta_{11}|s_1|^p \text{sign}(s_1) - \beta_{12} \int_0^t \text{sign}(s_1) dt \quad \dots (41)$$

$$u_{2stw_dis} = -\beta_{21}|s_2|^p \text{sign}(s_2) - \beta_{22} \int_0^t \text{sign}(s_2) dt \quad \dots (42)$$

Finally, the control law pitching angle and net buoyancy are written in Eqs. 43 and 44.

$$u_1 = u_{1stw_eq} + u_{1stw_dis} = \frac{1}{g_1} (f_1 - g_1 \delta_1 + c_1 \dot{e}_1 - \ddot{x}_{1d}) - \beta_{11}|s_1|^p \text{sign}(s_1) - \beta_{12} \int_0^t \text{sign}(s_1) dt \quad \dots (43)$$

$$u_2 = u_{2stw_eq} + u_{2stw_dis} = -\delta_2(x, t) - \beta_{21}|s_2|^p \text{sign}(s_2) - \beta_{22} \int_0^t \text{sign}(s_2) dt \quad \dots (44)$$

Where, c_i is positive constant and β_{11} , β_{12} , β_{21} and β_{22} are super twisting controller parameters which are positive constants.

B. Variable Gain Super Twisting SMC (VGSTW)

Variable Gain Super Twisting (VGSTW) is an algorithm proposed by Dávila *et al.*³¹. This algorithm is proposed to overcome the shortcomings of original super twisting algorithm which allows to deal with perturbations growing linearly in s . The proposed control law of VGSTW is defined is written in Eqs. 45 and 46.

$$u_{1vgstw} = u_{1vgstw_eq} + u_{1vgstw_dis} \quad \dots (45)$$

$$u_{2vgstw} = u_{2vgstw_eq} + u_{2vgstw_dis} \quad \dots (46)$$

In this study, the sliding surface is designed based on original STW as written in Eqs. 35 and 36 which results in same equivalent controls as written in Eqs. 39 and 40. The discontinuous controls (u_{1vgstw_dis} , u_{2vgstw_dis}) are defined as proposed in Dávila³¹. The discontinuous control laws for VGSTW are written in Eqs. 47 and 48.

$$u_{1vgstw_dis} = -\gamma_{11}(t, x)\varphi_{11}(s_1) - \int_0^t \gamma_{12}(t, x)\varphi_{12}(s_1) dt \quad \dots (47)$$

$$u_{2vgstw_dis} = -\gamma_{21}(t, x)\varphi_{21}(s_2) - \int_0^t \gamma_{22}(t, x)\varphi_{22}(s_2) dt \quad \dots (48)$$

Where,

$$\begin{aligned} \varphi_{11}(s_1) &= |s_1|^{1/2} \text{sign}(s_1) + \gamma_{13}s_1; \\ \varphi_{12}(s_1) &= \frac{1}{2} \text{sign}(s_1) + \frac{3}{2} \gamma_{13}|s_1|^{1/2} \text{sign}(s_1) + \gamma_{13}^2 s_1; \\ \varphi_{21}(s_2) &= |s_2|^{1/2} \text{sign}(s_2) + \gamma_{23}s_2; \\ \varphi_{22}(s_2) &= \frac{1}{2} \text{sign}(s_2) + \frac{3}{2} \gamma_{23}|s_2|^{1/2} \text{sign}(s_2) + \gamma_{23}^2 s_2 \end{aligned}$$

Finally, the VGSTW control laws are defined in Eqs. 51 and 52.

$$u_{1vgstw} = \frac{1}{g_1}(f_1 - g_1\delta_1 + c_1\dot{e}_1 - \ddot{x}_{1d}) - \gamma_{11}(t, x)\varphi_{11}(s_1) - \int_0^t \gamma_{12}(t, x)\varphi_{12}(s_1) dt \quad \dots (49)$$

$$u_{2vgstw} = -\delta_5(x, t) - \gamma_{21}(t, x)\varphi_{21}(s_2) - \int_0^t \gamma_{22}(t, x)\varphi_{22}(s_2) dt \quad \dots (50)$$

Where, $\gamma_{11}, \gamma_{12}, \gamma_{13}, \gamma_{21}, \gamma_{22}$ and γ_{23} are variable gain super twisting controller parameters which are positive constants.

C. Stability analysis

This section discusses the stability analysis of the proposed controller algorithm VGSTW. The stability analysis is important to ensure the convergence of the controlled parameters are stabilized at the desired value via ensuring the sliding mode using lyapunov stability theorem as explained in the following.

Theorem 1: Consider the nonlinear system in Eqs. 29 to 30 subjected to bounded uncertainty in equation 27 with assumptions, the system is proper ($m = p$) and minimum phase where the zero dynamic of the system is asymptotically stable. If the sliding manifolds (s_i) as written in Eqs. 35 and 36 the discontinuous controls (u_{idis}) as written in Eqs. 47 and 48 then the convergence conditions are satisfied.

Proof: Let consider the lyapunov functions, and their time derivatives in Eqs. 53, 54, 55 and 56, respectively.

$$V_1(s_1) = \frac{1}{2} s_1^2 \quad \dots (53)$$

$$V_2(s_2) = \frac{1}{2} s_2^2 \quad \dots (54)$$

$$\dot{V}_1(s_1) = s_1 \dot{s}_1 \quad \dots (55)$$

$$\dot{V}_2(s_2) = s_2 \dot{s}_2 \quad \dots (56)$$

Substitute the Eqs. 37 and 38 into Eqs. 55 and 56, obtain Eqs. 57 and 58.

$$\dot{V}_1(s_1) = s_1\{c_1\dot{e}_1 + f_1(x, t) - g_1u_{1vgstw}(x, t) - g_1\delta_1(x, t)\} \quad \dots (57)$$

$$\dot{V}_2(s_2) = s_2\{u_{2vgstw} + \delta_5(x, t)\} \quad \dots (58)$$

Now substitute the equation 49 into equation 57, and equation 50 into equation 58 which give

$$\dot{V}_1(s_1) = s_1\{-\gamma_{11}(t, x)\varphi_{11}(s_1) - \int_0^t \gamma_{12}(t, x)\varphi_{12}(s_1) dt\} \quad \dots (59)$$

$$\dot{V}_2(s_2) = s_2\{-\gamma_{21}(t, x)\varphi_{21}(s_2) - \int_0^t \gamma_{22}(t, x)\varphi_{22}(s_2) dt\} \quad \dots (60)$$

The following sufficient conditions for finite time convergence must be satisfied³²⁻³³.

$$\gamma_{i1}^2 \geq \frac{4C_0K_M(\beta_{i2} + C_0)}{K_m^2K_m(\beta_{i2} - C_0)}$$

$$\gamma_{i2} > \frac{C_0}{K_m}$$

$$0 < p \leq 0.5$$

$\gamma_{i3} > 0$ allows to deal with perturbations growing linearly in s , i.e. outside of the sliding surface, and the variable gains γ_{i1} and γ_{i2} makes it possible to render the sliding surface insensitive to perturbations growing with bounds given by known functions³¹.

Results and Discussion

In this section the performance of the proposed controller VGSTW is compared with the performance of original STW for the nominal system, existence of input matched disturbance and parameter variations. All the system parameters used in this study are adopted from Graver’s work². All the parameters are depicted in Table 2.

The controllers were simulated for the glide angle switching from 25° downward to 25° upward.

Table 2 — Parameter values of the glider

Parameter	Value	Unit
Hull mass, m_h	40	kg
Internal sliding mass, m_p	9	kg
Displaced fluid mass, m_{df}	50	kg
Added mass, m_{f1}, m_{f2}, m_{f3}	50, 60, 70	Kgm ²
Inertia, J_1, J_2, J_3	4, 12, 11	-
Lift coefficient, K_{LO}, K_L	0, 132.5	-
Drag coefficient, K_{DO}, K_D	2.15, 25	-
Moment coefficient, K_{MO}, K_M	0, -100	-
Constant coefficient, $K_{\omega_1^2}, K_{\omega_2^2}$	50, 50	-

The responses for nominal system are shown in Figures 2 – 4.

Both controllers stabilized in the vicinity of the desired values. From Figures 2 – 4, VGSTW provides the faster tracking at about $t = 8$ seconds and STW settling time about 10 seconds for pitching angle and glide angle. Both controllers provide less than 1 second settling time for net buoyancy. However, VGSTW provides smaller steady state error than STW. VGSTW demonstrates better control effort

and better convergence in sliding surface as shown in Figures 3 & 4. The chattering exhibit in control input u_1 . However, the responses are in downward trend and amplitude less than 20 with VGSTW provides the smaller amplitude.

In this study, water current is considered as input matched external disturbance. The input matched external disturbance of $\delta_1(x, t) = 5x_1 \sin(\pi t)$ and $\delta_2(x, t) = 0.1x_7 \sin(\pi t)$ were induced to input u_1 and u_2 , respectively. The

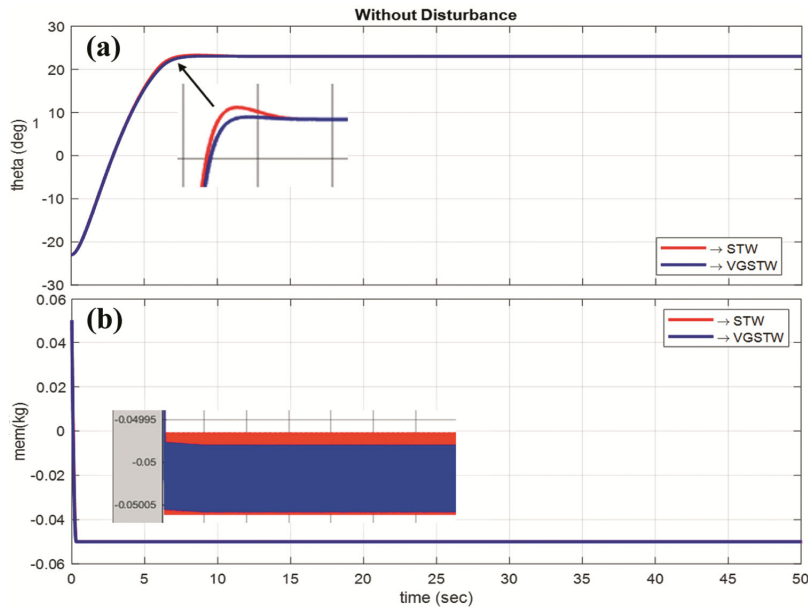


Fig. 2 — (a) Pitching angle, θ , and (b) Net buoyancy, m_{em} (without disturbance)

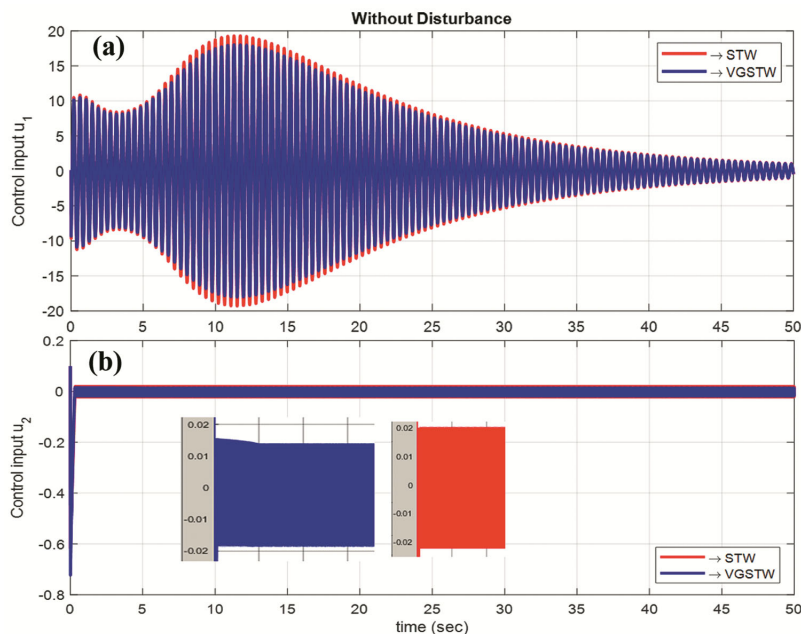


Fig. 3 — (a) Control inputs u_1 , and (b) Control input, u_2 (without disturbance)

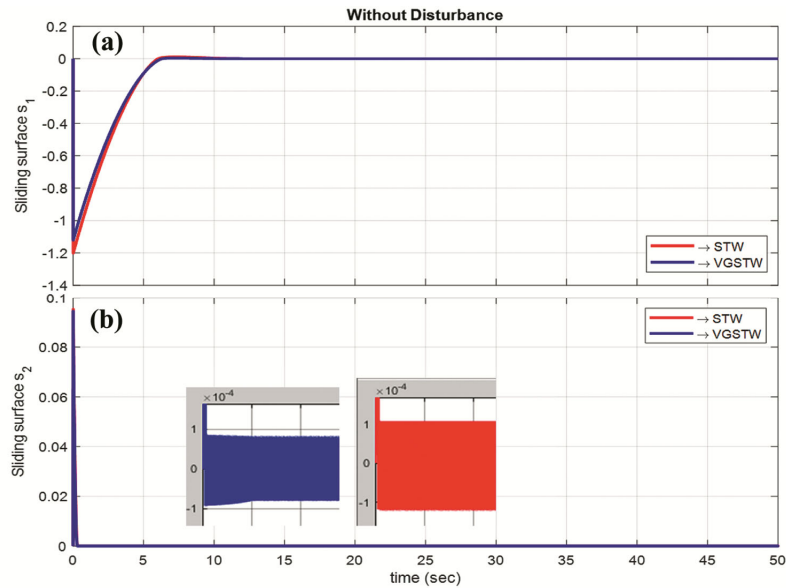


Fig. 4 — (a) Sliding surface s_1 , and (b) Sliding surface s_2 (without disturbance)

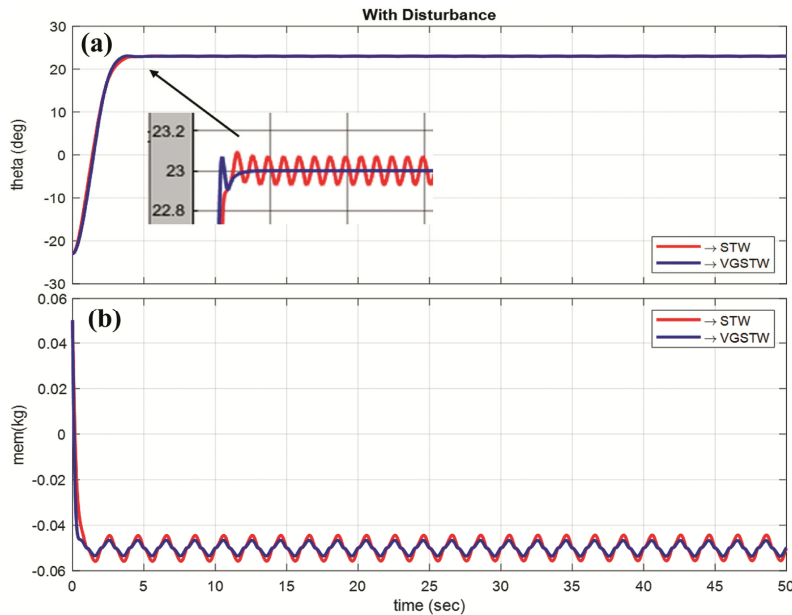


Fig. 5 — (a) Pitching angle, θ , and (b) Net buoyancy, m_{em} (with disturbance)

simulation results for induced disturbance are shown in Figures 5 – 7.

All the controllers are stabilized in the vicinity of the desired values. The VGSTW is able to retain the performance of nominal system for pitching and glide angles whereas STW provides small oscillation in the vicinity of desired angles. In net buoyancy both controllers demonstrate oscillation in vicinity of desired value with VGSTW provides lower amplitude. The control effort u_1 for both controllers reduced more than half of the control in nominal case

with VGSTW shows above half the control of STW. However, both controllers provide almost same control effort of u_2 . The sliding surface s_1 of both controllers are converged in vicinity of origin with VGSTW in range less than $\pm 5 \times 10^{-5}$ whereas STW in range of $\pm 5 \times 10^{-3}$. However, both controllers demonstrate similar performance in sliding surface s_2 where within less than $\pm 1 \times 10^{-2}$ with VGSTW gives slightly lower oscillation than STW. The VGSTW has improved about more than 50 % of performance of nominal case.

The parameter variations are applied to the system time $t = 30$ seconds. The parameters of all hydrodynamic coefficients were increased by 30 % of the original values. The simulation results are shown in Figures 8 & 9.

Both controllers able to retain the performance nominal even with 30 % of increment in

hydrodynamic coefficients. The effect of the increment only can be seen in glide and pitching angles since the dynamic of net buoyancy does not contains hydrodynamic coefficients. In this case VGSTW provides better performance as compared to original STW. The controller gains of the proposed controller for all cases are summarised in Table 3.

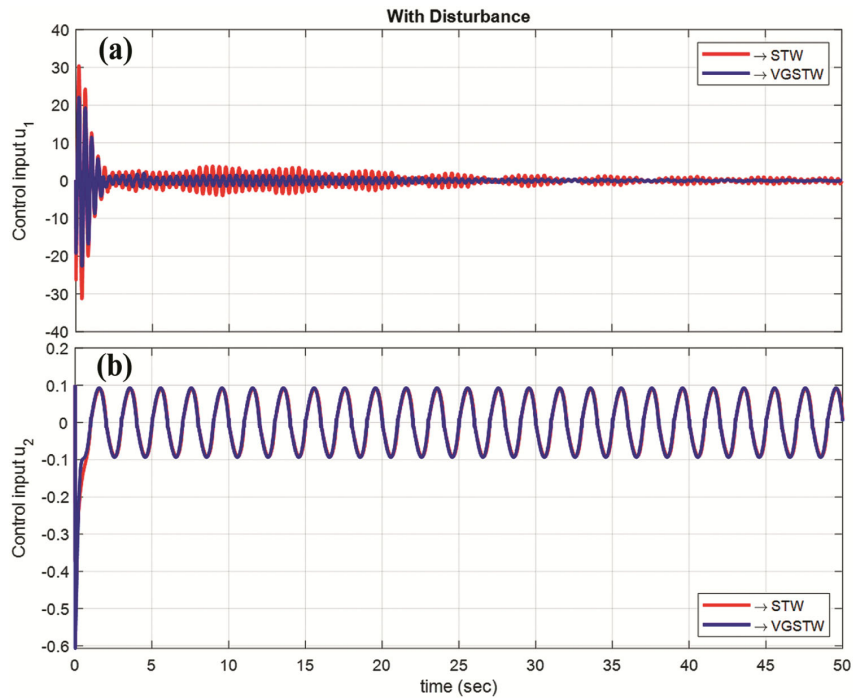


Fig. 6 — (a) Control input, u_1 , and (b) Control input, u_2 (with disturbance)

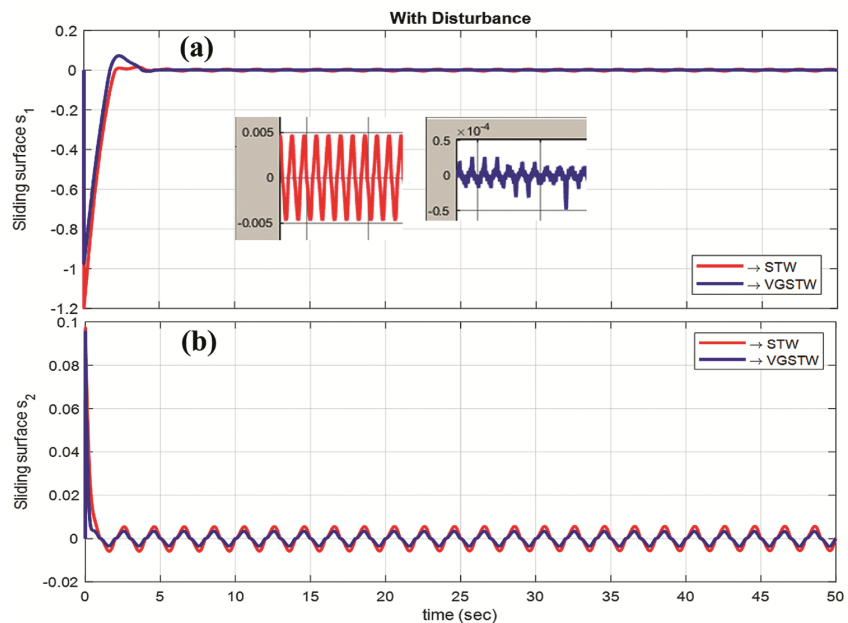


Fig. 7 — (a) Sliding surface, s_1 , and (b) Sliding surface, s_2 (with disturbance)

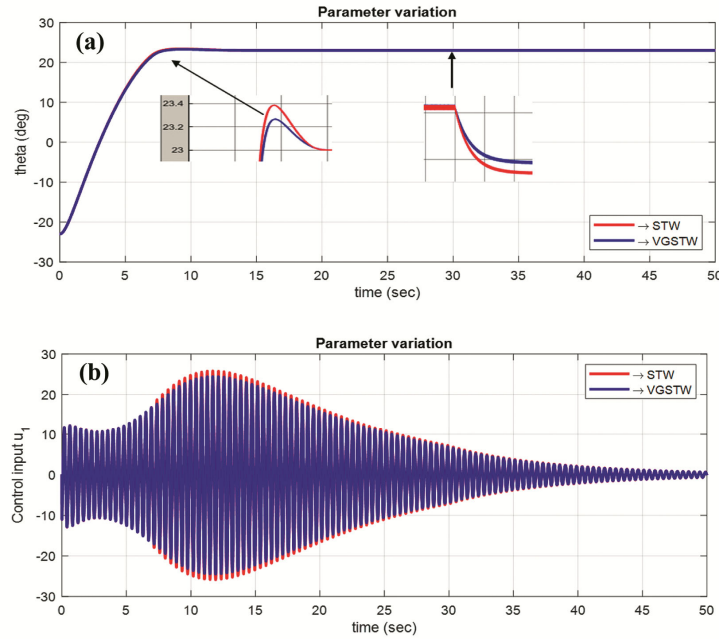


Fig. 8 — (a) Pitching angle, θ , and (b) Control input, u_1 (parameter variation)

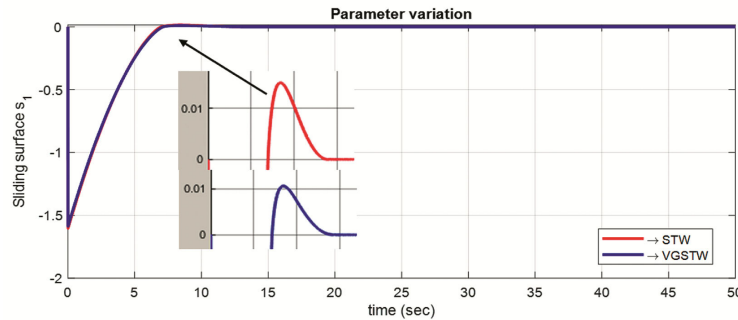


Fig. 9 — Sliding surface, s_1 (parameter variation)

Table 3 — The VGSTW controller gain

Parameter	Gain
	Nominal system
θ	$c_1 = 1.4, \gamma_{11} = 9, \gamma_{12} = 0.2, \gamma_{13} = 0.01$
m_{em}	$\gamma_{21} = 1.8, \gamma_{22} = 0.008, \gamma_{23} = 1$
	With disturbance
θ	$c_1 = 1.22, \gamma_{11} = 20, \gamma_{12} = 12, \gamma_{13} = 0.005$
m_{em}	$\gamma_{21} = 1.5, \gamma_{22} = 0.001, \gamma_{23} = 1$
	With parameter variations
θ	$c_1 = 1.98, \gamma_{11} = 9, \gamma_{12} = 0.32, \gamma_{13} = 0.001$
m_{em}	$\gamma_{21} = 1.8, \gamma_{22} = 0.008, \gamma_{23} = 1$

Conclusion

In this paper variable super twisting sliding mode control (VGSTW) is designed and proposed for robust tracking and robust rejection against disturbance and parameter variations of an autonomous underwater glider. The simulation results have shown that the proposed controller provides a good performance under existence of disturbance and parameter variations and chattering is reduced. Simulation results also shown that

the VGSTW can be further improved by introducing optimization method for tuning parameters of the controller which will directly improve the controller performance. This will become the next research work in future.

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Conflict of Interest

The authors confirm that they are not affiliated with or involved in any organization or institution that has a financial or non-financial interest in the subject matter or materials addressed in this work.

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