Indian Journal of Geo Marine Sciences
Vol. 49 (01), January 2020, pp. 34-43

A new method for detecting line spectrum of ship-radiated noise based on a new double duffing oscillator differential system

Guohui Li, Ke Zhao & Hong Yang
School of Electronic Engineering, Xi'an University of Posts and Telecommunications, Xi'an, Shaanxi 710121, China
[E-mail: lghcd@163.com; uestcyhong@163.com]

Recieved 24 May 2018;revised 06 August 2018

In order to detect line spectrum of ship-radiated noise under the ocean background noise and improve the method of detecting duffing oscillator intermittent chaos, a method of detecting intermittent chaos based on variable step size dual duffing oscillator differential system is proposed. Based on the duffing oscillator, two independent and incompletely coupled duffing oscillators can be differentiated based on the differential principle by using the proposed method, which reduces the computational complexity and makes the timing diagram more intuitive. In order to further improve the detection efficiency and reduce the computational complexity of the system, the author put forward that a sequence of solving steps can be built by using only one duffing oscillator and the method of detecting the unknown frequency signal can be achieved by changing the step size of the system. Simulation results show that compared with the conventional duffing oscillator detection method, the proposed method has improved the SNR (signal-to-noise ratio) by at least 10.6 dB. Comparing with duffing system chaotic oscillator column and double duffing system chaotic oscillator column detection method, the proposed method is most effective in detecting line spectrum of ship-radiated noise.

[**Keywords:** Double duffing oscillator; Intermittent chaos; Ship-radiated noise; Unknown frequency detection]

Introduction

The detection and recognition of underwater acoustic weak signal have always been the hot issues in the field of underwater acoustic, which have important application value in the field of national defense. The main signal of underwater acoustic weak signal detection is the ship-radiated noise. The radiated noise of the ship is the periodic noise that inevitably radiates to the surrounding sea areas due to the vibration of the rotating machinery of the power system on the ship and the rotation of the propeller blades during the actual navigation[1]. In recent years, due to the rapid development of the stealth technology of the ship and the complexity of the background noise of the sea, the difficulty of detecting the underwater acoustic signals is increasing. Conventional weak signal detection method such as matched filtering, coherent accumulation and optimal detection can only detect weak signal with SNR (signal-to-noise ratio) above -10dB, but it can't realize the detection of underwater acoustic weak signal under low SNR[2-4]. With further research on chaos theory, it is found that chaotic oscillators have significant advantages for the detection of weak periodic signal. The detection of extremely weak periodic signal by chaotic oscillators has become a hot topic in the field of weak signal detection[5]. Chaotic oscillators have the characteristics of being sensitive to weak signal of the same frequency and strong immune to noise[6,7], so the known weak signal under strong background noise can be detected by chaotic oscillator[8,9]. Zheng et al.[10] have confirmed that duffing oscillator can detect the weak underwater acoustic signal with known line spectral frequency. Cong et al.[11] have put forward adaptive step size duffing detection system with a small amount of calculation. Shi et al.[12] have proposed the use of more simple and intuitive frequency-phase trajectory diameter map for detection of underwater target radiated noise line spectrum. However, the signal frequency to be measured is mostly unknown in actual engineering application. Therefore, the detection of weak signal with unknown frequency is of top priority at present. Chen et al.[13] have proposed intermittent chaos oscillator column scanning method to detect unknown frequency signal. However, this method has some disadvantages: (1) Limited detection frequency range: if $|\Delta w / w| \le 0.03$, it can be detected, where Δw is the frequency difference between the measured signal and the internal motive power, and w is the frequency of internal motive power. Once this range is exceeded, it cannot be effectively detected; (2) A large number of chaotic oscillators: as long as the power frequency of the system is changed, chaotic oscillators must be reset, and each frequency must have a chaotic oscillator corresponding to it; (3) High computational complexity; for each chaotic oscillator, parameters must be reset, and the critical threshold of chaotic oscillator is difficult to calculate, therefore, it leads to high computational complexity. (4) Lack of intuitive and clear sequence diagram: the intermittent chaos sequence diagram is the alternating of chaotic state and periodic state, so the state of the sequence diagram cannot be determined by direct observation.

In view of the above problems, a variable step type double duffing oscillator differential detection system is proposed. Comparing with the conventional duffing system, the detection frequency range of double duffing oscillator differential system is increased, which reduces the amount of computation and makes the timing diagram more intuitive. In order to further improve the detection efficiency and reduce the computational complexity of the system, a double duffing oscillator differential system is improved. A sequence of solving steps can be built by using only one duffing oscillator, and the method of detecting the unknown frequency signal can be achieved by changing the step size of the system.

Materials and Methods

Duffing Oscillator Intermittent Chaos Detection Method

Holmes type duffing equation is as follows[14,15]:

$$\ddot{x}(\mathrm{t})+k\dot{x}(t)-x(t)+x^3(t)=\gamma\cos(wt) \quad \text{...(1)}$$

where $\gamma\cos(wt)$ is the internal motive power, γ is the amplitude of internal motive power, w is the angle frequency of internal motive power, $-x(t)+x^3(t)$ is the nonlinear restoring force, and k is the damping coefficient. Because of the existence of nonlinear terms, the duffing system is rich in nonlinear dynamics and is often used to study chaotic motion. The sensitivity of the system to the initial conditions is one of the characters of the chaotic system, that is, a slight change in the initial conditions of the system can also cause significant changes in the system state[16]. As a result of the sensitivity of duffing system to system parameter γ, when the system damping coefficient k is fixed, with the increase of internal motive power γ, the system will in turn experience attractor, homoclinic orbit, period doubling bifurcation, chaos, critical chaos and

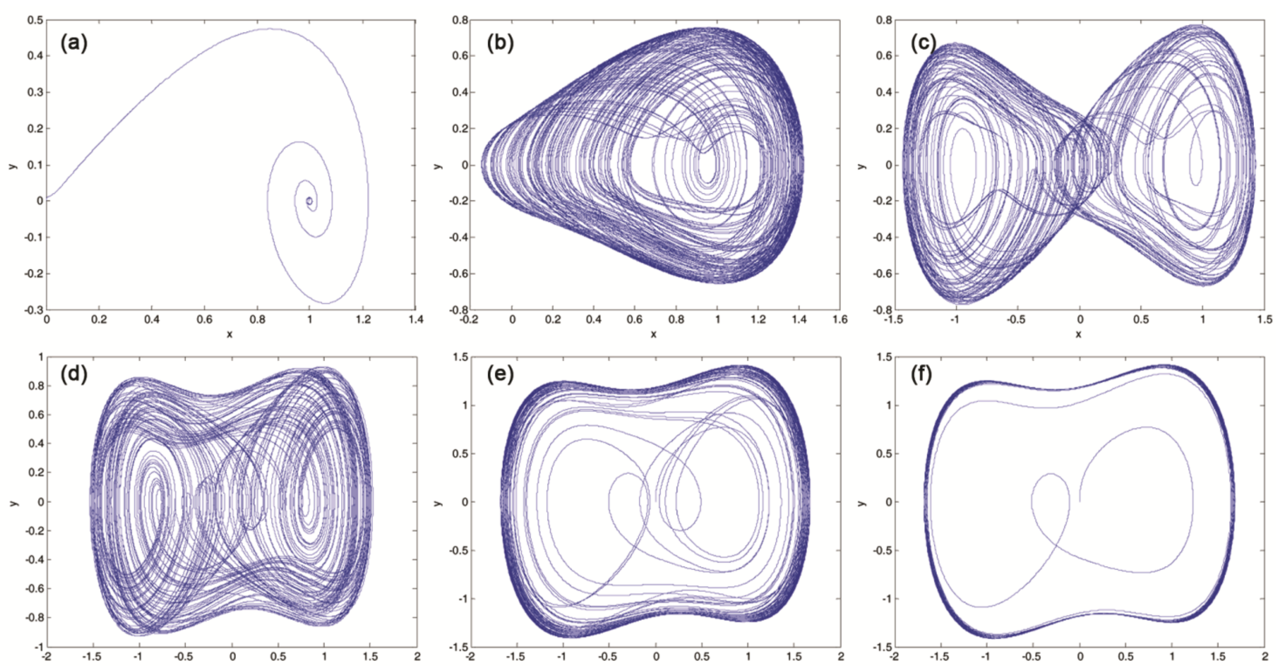


Fig. 1 — Duffing system phase diagram: (a) attractor, (b) homoclinic orbit, (c) period doubling bifurcation, (d) chaos, (e) critical chaos and (f) large-scale periodic

large-scale periodic state[17,18], as shown in Fig. 1. Let γ_d be the threshold for the transition from the critical chaos state to the chaotic state, and when $\gamma > \gamma_d$, the system enters the large-scale periodic state.

Using duffing oscillator to detect weak periodic signal, a detection method of intermittent chaotic oscillator column was proposed in the reference[13]. When the system changes from chaos state to large-scale periodic state, it is called the critical chaos state. The timing diagram of chaotic state and large-scale periodic state are shown respectively in Fig. 2 (a) and Fig. 2 (b). Duffing system is immune to noise[19,20]. Intermittent chaos can realized the weak periodic signal detection by using the immunity to noise and the sensitivity to small perturbation in the critical chaos state. In formula (2), adding the test signal $s(t)$, the system state equation can be described as:

$$\begin{cases} \dot{x} = wy \\ \dot{y} = w\left(-ky + x - x^3 + \gamma_d \cos(wt) + s(t)\right) \end{cases} \quad \ldots(2)$$

where, $s(t) = f\cos(w't)$, $w' = w + \Delta w$, Δw is the frequency difference between the measured signal and the internal motive power, φ is the initial phase of the signal under test, f is the amplitude of the signal under test. At this point, the total motive power of the system is as follows:

$$\begin{aligned} S(t) &= \gamma_d \cos(wt) + f\cos(w't + \varphi) \\ &= \gamma'(t)\cos(wt + \theta(t)) \end{aligned} \quad \ldots(3)$$

where, $\gamma'(t) = \sqrt{\gamma_d^2 + 2\gamma_d f \cos(\Delta wt + \varphi) + f^2}$. When $|\Delta w / w| \le 0.03$, the system state is intermittent chaos[21], that is, the chaotic state and the periodic state are regularly alternated, as shown in Fig. 2(c). It is proposed in the reference[13] that the chaos oscillator column is constructed with 1.03 as the common ratio, the unknown frequency signal can be detected by frequency sweep method, the frequency detection range of each vibrator is $\pm 0.03w$ and the frequency range of the signal under test can be determined when intermittent chaos occurs between two adjacent oscillators. Although this method can solve the problem that the signal frequency to be measured is unknown, the corresponding parameters need to be set for the duffing oscillator at each frequency point in the detection process. When the frequency range of the signal to be measured is large, the method increases the complexity of the system detection. Therefore, the existing method is improved in this paper.

A New Intermittent Chaotic Detection Method for Differential System with Double Duffing Oscillator

Because of the long critical chaos process and other errors in the duffing system, the critical state may be regarded as a chaotic state in some cases. Therefore, the Holmes-type duffing equation is revised to improve the shortcoming. Differentiation of the equation (1) is as follows:

$$x^{(3)}(t) + k\ddot{x}(t) - \dot{x}(t) + 3x^2\dot{x}(t) = -\gamma \sin(wt) \quad \ldots(4)$$

Two sides of formula (4) divided by two sides of formula (1), and a third-order system can be got:

$$\begin{aligned} &x^{(3)}(t) + [k + \tan(wt)]\ddot{x}(t) + [3x^2 - 1 + \\ &k\tan(wt)]\dot{x}(t) + \tan(wt)(x^3 - x) = 0 \end{aligned} \quad \ldots(5)$$

The initial phase point of the third-order system is $(x(0), \dot{x}(0), \ddot{x}(0))$. When $t = 0$, formula (1) is:

$$\ddot{x}(0) = -k\dot{x}(0) + x(0) - x^3(0) + \gamma \quad \ldots(6)$$

It can be obtained by formula (6) that the magnitude γ of the internal dynamic force in the

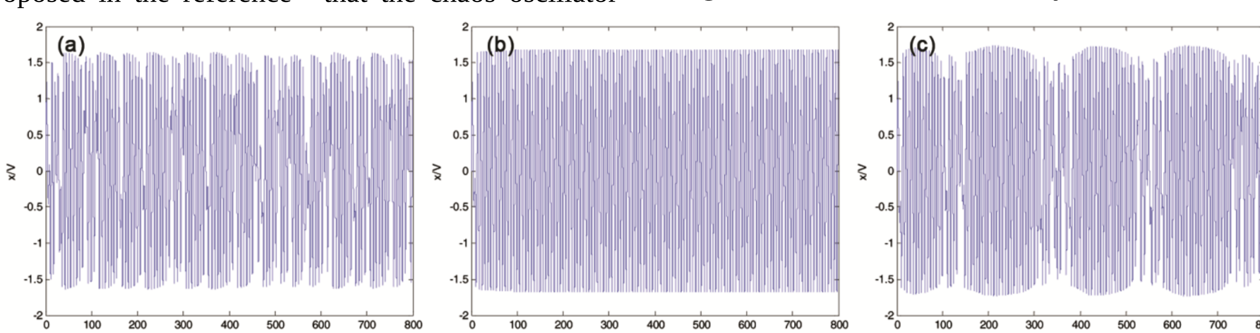


Fig. 2 — Sequence diagrams of the duffing system in different states (a) chaos, (b) large-scale periodic state and (c) intermittent chaos

system exists in the initial phase point of the third-order system, small change of the value γ in the chaotic state will cause significant difference of the system state, but it has no effect on the periodic state. The two oscillators with different amplitude of internal dynamic force can be used to differentiate in order to highlight the chaos state, suppress the periodic state, and make the intermittent chaos phenomenon more obvious. The construction of double duffing oscillator difference equation is:

$$\begin{cases} \ddot{x}_1(\mathrm{t})+k\dot{x}_1(t)-x_1(t)+x_1^{\,3}(t)=\gamma\cos(wt) \\ \ddot{x}_2(\mathrm{t})+k\dot{x}_2(t)-x_2(t)+x_2^{\,3}(t)=\alpha\gamma\cos(wt) \end{cases} \quad ...(7)$$

where $\alpha \neq 0$, $k = 0.5$, $\gamma=\gamma_d = 0.826$. In addition to different amplitude of the system internal dynamic force, the remaining parameters are the same. Two oscillators are independent of each other, not fully coupled, and take the difference between x_1 and x_2 in the output. When $\alpha = 1$, the output of x_1 and x_2 is the same. When $\alpha \neq 1$, the output of x_1 and x_2 has phase difference, and the value of α directly affects the result of the difference. In this paper, α is 1.001. Adding the signal $f\cos((w+\Delta w)t+\varphi)$ to be tested to formula (7), formula (7) is as follows:

$$\begin{cases} \ddot{x}_1(\mathrm{t})+k\dot{x}_1(t)-x_1(t)+x_1^{\,3}(t)=\gamma\cos(wt)+f\cos(w't+\varphi) \\ \ddot{x}_2(\mathrm{t})+k\dot{x}_2(t)-x_2(t)+x_2^{\,3}(t)=\alpha\gamma\cos(wt)+f\cos(w't+\varphi) \end{cases} \quad ...(8)$$

When $\gamma = \gamma_d = 0.826$, oscillator 1 is a critical chaotic state, and oscillator 2 is a large-scale periodic state. When the signal to be tested is not entered, it is a critical chaos state shown in Fig. 3 (a). When $\Delta w = 0$, it is a large-scale periodic state shown in Fig. 3 (b). When $\left|\frac{w'}{w}\right| \leq 1.06$ [22], it is an intermittent chaos state shown in Fig. 3 (c). In summary, the system output difference sequence diagram has three cases. 1) If the timing diagram is the regular waveform diagram whose amplitude is small, then it is a large-scale periodic state. 2) If the straight line and the irregular waveform regularity alternately appear, then it is an intermittent chaos state. 3) Otherwise, it is a chaotic state. By comparing Fig. 2(c) with Fig. 3(c), it can be seen that the intermittent chaotic state of Fig. 3(c) is more intuitive and clear.

Principle of Variable Step Size Intermittent Chaos Detection

The detection signal by using the dual duffing oscillator differential system is still using the intermittent chaos detection principle which has some characteristics such as replacing the single oscillator with two oscillators, suppressing the common mode interference while highlighting the chaotic state, keeping the immunity of the intermittent chaos detection to the Gaussian noise, and that the system detection accuracy is not affected by the initial phase of the signal under test. Hu et al.[22], Zhao et al.[23], Wang et al.[24] have used the intermittent chaotic oscillator column method to detect the signal based on the double duffing oscillator differential system. Compared to the conventional chaotic oscillator column detection method, the range of the ratio of chaotic oscillator column is increased from (1.01, 1.03) to (1.06, 1.08), and the detection range of a single oscillator is increased from $0.01 \leq |\Delta w| \leq 0.03$ to $0.06 \leq |\Delta w| \leq 0.08$. It reduces the number of chaotic oscillator column and improves system detection efficiency. However, it is still necessary to adjust the parameter for each oscillator to find the internal dynamic threshold that makes the system in the critical chaos state. The internal

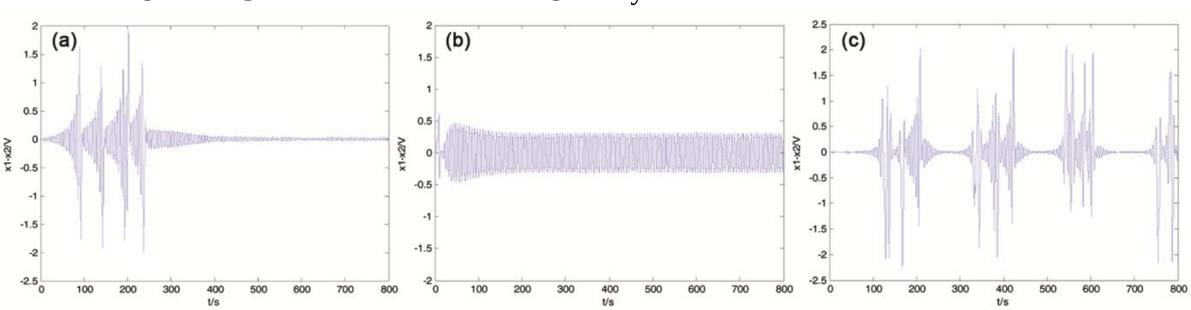


Fig. 3 — Various states of the double duffing oscillator differential system (a) chaos, (b) large-scale periodic state and (c) intermittent chaos

dynamic threshold of the system can only be approximated by simulation continuously, and the exact value can not be directly calculated[25]. These increase the difficulty to implement this method. So we propose a variable step size detection method based on double duffing differential oscillator in this paper.

When using double duffing oscillator to detect the signal, it can judge whether the signal to be measured is detected by observing the differential timing diagram of the system output. Zhang et al.[26] pointed out that the solution of duffing oscillator equation is very complicated and can only be solved by numerical fraction method. In this paper, the fourth order Runge-Kutta method is used to solve the duffing oscillator. In the solving process, the system motivation terms (internal motivation and signal under test) are transformed into a set of discrete sequences by setting the solving step. The sequence interval of internal motivation terms is the system solution step. The signal sequence interval to be tested is T_s ($T_s = 1/f_s$; f_s is the sampling frequency). Since the interval of signal sequence to be measured is independent of the solution step, intermittent chaotic phenomena are caused by changing the solution step in this paper. Let $\gamma = \gamma_d$, the two oscillator terms of motion are respectively $\gamma_d \cos(wt)$ and $\alpha\gamma_d \cos(wt)$ in formula (7), the signal to be tested $f \cos(w't)$ is joined, $w' = w + \Delta w$ is the frequency of the signal to be measured, the system solution step is $h = \frac{w'}{lwf_s}$, where $l \in (0.94, 1.06)$, f_s is the sampling frequency of the signal to be measured. In the system, the driving forces of the two oscillators are as follows:

$$s_{n1} = \gamma_d \cos\left(\frac{nw'}{lf_s}\right) + f \cos\left(\frac{nw'}{f_s}\right) \qquad ...(9)$$

$$s_{n2} = \alpha\gamma_d \cos\left(\frac{nw'}{lf_s}\right) + f \cos\left(\frac{nw'}{f_s}\right) \qquad ...(10)$$

where $n = 1, ..., N$. At this point the system output differential timing diagram is intermittent chaos.

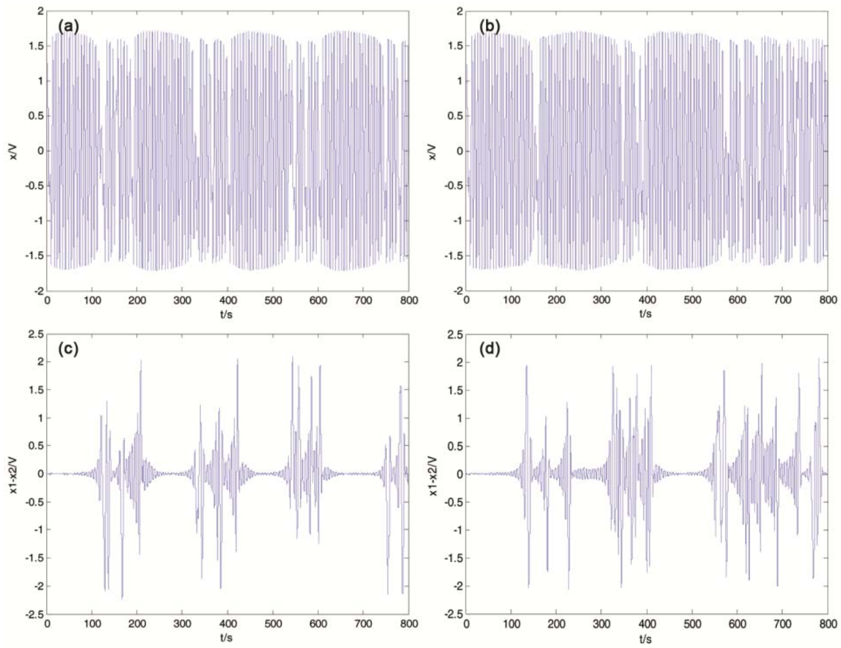
In order to verify the feasibility of the method, adjust $\gamma = 0.826$ to make the system in the critical chaos state. The sampling frequency of 1KHz sine signal $a\cos(10t) + 0.03randn(t)$ as the signal to be tested were added to formula (7) and formula (1) for detection. System solution step is $\frac{w'}{lwf_s} = \frac{10}{1.06 \times 1 \times 1000} = \frac{1}{106}$. The output timing diagrams of two systems are shown in Fig. 4. When a decreases from 0.017 to 0.015, the output timing diagram of duffing oscillator detection system is no longer a regular intermittent chaos shown in Fig. 4 (a) & (b). At this point, the minimum detection threshold of the system is -40.75 dB, and SNR is -18.29 dB. When $a = 0.01$, the output of the double duffing oscillator differential system can still see the regular intermittent chaos until the intermittent chaos disappears at $a = 0.009$, as shown in Fig. 4 (c) & (d). The minimum detection threshold of the system is -40.75 dB, and SNR is -28.90 dB. In summary, it is proved that the signal can be detected by setting a set of solving step sequence by fixing internal dynamic frequency of the system. In this paper, the system's internal dynamic frequency is 1rad/s. Comparing with the conventional duffing oscillator system, this method increases the detection bandwidth of duffing oscillator and reduces the complexity of system detection. The detection performance of the system is improved by 10.6 dB, and the output timing diagram of the system is clearer. SNR is defined as follows:

$$SNR = 10\log\frac{P_s}{P_n} = 10\log\frac{a^2}{2\delta^2} \qquad ...(11)$$

Results and Discussion

Analog Signal Simulation

In order to verify the detection effect of the variable step double duffing oscillator differential system on the unknown frequency signal, the sinusoidal signals $0.03\cos(10t)$ and $0.03\cos(20t)$ with the sampling frequency of 1000Hz are substituted into the formula (7). The step sequence of the system is constructed with 1.06 as the common ratio. Experiments show that $0.03\cos(10t)$ produces intermittent chaos between a_{39} and a_{40} , and $0.03\cos(20t)$ produces intermittent chaos between a_{52} and a_{53} . System differential timing diagrams are shown in Fig. 5 and Fig. 6. Signal frequencies to be measured of corresponding steps are: 9.7035, 10.9092, 19.5254 and 20.6969. The feasibility of the method is proved. Based on the double duffing oscillator differential system and the duffing oscillator



(a) $a = 0.017$, (b) $a = 0.015$, (c) $a = 0.01$ and (d) $a = 0.009$

Fig. 4 — Comparison between duffing oscillator and double duffing oscillator: (a)(b): duffing oscillator output; (c)(d): double duffing oscillator output

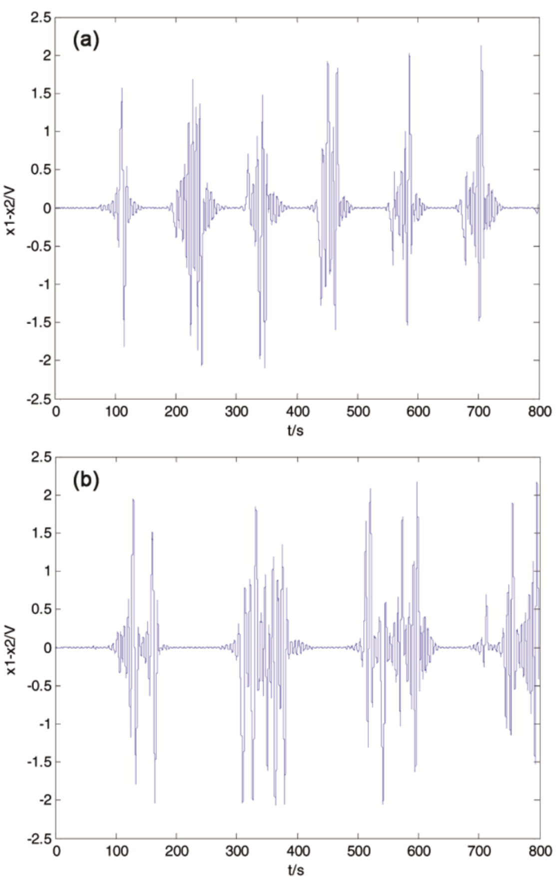


Fig. 5 — 0.03 cos(10t) test results: (a) $w = 9.7035$, (b) $w = 10.9029$

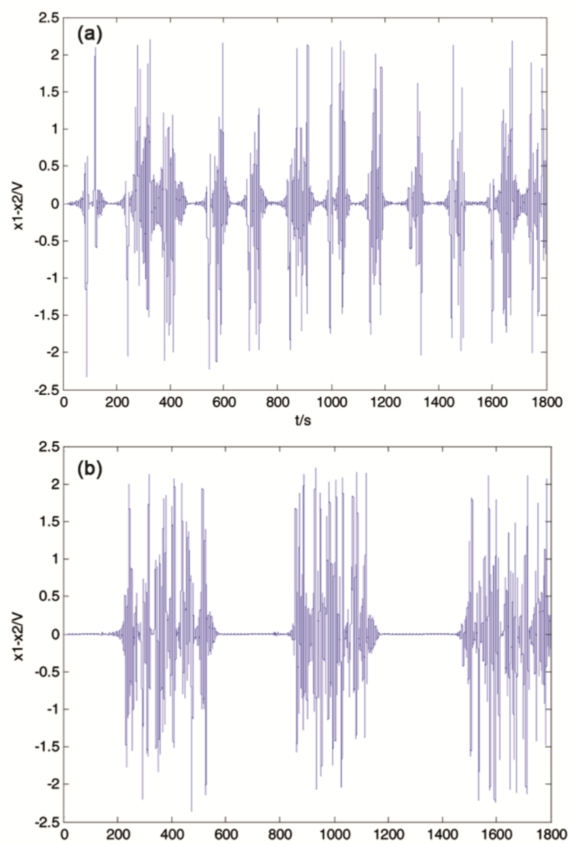


Fig. 6 — 0.03 cos(20t) test results: (a) w = 19.5254 and (b) w = 20.6969

system, the variable step method is used to detect $0.03\cos(20t)$. It can be seen from Table 1 that the number of steps required for double duffing oscillator is much less than that of duffing oscillator, the detection bandwidth is much larger, and the calculation amount is smaller.

Ship-Radiated Noise Signal Simulation

The ship-radiated noise data is measured in the South China Sea and the sampling frequency of sonar collecting signal is 20kHz. In this paper, the sampling frequency of 1 kHz is used to get about 20s of the ship-radiated noise signal in the measurement data as the signal to be measure. Fig. 7 (a) is the timing diagram of the signal under the test. The frequency of the signal under test is unknown. In order to verify the superiority of the variable step size double duffing oscillator differential detection method, the ship-radiated noise signal is detected by using the duffing oscillator chaotic column detection method, the double duffing oscillator differential system chaotic oscillator detection method and the variable step double duffing oscillator differential detection method respectively. The specific steps of the variable step double duffing oscillator differential detection method are as follows:

1) In formula (7), the system internal dynamic frequency w is 1 rad / s, and the amplitude γ is 0.826, which makes the system in the critical chaos state.

2) Divide the frequency band of the line spectrum into a set of geometric sequence w_n with 1.06 as the ratio, and solve the duffing oscillator with a fourth-order Runge-Kutta, and the solve step sequence is $a_n = \frac{2\pi \times 1.06^n}{1.06 \times 1 \times f_s}$, where $f_s = 1000\text{Hz}$.

3) Add the signal to be tested into formula (7), and then adjust the system step-by-step and observe the system output differential timing diagram. If there is intermittent chaos between two adjacent steps, then it is proved that the signal line spectrum under the test exists.

Table 1 — Comparison between duffing oscillator system and double duffing oscillator differential system

method	Index		
	common ratio	the number of solving steps	detection bandwidth
Duffing oscillator system	1.01	302	(0.99,1.01).*w*
	1.02	152	(0.98,1.02).*w*
	1.03	102	(0.97,1.03).*w*
Double duffing oscillator differential system	1.06	52	(0.94,1.06).*w*
	1.07	45	(0.93,1.07).*w*
	1.08	39	(0.92,1.08).*w*

Firstly, the variable step size double duffing oscillator difference method is used to detect the ship-radiated noise signal, and then the solution step length a_n is adjusted one by one. The intermittent chaos phenomenon appears at a_{44} and a_{45}, as shown in Fig. 8. According to $a_n = \frac{2\pi \times 1.06^n}{1.06 \times 1 \times f_s}$, a_{44}=0.0770 and a_{45}=0.0816 can be obtained. The corresponding frequencies are $f_{44} = 12.9855$ and $f_{45} = 13.7646$ respectively. According to the formula (12), there is a line spectrum in 13.375 Hz:

$$f = \frac{f_{44} + f_{45}}{2} \approx 13.375 \qquad \text{...(12)}$$

In order to verify the correctness of this method and draw the spectrum of the ship-radiated noise, as shown in Fig. 7 (b), we can see that there is a clear

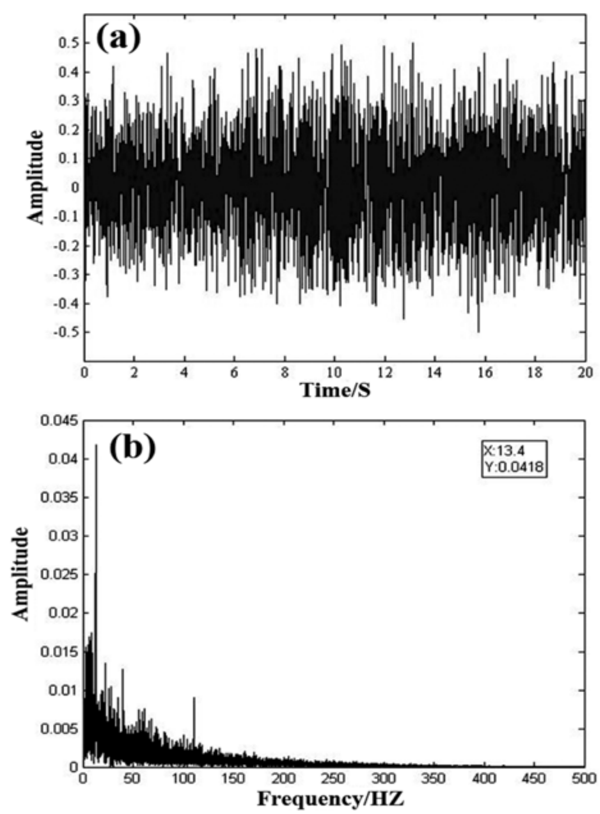


Fig. 7 — Ship-radiated noise signal: (a) timing diagram (b) spectrum diagram

line spectrum near 13.09, which can prove the validity of this method.

Secondly, using the double duffing oscillator differential system chaotic oscillator column detection method to detect, it can be seen from Fig. 9 that no obvious intermittent chaos occurs between the system's internal motive power w_{44} and w_{45}.

Finally, by using the conventional duffing oscillator chaotic oscillator column, we can see that there is no intermittent chaos between the system's internal power w_{44} and w_{45} as can be seen in Fig. 10.

Comparison of the above three methods, it has shown that the variable step double duffing oscillator differential system has the best detection performance. The main reasons are that the method suppresses the periodic state, highlights the chaos state, and greatly increases the energy of the chaotic state after difference. This can be seen by comparing (a)(b) and (c)(d) in Fig. 4. In (c)(d), the periodic states

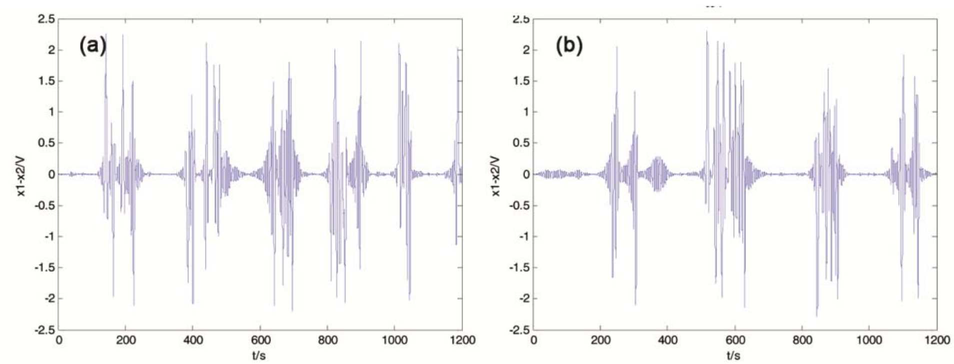


Fig. 8 — Variable step double duffing oscillator differential detection: (a) step size is a_{44}, (b) step size is a_{45}

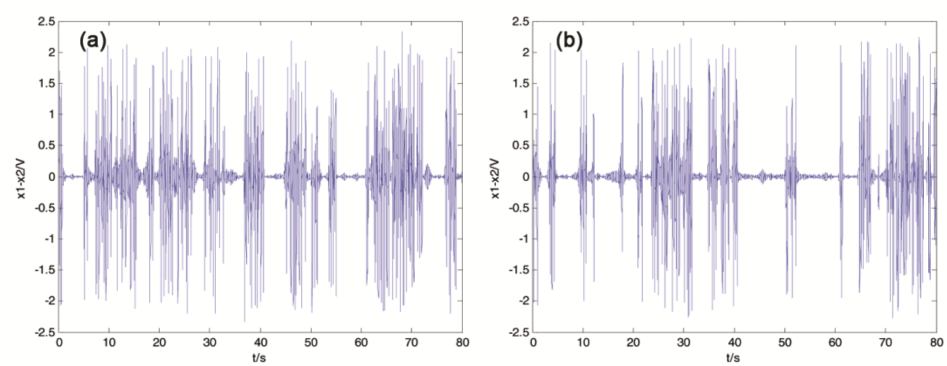


Fig. 9 — Double duffing oscillator chaotic oscillator column detection: (a) internal motivation frequency is w_{44} and (b) internal motivation frequency is w_{45}

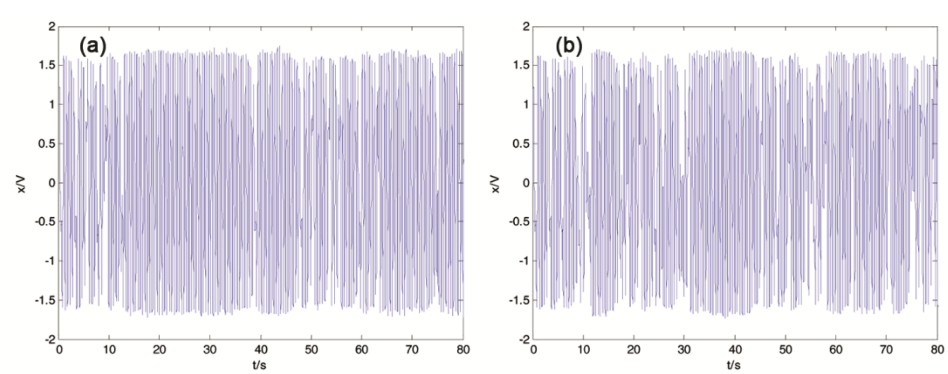


Fig. 10 — Duffing oscillator chaotic oscillator column detection: (a) internal motivation frequency is W_{44} and (b) internal motivation frequency is W_{45}

cancel each other out to approximately zero, and the chaos state still exists. The method of detecting the signal to be measured by changing the step size avoids troubles that the conventional chaotic oscillator column method needs to constantly look for the corresponding parameter of the system, increases the detection bandwidth of the system, reduces the calculation amount of the system, and improves the detection efficiency.

Conclusion

In this paper, a method of detecting line spectrum of ship-radiated noise based on the variable step size double duffing oscillator differential detection system is proposed. The method outputs the system differential timing diagram to detect the unknown signal of line spectrum, and provides a new method for the chaotic oscillator to detect weak targets in water. Comparing with the duffing oscillator chaotic column detection method, we can see that the proposed method can improve the system detection performance by at least 10.6dB. The experimental results show that the proposed method has some advantages such as small number of chaotic oscillator, low computational complexity, intuitive and clear intermittent chaotic time sequence diagram. The characteristics of this paper are as follows:

1) Based on the analysis and improvement of the duffing oscillator detection system, a double duffing oscillator differential detection system is proposed. The output differential timing diagram of the system under different states is analyzed. The proposed method can increase the detection bandwidth of the system and reduce the system calculation.

2) Based on the double duffing oscillator differential detection system, a variable step detection method is proposed, that is, the unknown frequency signal can be detected by only one duffing oscillator. The unknown frequency signal is detected by changing the system solution step, which avoids the shortcoming of the chaotic column detection method that the system parameters need to be adjusted continuously so as to further reduce the system complexity.

3) The feasibility of this method is verified by the detection of analog signals, which can be used to detect the actual unknown frequency of ship-radiated noise.

Conflict of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

Acknowledgment

This work was supported by the National Natural Science Foundation of China (No. 51709228).

References

1 Wu, S. Y. & Li, S., Numerical analysis for induced by vibration of propeller blades. *J. Vibr. Shock*, 33(12) (2014): 207-210.
2 Zhang, G. G., Real-time beam detection technology based on adaptive beamforming. Shipborne *Elect. Engg.*, 37(3) (2017): 122-126.
3 Zhou, K. B., Dou, C. Q. & Chen, T., A comparative study on two methods of weak signal detection. *J. Wuhan Uni. Tech.* (Information and Management Engineering), 29(4) (2007): 53-55.
4 Sun, Y. J. & Lu, F. P., Weak signal detection based on coherent accumulation by wavelet transform. *Electronic Info. Info. Tech.*, 18(5) (2003): 7-10.
5 Haykin, S. & Li, X. B., Detection of signals in chaos. *Proceedings Ins. Elect. Electro. Engg.*, 83(1) (1995): 95-122.
6 Li, Y., Yang, B. J. & Shi, Y. W., Chaos-based weak sinusoidal signal detection approach under colored noise background. *Acta Phys. Sin.*, 52(3) (2003): 526-530.
7 Li, Z. B., Ru, G. G., Li, F. Q., Tao, W., Wu, Z. H. & Xu, Z.Y., Weak sinusoidal signal detecting based on duffing oscillator. *J. Xuzhou Ins. Tech.* (Natural Science Edition), 30(3) (2015): 59-63.
8 Li, Y., Shi, Y. W. & Ma, H. T., Submerged chaos detection of weak square wave signal in colored noise background. *J. Electro*., 32(1) (2004): 87-90.
9 Li, Y., Yang, B. J., Shi, Y. W., Yu, G. M. & Zhang, Z. S., Using chaotic oscillators to detect square wave signals submerged in strong background noise. *J. Jilin Uni. Natur. Sci*., (2) (2001): 65-68.
10 Zheng, S. Y., Guo, H. X., Li, Y. A., Zhang, P. Y. & Wang, B. H., A new method of detecting ship radiation noise profile with duffing oscillator. *Scientific Bull*., 52(3) (2007): 258-263.
11 Cong, C., Li, X. K. & Song, Y., A method of detecting line spectrum of ship-radiated noise using a new intermittent chaotic oscillator. *Acta Phys. Sin*., 2014, 63 (6): 064301(1-12).
12 Shi M. & Xu X., Detection of underwater target radiated noise line-spectra based on chaos system. *Ship Sci. Tech*., 35(5) (2013): 30-33.
13 Chen, Z. G., Li, Y. A. & Chen, X., Underwater acoustic weak signal detection based on Hilbert transform and intermittent chaos. *Acta Phys. Sin.*, 64(20) (2015): 200502(1-8).
14 Fan, J., Zhao, W. L. & Wang, W. Q., Study on the weak sinusoidal signal detection property using duffing chaos system. *Acta Phys. Sin.*, 62(18) (2013): 180502(1-6).
15 Zhang, Y. H., Mao, H. L., Mao, H. Y. & Huang, Z. F., Detection the nonlinear ultrasonic signals based on modified duffing equations. Results in Physics, 7(2017): 3243-3250.

16 Shi, J., Hao, J. P. & Wu, Z. Y., Exact solution for a class of nonlinear vibration duffing equation. *Mech. Sci. Technol. Aerosp. Eng*., 27(6) (2008): 764-765.

17 Lai, Z. H. & Leng, Y. G., Generalized parameter-adjusted stochastic resonance of duffing oscillator and its application to weak-signal detection. *Sensors*, 15(9) (2015): 21327-21349.

18 Zhang, L., Wang, L. F., Ning, X. L. & Chen, M. H., Research and simulation on weak sine signal detection by using duffing oscillator. *Chinese J. Sens. Actua*., 22(8) (2009): 1137-1141.

19 Wu, D. H. & Wu, X., Research on the influence of noise to weak signal detection based on duffing equation. *Applied Mech. Mat*., 65 (2011): 5-8.

20 Bobryk, R. V. & Chrzeszczyk, A., Transitions in a duffing oscillator excited by random noise. *Nonlinear Dyn*., 51(4) (2008): 541-550.

21 Niu, D. Z., Chen, C. X., Chen, T., Ren, X. Y., Wang, Z., Cheng, M. J. C. & Jiang, J., Weak signal detection with duffing oscillator based on short time Fourier transform. *Acta Aeronaut. Astronaut. Sin*., 36(10) (2015): 3418-3429.

22 Hu, Y. Y., Zeng, F. L. & Zhang, S. G., New method of complex-frequency signal detection based on double oscillator difference system. *Com. Engg. App.*, 50(16) (2014): 206-210.

23 Zhao, H., Yin, C. Q., Shang, Q. F. & Geng Y. J., Two oscillator difference chaotic identification method. *Proc. Chin. Soc. Elect. Eng*., 26(23) (2006): 32-35.

24 Wang, M., Yan, Y. & Zhang, H. F., Frequency detection of weak signal based on double differential duffing oscillators. *J. Circuits Sys*., 15(2) (2010): 118-121.

25 Ahmad, B., Alsaedi, A. & Alghamdi, B. S., Analytic approximation of solutions of the forced duffing equation with integral boundary conditions. *Nonlinear Anal. Real World Appl*., 9(4) (2008): 1727-1740.

26 Zhang, S., Rui, G. S., Sun, W. J., Zhang, Y. & Cui, W., Euler-maruyama method for stochastic differential equations of duffing oscillator. *Elect. Meas. Tech*., 34(3) (2011): 37-40.