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Assisted cloning of an arbitrary unknown two-qubit state via a genuine four-qubit entangled state and positive operator-valued measure

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A protocol for cloning an arbitrary unknown two-qubit state and its orthogonal complement state with the assistance of a state preparer is presented. In this paper, a genuine four-particle entangled state which cannot be reduced to the product of two Bell states is used as the quantum channel. Moreover, positive operator-valued measure (POVM) instead of usual projective measurement is employed. In one hand, the usual teleportation is required. And in another hand, the perfect copies and complement copies of an unknown two-particle state can be implemented with a certain probability in the help of the state preparer.

Keywords: Quantum cloning, Genuine four-particle entangled state, Two-particle measurement, Positive operator-valued measure

1 Introduction

Many interesting applications in the regime of information have produced due to the principle of quantum mechanics in recent years, such as quantum key distribution[1,2], quantum secret sharing[3-5], quantum teleportation[6-8], remote state preparation[9-12], and so on. According to the nocloning theorem[13,14], a quantum state cannot be cloned perfectly in quantum information, while classical information can. However, quantum cloning approximately is necessary in quantum information[15]. In literature, various approximate cloning machines have been demonstrated[16-30].

Pati[22] proposed a protocol of producing perfect copies and orthogonal-complement copies of an unknown state with the assistance from a state preparer. In his protocol, usual teleportation is required in the first stage, and in the second stage the preparer performs a single-particle measurement and communicates some classical information to different parties. Therefore, Pati's protocol can realize perfect cloning and complementing of an unknown state with a probabilistic manner. In some previous cloning protocols, Bell state[22,27,31], GHZ state[22] and the cluster state[22] are employed as the quantum channels.

Recently, Yeo and Chua[32] proposed a novel four-particle entanglement $\left|\bar{\chi}^{00}\right\rangle$ state. This state cannot be decomposed into a pair of Bell states. Differ from those of four-party GHZ and *W* states, it is a genuine four-particle entangled state. It could play an analogous role to Bell state in the theory of multipartite entanglement. The state can be used in the teleportation[32,33], dense coding[32,34], quantum secure direct communication[35], quantum state splitting[36] and remote state preparation[37]. In the present paper, the genuine four-particle entangled state in the regime of assisted cloning using POVM instead of usual projective measurement, has been studied. POVM has been extensively studied in the field of quantum communication[38-41]. A protocol will be put forward of cloning an unknown two-qubit entangled state via assistance provided by the state preparer. Previously different assisted cloning schemes of an unknown two-particle state[28-30] are used. Now, the genuine four-particle entangled state is used as the quantum channel in this protocol. The assisted cloning protocol includes two steps. In the first step, the ordinary teleportation is needed. In the second step, the state preparer makes two two-particle measurements. Two-particle measurement can be realized by quantum nondemolition(QND) measurement in experiment[42-44]. The act of measuring a quantum system must disturb the system. Nevertheless, QND measurement is designed to avoid the back action produced by a measurement[45,46], it has been widely studied in quantum information processing[42-48]. Then, the state preparer communicates the result to the partner through the classical channel. According to the state preparer's measurement result, the partner performs a POVM on the particles so that the perfect copies and complement copies of an unknown two-particle state can be produced with some certain probability.

2 Assisted Cloning of an Arbitrary Unknown Two-Qubit Entangled State and Its Orthogonal Complement State

Now let us consider the case that assisted cloning of an arbitrary unknown two-particle entangled state by employing a genuine four-particle entangled state as quantum channel. Suppose there are three participants, Victor, Alice and Bob. Victor is the state preparer. He prepares an arbitrary unknown two-particle entangled state.

$$|\Sigma\rangle_{12} = \alpha|00\rangle_{12} + \beta|01\rangle_{12} + \gamma|10\rangle_{12} + \delta|11\rangle_{12} \quad \ldots(1)$$

where α, β, γ and δ are all the complex numbers. They satisfy $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$, and which are known completely to Victor but unknown to Alice. Then he transmits the two particles to Alice. After receiving the state, Alice uses it as her input state. The input state is unknown to both Alice and Bob. Alice wishes to create either a copy or an orthogonal copy of the unknown state $|\Sigma\rangle_{12}$ at her place with the assistance of Victor. Assume that Alice and Bob share a genuine four-particle entangled state as the quantum channel, given by:

$$\left|\bar{\chi}^{00}\right\rangle_{3456} = \frac{1}{2}\sum_J |J\rangle_{34} \otimes |J'\rangle_{56} \quad \ldots(2)$$

The $|J\rangle$'s constitute an orthonormal basis, and explicitly:

$$|0\rangle = \cos\theta_1|00\rangle + \sin\theta_1|11\rangle$$
$$|1\rangle = \cos\phi_1|01\rangle + \sin\phi_1|10\rangle$$
$$|2\rangle = -\sin\phi_1|01\rangle + \cos\phi_1|10\rangle$$
$$|3\rangle = -\sin\theta_1|00\rangle + \cos\theta_1|11\rangle \quad \ldots(3)$$

The $|J'\rangle$'s constitute another orthonormal basis:

$$|0'\rangle = \cos\theta_2|00\rangle + \sin\theta_2|11\rangle$$
$$|1'\rangle = \cos\phi_2|01\rangle + \sin\phi_2|10\rangle$$
$$|2'\rangle = -\sin\phi_2|01\rangle + \cos\phi_2|10\rangle$$
$$|3'\rangle = -\sin\theta_2|00\rangle + \cos\theta_2|11\rangle \quad \ldots(4)$$

where $0 < \theta_1, \theta_2, \phi_1, \phi_2 < \frac{\pi}{2}$ and assume that $\theta_1 \neq \theta_2$, $\phi_1 \neq \phi_2$. Alice owns the particles 3 and 4, and the particles 5 and 6 belong to Bob. The state of the six qubit is:

$$|\Theta\rangle = |\Sigma\rangle_{12}\left|\bar{\chi}^{00}\right\rangle_{3456} \quad \ldots(5)$$

On receiving the state, Alice carries out two two-particle measurements on the particle pairs (1, 3) and (2, 4) in the basis $\{|\phi^{\pm}\rangle, |\psi^{\pm}\rangle\}$.

$$|00\rangle = a|\Phi^{+}\rangle + b|\Phi^{-}\rangle$$
$$|11\rangle = b|\Phi^{+}\rangle - a|\Phi^{-}\rangle$$
$$|10\rangle = a|\Psi^{+}\rangle + b|\Psi^{-}\rangle$$
$$|01\rangle = b|\Psi^{+}\rangle - a|\Psi^{-}\rangle \quad \ldots(6)$$

where a and b are real numbers, $|a|^2 + |b|^2 = 1$ and $|a| \geq |b|$. Thus, the state particles 5 and 6 become:

$$_{24}\langle\cap|_{13}\langle\cup|\Theta\rangle_{123456} = \frac{1}{2}[(G\cos\theta + H\sin\theta)|00\rangle_{56} + (I\sin\varphi + K\cos\phi)|01\rangle_{56} + (R\cos\phi + S\sin\phi)|10\rangle_{56} + (X\sin\theta + Y\cos\theta)|11\rangle_{56} \quad \ldots(7)$$

where $\theta \equiv \theta_1 - \theta_2$ and $\phi \equiv \phi_1 - \phi_2$, and *G, H, I, K, R, S, X, Y* can be seen in Table 1.

Table 1 — $_{24}\langle\cap|$ and $_{13}\langle\cup|$ represent Alice's measurement results on the particle pairs (2,4) and (1,3), respectively

$_{24}\langle\cap\|$	$_{13}\langle\cup\|$	*G*	*H*	*I*	*K*	*R*	*S*	*X*	*Y*
$_{24}\langle\phi^+\|$	$_{13}\langle\phi^+\|$	αa^2	δb^2	$-\beta ab$	γab	βab	γab	$-\alpha a^2$	δb^2
$_{24}\langle\phi^-\|$	$_{13}\langle\phi^+\|$	αab	$-\delta ab$	βa^2	γb^2	$-\beta a^2$	γb^2	$-\alpha ab$	$-\delta ab$
$_{24}\langle\psi^+\|$	$_{13}\langle\phi^+\|$	βa^2	γb^2	$-\alpha ab$	δab	αab	δab	$-\beta a^2$	γb^2
$_{24}\langle\psi^-\|$	$_{13}\langle\phi^+\|$	βab	$-\gamma ab$	αa^2	δb^2	δb^2	$-\alpha a^2$	$-\beta ab$	$-\gamma ab$
$_{24}\langle\phi^+\|$	$_{13}\langle\phi^-\|$	αab	$-\delta ab$	$-\beta b^2$	$-\gamma a^2$	βb^2	$-\gamma a^2$	$-\alpha ab$	$-\delta ab$
$_{24}\langle\phi^-\|$	$_{13}\langle\phi^-\|$	αb^2	δa^2	βab	$-\gamma ab$	$-\beta ab$	$-\gamma ab$	$-\alpha b^2$	δa^2
$_{24}\langle\psi^+\|$	$_{13}\langle\phi^-\|$	βab	$-\gamma ab$	$-\alpha b^2$	$-\delta a^2$	αb^2	$-\delta a^2$	$-\beta ab$	$-\gamma ab$
$_{24}\langle\psi^-\|$	$_{13}\langle\phi^-\|$	βb^2	γa^2	αab	$-\delta ab$	$-\alpha ab$	$-\delta ab$	$-\beta b^2$	γa^2
$_{24}\langle\phi^+\|$	$_{13}\langle\psi^+\|$	γa^2	βb^2	$-\delta ab$	αab	δab	αab	$-\gamma a^2$	$-\beta b^2$
$_{24}\langle\phi^-\|$	$_{13}\langle\psi^+\|$	γab	$-\beta ab$	δa^2	αb^2	$-\delta a^2$	αb^2	$-\gamma ab$	$-\beta ab$
$_{24}\langle\psi^+\|$	$_{13}\langle\psi^+\|$	δa^2	αb^2	$-\gamma ab$	βab	γab	βab	$-\delta a^2$	αb^2
$_{24}\langle\psi^-\|$	$_{13}\langle\psi^+\|$	δab	$-\alpha ab$	γa^2	βb^2	$-\gamma a^2$	βb^2	$-\delta ab$	$-\alpha ab$
$_{24}\langle\phi^+\|$	$_{13}\langle\psi^-\|$	γab	$-\beta ab$	$-\delta b^2$	$-\alpha a^2$	δb^2	$-\alpha a^2$	$-\gamma ab$	$-\beta ab$
$_{24}\langle\phi^-\|$	$_{13}\langle\psi^-\|$	γb^2	βa^2	$-\delta ab$	$-\alpha ab$	βa^2	$-\delta ab$	αab	$-\gamma b^2$
$_{24}\langle\psi^+\|$	$_{13}\langle\psi^-\|$	δab	$-\alpha ab$	$-\gamma b^2$	$-\beta a^2$	γb^2	$-\beta a^2$	$-\delta ab$	$-\alpha ab$
$_{24}\langle\psi^-\|$	$_{13}\langle\psi^-\|$	δb^2	αa^2	γab	$-\beta ab$	$-\gamma ab$	$-\beta ab$	$-\delta b^2$	αa^2

Without loss of generality, given that Alice's measurement outcome is $|\varphi^-\rangle_{13}|\psi^+\rangle_{24}$, the qubits 5 and 6 will collapse into the state:

$$\begin{aligned}|\Pi\rangle_{56} = \frac{1}{2}[&(\beta ab\cos\theta - \gamma ab\sin\theta)|00\rangle_{56} \\ &-(\alpha b^2\sin\phi + \delta a^2\cos\phi)|01\rangle_{56} \\ &+(ab^2\cos\phi - \delta a^2\sin\phi)|10\rangle_{56} \\ &+(-\beta ab\sin\theta - \gamma ab\cos\theta)|11\rangle_{56}]\end{aligned} \quad \ldots(8)$$

Bob executes an unitary operation *U* on his particles 5 and 6. *U* takes the following form:

$$U = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{34} & A_{44} \end{pmatrix} \quad \ldots(9)$$

Consider the case of A_{11}=0, A_{12}=$-\sin\phi$ A_{13}=$-\cos\phi$ A_{14}=0, A_{21}=$\cos\theta$, A_{22}=0, A_{23}=0, A_{24}=$\sin\theta$, A_{31}=$\sin\theta$, A_{32}=0, A_{33}=0, A_{34}=$\cos\theta$, A_{41}=0, A_{42}=$-\cos\phi$, A_{43}=$-\sin\phi$, A_{44}=0.

Hence, the state of the qubits 5 and 6 becomes:

$$|\Pi\rangle'_{56} = \frac{1}{2}(\alpha b^2|00\rangle + \beta ab|01\rangle + \gamma ab|10\rangle + \delta a^2|11\rangle)_{56}$$

To the other cases, the operation U can be found in Table 2. Ultimately, Bob introduces two auxiliary particles m, n with the initial state $|00\rangle_{mn}$, and performs two controlled-not (CNOT) operations on particles (5, m) and (6, n), respectively (the particles 5 and 6 are the controlled bits, while the auxiliary particles m and n are the target ones). Consequently, the state of particles 56mn becomes:

$$|\Xi\rangle_{56mn} = \frac{1}{8}(|K_1\rangle_{56}|Q_1\rangle_{mn} + |K_2\rangle_{56}|Q_2\rangle_{mn} + |K_3\rangle_{56}|Q_3\rangle_{mn} + |K_4\rangle_{56}|Q_4\rangle_{mn}) \quad \ldots(10)$$

where

$$|K_1\rangle = (\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle)_{56}$$
$$|K_2\rangle = (\alpha|00\rangle + \beta|01\rangle - \gamma|10\rangle - \delta|11\rangle)_{56}$$
$$|K_{31}\rangle = (\alpha|00\rangle - \beta|01\rangle + \gamma|10\rangle - \delta|11\rangle)_{56}$$
$$|K_4\rangle = (\alpha|00\rangle - \beta|01\rangle - \gamma|10\rangle + \delta|11\rangle)_{56}$$
$$|Q_1\rangle = (b^2|00\rangle + ab|01\rangle + ab|10\rangle + a^2|11\rangle)_{mn}$$
$$|Q_2\rangle = (b^2|00\rangle + ab|01\rangle - ab|10\rangle - a^2|11\rangle)_{mn}$$
$$|Q_3\rangle = (b^2|00\rangle - ab|01\rangle + ab|10\rangle - a^2|11\rangle)_{mn}$$
$$|Q_4\rangle = (b^2|00\rangle - ab|01\rangle - ab|10\rangle + a^2|11\rangle)_{mn} \quad \ldots(11)$$

Apparently, Bob will obtain the states $|K_i\rangle_{56}\ (i = 1,2,3,4)$ if the states $|Q_i\rangle_{mn}$ can be distinguished. In general, the four states $|Q_i\rangle_{mn}$ cannot be differentiated deterministically by using a usual projective measurement because they are not

Table 2 — AMRs denotes Alice's measurement results. Suppose that $C=\sin\theta$, $D=\cos\theta$, $E=\sin\phi$ and $F=\cos\phi$

AMRs	A_{11}	A_{12}	A_{13}	A_{14}	A_{21}	A_{22}	A_{23}	A_{24}	A_{31}	A_{32}	A_{33}	A_{34}	A_{41}	A_{42}	A_{43}	A_{44}
$\lvert\phi^+\rangle_{13}\lvert\phi^+\rangle_{24}$	*D*	0	0	-*C*	0	-*E*	*F*	0	0	*F*	*E*	0	*C*	0	0	*D*
$\lvert\phi^+\rangle_{13}\lvert\phi^-\rangle_{24}$	*D*	0	0	-*C*	0	*E*	-*F*	0	0	*F*	*E*	0	-*C*	0	0	-*D*
$\lvert\phi^+\rangle_{13}\lvert\psi^+\rangle_{24}$	0	-*E*	*F*	0	D	0	0	-*D*	*C*	0	0	*C*	0	*F*	*E*	0
$\lvert\phi^+\rangle_{13}\lvert\psi^-\rangle_{24}$	0	*E*	-*F*	0	D	0	0	-*C*	-*C*	0	0	-*D*	0	*F*	*E*	0
$\lvert\phi^-\rangle_{13}\lvert\varphi^+\rangle_{24}$	*D*	0	0	-*C*	0	-*E*	*F*	0	0	-F	E	0	-*C*	0	0	-*D*
$\lvert\phi-\rangle_{13}\lvert\varphi^-\rangle_{24}$	*D*	0	0	-*C*	0	*E*	-*F*	0	0	-F	-E	0	*C*	0	0	*D*
$\lvert\phi^-\rangle_{13}\lvert\psi^+\rangle_{24}$	0	-*E*	*F*	0	*D*	0	0	-*C*	-*C*	0	0	-*D*	0	-*F*	-*E*	0
$\lvert\phi^-\rangle_{13}\lvert\psi^-\rangle_{24}$	0	*E*	-*F*	0	*D*	0	0	-*C*	*C*	0	0	*D*	0	-*F*	-*E*	0
$\lvert\psi^+\rangle_{13}\lvert\phi^+\rangle_{24}$	0	*F*	*E*	0	*C*	0	0	*D*	*D*	0	0	-*C*	0	-*E*	*F*	0
$\lvert\psi^+\rangle_{13}\lvert\phi^-\rangle_{24}$	0	*F*	*E*	0	-*C*	0	0	-*D*	*D*	0	0	-*C*	0	*E*	-*F*	0
$\lvert\psi^+\rangle_{13}\lvert\psi^+\rangle_{24}$	*C*	0	0	*D*	0	*F*	*E*	0	0	-*E*	*F*	0	*D*	0	0	-*C*
$\lvert\psi^+\rangle_{13}\lvert\psi^-\rangle_{24}$	-*C*	0	0	-*D*	0	*F*	*E*	0	0	*E*	-*F*	0	*D*	0	0	-*C*
$\lvert\psi^-\rangle_{13}\lvert\varphi^+\rangle_{24}$	0	-*F*	-*E*	0	-C	0	0	-*D*	*D*	0	0	-C	0	-*E*	*F*	0
$\lvert\psi^-\rangle_{13}\lvert\phi^-\rangle_{24}$	0	-*F*	-*E*	0	C	0	0	*D*	*D*	0	0	-C	0	*E*	-*F*	0
$\lvert\psi^-\rangle_{13}\lvert\psi^+\rangle_{24}$	-*C*	0	0	-*D*	0	-*F*	-*E*	0	0	-*E*	*F*	0	*D*	0	0	-*C*
$\lvert\psi^-\rangle_{13}\lvert\psi^-\rangle_{24}$	-*C*	0	0	-*D*	0	-*F*	-*E*	0	0	*E*	-*F*	0	*D*	0	0	-*C*

orthogonal. Nevertheless, the discrimination can be achieved as long as Bob carries out an optimal POVM on the auxiliary particles m and n as follows:

$$P_1 = \frac{1}{v}|N_1\rangle\langle N_1|,\ P_2 = \frac{1}{v}|N_2\rangle\langle N_2|,\ P_3 = \frac{1}{v}|N_3\rangle\langle N_3|$$

$$P_4 = \frac{1}{v}|N_4\rangle\langle N_4|,\ P_5 = I - \frac{1}{v}\sum_{i=1}^{4}|N_i\rangle\langle N_i|$$

where

$$|N_1\rangle = \frac{1}{\sqrt{\zeta}}\left(\frac{1}{b^2}|00\rangle + \frac{1}{ab}|01\rangle + \frac{1}{ab}|10\rangle + \frac{1}{a^2}|11\rangle\right)_{mn}$$

$$|N_2\rangle = \frac{1}{\sqrt{\zeta}}\left(\frac{1}{b^2}|00\rangle + \frac{1}{ab}|01\rangle - \frac{1}{ab}|10\rangle - \frac{1}{a^2}|11\rangle\right)_{mn}$$

$$|N_3\rangle = \frac{1}{\sqrt{\zeta}}\left(\frac{1}{b^2}|00\rangle - \frac{1}{ab}|01\rangle + \frac{1}{ab}|10\rangle - \frac{1}{a^2}|11\rangle\right)_{mn}$$

$$|N_4\rangle = \frac{1}{\sqrt{\zeta}}\left(\frac{1}{b^2}|00\rangle - \frac{1}{ab}|01\rangle - \frac{1}{ab}|10\rangle + \frac{1}{a^2}|11\rangle\right)_{mn}$$

$$\zeta = \frac{1}{a^4} + \frac{2}{a^2b^2} + \frac{1}{b^4} \qquad \ldots(12)$$

where I is an identity operator, and the parameter v closely relates to a and b, and P_5 must be positive. To gain the parameter v, the five operators $P_j\,(j = 1,2,3,4,5)$ can be denoted by the forms:

$$P_1 = \frac{1}{v\zeta}\begin{pmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{21} & B_{22} & B_{23} & B_{24} \\ B_{31} & B_{32} & B_{33} & B_{34} \\ B_{41} & B_{42} & B_{43} & B_{44} \end{pmatrix}$$

$$P_2 = \frac{1}{v\zeta}\begin{pmatrix} B_{11} & B_{12} & -B_{13} & -B_{14} \\ B_{21} & B_{22} & -B_{23} & -B_{24} \\ -B_{31} & -B_{32} & B_{33} & B_{34} \\ -B_{41} & -B_{42} & B_{43} & B_{44} \end{pmatrix}$$

$$P_3 = \frac{1}{v\zeta}\begin{pmatrix} -B_{11} & B_{12} & B_{13} & -B_{14} \\ -B_{21} & B_{22} & -B_{23} & B_{24} \\ B_{31} & -B_{32} & B_{33} & -B_{34} \\ -B_{41} & B_{42} & -B_{43} & B_{44} \end{pmatrix}$$

$$P_4 = \frac{1}{v\zeta}\begin{pmatrix} B_{11} & -B_{12} & -B_{13} & B_{14} \\ -B_{21} & B_{22} & B_{23} & -B_{24} \\ -B_{31} & B_{32} & B_{33} & -B_{34} \\ B_{41} & -B_{42} & -B_{43} & B_{44} \end{pmatrix}$$

$$P_5 = \frac{1}{v\zeta}\begin{pmatrix} 1-\frac{4B_{11}}{v\zeta} & 0 & 0 & 0 \\ 0 & 1-\frac{4B_{22}}{v\zeta} & 0 & 0 \\ 0 & 0 & 1-\frac{4B_{33}}{v\zeta} & 0 \\ 0 & 0 & 0 & 1-\frac{4B_{44}}{v\zeta} \end{pmatrix} \qquad \ldots(13)$$

The parameter v should be chosen as $1 \le v \le 4$ as all the diagonal elements of P_5 are positive. Based on the POVM's outcome, Bob can positively deduce the state $|Q_i\rangle_{mn}$ of qubits m and n. In the case that Bob knows the state of qubits 5 and 6, he can reconstruct the original state without any operation or performing appropriate unitary operation on the particles 5 and 6. However, Bob cannot infer which state the particles 5 and 6 is in if his POVM outcome is P_5 with the probability $1-\frac{1}{v\zeta}$ as shown in Table 3. Under this circumstance, the task fails.

Up to now, we have showed one case of Alice's and Bob's BSM results $|\phi^-\rangle_{13}|\psi^+\rangle_{24}$. For the other 15 cases, the protocol also can be achieved. The total success probability of this scheme is:

$$p = \frac{16}{v\zeta} \qquad \ldots(14)$$

If $|a| = |b| = \sqrt{2}/2$, one can see $v = 1$. The total success probability equals 1. Obviously, the scheme becomes to a deterministic one. In essence, it equals to the case of teleporating an arbitrary unknown two-qubit state.

Afterwards Alice sends the particles 1 and 2 to Victor. Victor measures the particles 1 and 2 in the set of mutually orthogonal basis vectors $\{|\Omega_0\rangle, |\Omega_1\rangle, |\Omega_2\rangle, |\Omega_3\rangle\}$, where

Table 3 — Bob's POVM result is P_i with the probability $p=\frac{1}{4\nu\zeta}$

AMRs	B_{11}	B_{12}	B_{13}	B_{14}	B_{21}	B_{22}	B_{23}	B_{24}	B_{31}	B_{32}	B_{33}	B_{34}	B_{41}	B_{42}	B_{43}	B_{44}
$\left\|\phi^+\right\rangle_{13}\left\|\phi^+\right\rangle_{24}$	$\frac{1}{a^4}$	$\frac{1}{a^3b}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{ab^3}$	$\frac{1}{b^4}$
$\left\|\phi^+\right\rangle_{13}\left\|\phi^-\right\rangle_{24}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{a^4}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$	$\frac{1}{b^4}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$
$\left\|\phi^+\right\rangle_{13}\left\|\psi^+\right\rangle_{24}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{a^4}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$	$\frac{1}{b^4}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$
$\left\|\phi^+\right\rangle_{13}\left\|\psi^-\right\rangle_{24}$	$\frac{1}{a^4}$	$\frac{1}{a^3b}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{ab^3}$	$\frac{1}{b^4}$
$\left\|\phi^-\right\rangle_{13}\left\|\phi^+\right\rangle_{24}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{b^4}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^4}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$
$\left\|\phi -\right\rangle_{13}\left\|\phi^-\right\rangle_{24}$	$\frac{1}{b^4}$	$\frac{1}{ab^3}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{a^3b}$	$\frac{1}{a^4}$
$\left\|\phi^-\right\rangle_{13}\left\|\psi^+\right\rangle_{24}$	$\frac{1}{b^4}$	$\frac{1}{ab^3}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{a^3b}$	$\frac{1}{a^4}$
$\left\|\phi^-\right\rangle_{13}\left\|\psi^-\right\rangle_{24}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{b^4}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^4}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$
$\left\|\psi^+\right\rangle_{13}\left\|\phi^+\right\rangle_{24}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{b^4}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^4}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$
$\left\|\psi^+\right\rangle_{13}\left\|\phi^-\right\rangle_{24}$	$\frac{1}{b^4}$	$\frac{1}{ab^3}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{a^3b}$	$\frac{1}{a^4}$
$\left\|\psi^+\right\rangle_{13}\left\|\psi^+\right\rangle_{24}$	$\frac{1}{b^4}$	$\frac{1}{ab^3}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{a^3b}$	$\frac{1}{a^4}$
$\left\|\psi^+\right\rangle_{13}\left\|\psi^-\right\rangle_{24}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{b^4}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^4}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$
$\left\|\psi^-\right\rangle_{13}\left\|\phi^+\right\rangle_{24}$	$\frac{1}{a^4}$	$\frac{1}{a^3b}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{ab^3}$	$\frac{1}{b^4}$
$\left\|\psi^-\right\rangle_{13}\left\|\phi^-\right\rangle_{24}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{a^4}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$	$\frac{1}{b^4}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$
$\left\|\psi^-\right\rangle_{13}\left\|\psi^+\right\rangle_{24}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{a^4}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$	$\frac{1}{b^4}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$
$\left\|\psi^-\right\rangle_{13}\left\|\psi^-\right\rangle_{24}$	$\frac{1}{a^4}$	$\frac{1}{a^3b}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{a^3b}$	$\frac{1}{a^2b^2}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{a^2b^2}$	$\frac{1}{ab^3}$	$\frac{1}{ab^3}$	$\frac{1}{b^4}$

$$
\begin{aligned}
&\left|\Omega_0\right\rangle=(\alpha\left|00\right\rangle+\beta\left|01\right\rangle+\gamma\left|10\right\rangle+\delta\left|11\right\rangle)\equiv\left|\Sigma\right\rangle_{12}\\
&\left|\Omega_1\right\rangle=(\eta\alpha\left|00\right\rangle-\eta^-\beta\left|01\right\rangle-\eta^-\gamma\left|10\right\rangle+\eta\delta\left|11\right\rangle)\\
&\left|\Omega_2\right\rangle=(\delta^*\left|00\right\rangle+\gamma^*\left|01\right\rangle-\beta^*\left|10\right\rangle-\alpha^*\left|11\right\rangle)\\
&\left|\Omega_3\right\rangle=(\eta\delta^*\left|00\right\rangle-\eta^-\gamma^*\left|01\right\rangle+\eta^-\beta^*\left|10\right\rangle-\eta\alpha^*\left|11\right\rangle)
\end{aligned}
\qquad \ldots(15)
$$

where

$\eta=\sqrt{\frac{|\beta|^2+|\gamma|^2}{|\alpha|^2+|\delta|^2}}$, and $\{|00\rangle_{12},|01\rangle_{12},|10\rangle_{12},|11\rangle_{12}\}$ is the computation basis. Victor holds qubits 1 and 2, Alice owns qubits 3 and 4. $\left|\varphi^-\right\rangle_{13}\left|\psi^+\right\rangle_{24}$ can be given by:

$$
\begin{aligned}
{}_{12}\left\langle\Omega_0\middle|\varphi^-\right\rangle_{13}\left|\psi^+\right\rangle_{24}&=(ab\beta^*\left|00\right\rangle+b^2\alpha^*\left|01\right\rangle\\
&\quad-a^2\delta^*\left|10\right\rangle-ab\gamma^*\left|11\right\rangle)_{34}\\
{}_{12}\left\langle\Omega_1\middle|\varphi^-\right\rangle_{13}\left|\psi^+\right\rangle_{24}&=(-ab\eta^-\beta^*\left|00\right\rangle+b^2\eta\alpha^*\left|01\right\rangle\\
&\quad-a^2\eta\delta^*\left|10\right\rangle+ab\eta^-\gamma^*\left|11\right\rangle)_{34}\\
{}_{12}\left\langle\Omega_2\middle|\varphi^-\right\rangle_{13}\left|\psi^+\right\rangle_{24}&=(ab\gamma\left|00\right\rangle+b^2\delta\left|01\right\rangle\\
&\quad+a^2\alpha\left|10\right\rangle+ab\beta\left|11\right\rangle)_{34}\\
{}_{12}\left\langle\Omega_3\middle|\varphi^-\right\rangle_{13}\left|\psi^+\right\rangle_{24}&=(-ab\eta^-\gamma\left|00\right\rangle+b^2\eta\delta\left|01\right\rangle\\
&\quad+a^2\eta\alpha\left|10\right\rangle-ab\eta^-\beta\left|11\right\rangle)_{34}
\end{aligned}
\qquad \ldots(16)
$$

If Victor's measurement outcome is $|\Omega_2\rangle_{12}$, the state of the particles 3 and 4 becomes:

$$|\Gamma\rangle_{34} = \frac{1}{2}(ab\gamma|00\rangle + b^2\delta|01\rangle + a^2\alpha|10\rangle + ab\beta)_{34} \quad \ldots(17)$$

Victor sends the measurement outcome to Alice with two bits through a classical channel. Then Alice performs σ_3^x on the particle 3, thus, the state of the particles 3 and 4 evolves to $|\Gamma\rangle'_{34} = \frac{1}{2}(\alpha^2 a|00\rangle + ab\beta|01\rangle + ab\gamma|10\rangle + b^2\delta|11\rangle)_{34}$. Next, Alice introduces two auxiliary particles *g*, *h* with the initial state $|00\rangle_{gh}$, and performs two CNOT operations on particles (3, *g*) and (4, *h*), respectively (the particles 3 and 4 are the controlled bits, while the auxiliary particles *g* and *h* are the target ones). Afterwards, Alice carries out σ_g^x and σ_h^x on the particle *g* and *h*, respectively. So the state of particles 3, 4, *g* and *h* turns into:

$$\begin{aligned}|\Lambda\rangle_{34gh} = \frac{1}{8}(&|K_1\rangle_{34}|Q_1\rangle_{gh} - |K_2\rangle_{34}|Q_2\rangle_{gh} \\ &-|K_3\rangle_{34}|Q_3\rangle_{gh} + |K_4\rangle_{34}|Q_4\rangle_{gh})\end{aligned} \quad \ldots(18)$$

Alice performs POVM which is the same as the form of the formula given in Eq. (13) on the particles 3 and 4, we choose:

$$\begin{aligned}&B_{11} = \frac{1}{a^4}\\ &B_{12} = B_{13} = B_{21} = B_{31} = \frac{1}{a^3 b}\\ &B_{24} = B_{34} = B_{42} = B_{43} = \frac{1}{ab^3}\\ &B_{14} = B_{22} = B_{23} = B_{32} = B_{33} = B_{41} = -\frac{1}{a^2b^2}\\ &B_{44} = \frac{1}{b^4}\end{aligned} \quad \ldots(19)$$

Only if she knows that the state of qubits 3 and 4, she can get the original state. However, if her POVM outcome is P_5 with the probability $1-\frac{1}{\nu\zeta}$, Alice cannot deduce the state of the particles 3 and 4. As a result, the scheme fails.

3 Discussion and Conclusions

A perfect copy or orthogonal-complement copies of an unknown two-qubit entangled state with the assistance of the state preparer Victor, has been demonstrated. In the present paper, the genuine four-particle entangled state is used as the quantum channel, which has some special characteristics that differ from other four-particles entangled states. Furthermore, the state has been realized in some systems[49,50], for example, cavity QED, ion trap[51] and linear optical elements[52]. Therefore, our protocol can be realized in experiment. Additionally, POVM instead of usual projective measurement is used. The assisted cloning protocol includes two steps. In the first step, the ordinary teleportation is needed. In the second step, the state preparer measures two particle in the set of mutually orthogonal basis vectors and informs the result to the partner through the classical channel. On the basis of the state preparer's measurement outcome, the partner introduces two auxiliary particles, and does two CNOT operations on two particle pairs. In the following, the partner performs POVM on the particles so that the perfect copies and complement copies of an unknown two-particle state can be achieved with some certain probability.

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