# Stability of a large amplitude plasma wave to oscillating two-stream instability

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The stability of a long wavelength large amplitude plasma wave, generated in a laser wakefield accelerator at moderately relativistic laser intensity, to oscillating two-stream instability has been examined. In the limit when the oscillatory velocity of electrons due to the plasma wave, $\vec{V}_0$, exceeds the electron thermal speed, the short wavelength plasma wave turns out to be $\omega^2 = \omega_p^2 + \left| \vec{k} \cdot \vec{V}_0 \right|^2 / 2$. In the four wave parametric process, involving the pump plasma wave, a short wavelength low frequency quasimode and two short wavelength plasma wave sidebands, the pump and the sidebands exert a ponderomotive force on the electrons driving a low frequency quasimode. The electron density perturbation associated with this mode couples with the pump driven electron oscillatory velocity to produce nonlinear currents driving the sidebands. We find that this process has no growth when the ion motion is ignored. However, with the inclusion of ion motion the parametric instability is important on the time scale of an ion plasma period.

Keywords: Oscillating two-stream instability, Laser driven plasma wave, Sideband waves

## 1 Introduction

The large amplitude, long wavelength plasma waves are excited in many situation, including laser based charged particle accelerators[1-6]. The main requirement for gaining greater acceleration is the large amplitude of the plasma wave. The plasma wave can be driven by the ponderomotive force due to an intense short laser pulse or short duration electron beam. The laser pulse period $\tau$ in a laser wake field accelerator is of the order of plasma period $\omega_p^{-1}$. In the case of a beat wave accelerator (PBWA), plasma waves are generated by employing two lasers with frequency difference equal to the plasma frequency. In both the cases, these plasma waves propagate with large phase velocity, equal to the group velocity of the laser and can accelerate charge particle to relativistic energies. Moreover, when the plasma wave amplitude becomes very large, it becomes susceptible to the oscillating two-stream instability. Experimental and theoretical investigations have been done all over the world and different schemes have been proposed for achieving effective electron acceleration[7-11]. Kumar et al[12] have examined the effect of a relativistic intense laser pulse on the propagation of electron plasma

wave and particle acceleration. Baiwen et al[13] have observed the electron acceleration by an intense laser pulse in low density plasma and detected a well collimated relativistic electron beam in the direction opposite to the laser propagation. Gorbunov et al[14], have investigated the electron acceleration up to GeV energies by using an ultrashort petawatt laser by exciting a nonlinear plasma wakefield. One of the prominent applications of intense accelerated particles is in inertial fusion, different features of which have been discussed by several authors[15-17]. Ramachandran et al.[18] have studied the Oscillating two-stream instability (OTSI) of a plasma wave in a plasma channel, in which the growth rate increases with the width of the plasma density channel and decreases with the mode number. Ahmad et al.[19] have investigated OTSI of laser wakefield-driven plasma wave in a low density plasma under local and non-local effects. Malik[20] have investigated the OTSI of a plasma wave in plasma, which has hot and cold positive ions, negative ions, and the electrons. He found that mass of the ions and effects of charge number are significant on the instability. Ferdous et al.[21] have studied the OTSI of beat waves in a hot magnetized plasma, in which they found that the maximum growth rate of the instability is about two orders higher when ion motion is taken into account. In this context, the stability of the

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plasma wave is a relevant issue. Earlier studies on modest intensity of laser interaction with plasmas and PIC simulations of laser wakefield acceleration indicate that the plasma wave develops short wavelength distortions on a time scale comparable to ion plasma period. These studies have revealed the excitation of short wavelength plasma waves near the critical layer that can be accredited to the excitation of oscillating two-stream instability. In OTSI, a long wavelength pump wave (or plasma wave) excites two short wavelength Langmuir wave sidebands and a purely growing density perturbation. In the region where the electric field of the pump and Langmuir waves are parallel, the plasma is pushed away to the regions where the fields are antiparallel. The depressed density regions attract more electric field energy from the neighborhood leading to deeper density depressions and enhancement of the short wavelength Langmuir waves[22-23]. In this paper, we examine the oscillating two-stream instability of long wavelength plasma wave generated in a laser wakefield accelerator at mildly relativistic intensity. First, the plasma wave of frequency $\omega_0 \approx \omega_p$ and wave number $k_0 \approx \omega_p / c$ (where $\omega_p$ is the plasma frequency) modifies the short wavelength Langmuir eigen mode dispersion relation. Second, it couples to them through a four wave parametric process. The dynamics of the process is as follows. The pump plasma wave imparts oscillatory velocity to electrons. The latter couples a short wavelength low frequency mode $\omega, \vec{k}; \vec{k} > \vec{k}_0, \omega << \omega_0$, to two Langmuir wave sidebands $(\omega_{1,2}, \vec{k}_{1,2})$. The pump and Langmuir wave sidebands exert a low frequency ponderomotive force on electrons, driving the low frequency mode. The density perturbation in conjunction with the oscillating electron velocity $\vec{v}_0$ at $(\omega_0, \vec{k}_0)$ produces nonlinear density perturbations $n_{1,2}^{NL}$ at $(\omega_{1,2}, \vec{k}_{1,2})$ that drive the sidebands.

## 2 Short Wavelength Plasma Susceptibility

Consider a large amplitude plasma wave of electrostatic potential in a plasma of equilibrium density $n_0^0$:

$$\varphi_0 = A_0 e^{-i(\omega_0 t - k_0 z)} \qquad \dots (1)$$

It gives rise to electron velocity and density perturbations:

$$\vec{v}_0 = -\frac{e\vec{k}_0 \phi_0}{m\omega_0} \qquad \dots (2)$$

$$n_0 = n_0^0 \frac{\vec{k}_0 . \vec{v}_0}{\omega_0} = -n_0^0 \frac{ek_0^2 \phi_0}{m\omega_0^2} \qquad \dots (3)$$

where $-e$ and m are the electron charge and mass.

Now we consider a short wavelength plasma wave in the presence of large amplitude plasma wave with $\Omega \sim \omega_0 \sim \omega_p$, $q > k_0$:

$$\phi_\Omega = A e^{-(\Omega t - \vec{q}.\vec{r})} \qquad \dots (4)$$

The coupling between the two waves produces a driven wave at $\Omega + \omega_0$, $\vec{q} + \vec{k}_0$ with potential:

$$\phi_{\Omega+\omega_0} = A_{\Omega+\omega_0} e^{-i[(\Omega+\omega_0)t - (\vec{q}+\vec{k}_0)\vec{r}]} \qquad \dots (5)$$

As a consequence of these waves the velocity and density perturbations of electrons can be written as:

$$\vec{v} = \vec{v}_0 + \vec{v}_\Omega + \vec{v}_{\Omega+\omega_0} , \; n = n_0^0 + n_0 + n_\Omega + n_{\Omega+\omega_0} \quad \dots (6)$$

The waves exert a ponderomotive force on electrons at $\Omega + \omega_0$ and $\Omega$:

$$\vec{F}_{P\,\Omega+\omega_0} = e\nabla\phi_{P\,\Omega+\omega_0}, \; \phi_{P\,\Omega+\omega_0} = -\frac{m}{2e}\vec{v}_0.\vec{v}_\Omega \quad \dots (7)$$

$$\vec{F}_{P\Omega} = e\nabla\phi_{p\Omega}, \; \phi_{P\Omega} = -\frac{m}{2e}\vec{v}_0^*.\vec{v}_{\Omega+\omega_0} \qquad \dots (8)$$

The electron velocities due to $(\phi_\Omega$ and $\phi_{P\Omega})$ and $(\phi_{\Omega+\omega_0}$ and $\phi_{P(\Omega+\omega_0)})$ are:

$$\vec{v}_\Omega = -\frac{e\vec{q}}{m\Omega}(\phi_\Omega + \phi_{P\Omega}) \qquad \dots (9)$$

Similarly:

$$\vec{v}_{\Omega+\omega_0} = -\frac{e(\vec{q}+\vec{k}_0)}{m(\Omega+\omega_0)}(\phi_{\Omega+\omega_0} + \phi_{P\Omega+\omega_0}) \qquad \dots (10)$$

Using these in the equation of continuity, $\partial n / \partial t + \nabla.(n\vec{v})$, one obtains:

$$n_{\Omega+\omega_0} = -\frac{n_0^0 e(\vec{q}+\vec{k}_0)^2}{m(\Omega+\omega_0)^2}(\phi_{\Omega+\omega_0} + \phi_{P\Omega+\omega_0}) +$$

$$\frac{n_0(\vec{q}+\vec{k}_0).\vec{v}_\Omega}{2(\Omega+\omega_0)} + \frac{n_\Omega(\vec{q}+\vec{k}_0).\vec{v}_0}{2(\Omega+\omega_0)} \qquad \dots (11)$$

$$n_\Omega = \frac{\vec{q}.}{\Omega}\left(n_0^0\vec{v}_\Omega + \frac{1}{2}n_{\Omega+\omega_0}\vec{v}_0^* + \frac{1}{2}n_0^*\vec{v}_{\Omega+\omega_0}\right) \quad \ldots (12)$$

We are looking for $\Omega, \vec{q}$ as an eigen mode, hence in the evaluation of nonlinear terms at $\Omega+\omega_0$, we may presume $\phi_{P\Omega} << \phi_\Omega$ and write:

$$\vec{v}_\Omega \approx -\frac{e\vec{q}}{m\Omega}\phi_\Omega, \quad n_\Omega = -\frac{n_0^0 eq^2\phi_\Omega}{m\Omega^2}, \quad \phi_{P\Omega+\omega_0} = \frac{\vec{q}.\vec{v}_0}{2\Omega}\phi_\Omega$$

$$n_{\Omega+\omega_0} = -\frac{n_0^0 e(\vec{q}+\vec{k}_0)^2}{m(\Omega+\omega_0)^2}\phi_{\Omega+\omega_0} - \frac{n_0^0 e\phi_\Omega v_{0z}}{2m\Omega(\Omega+\omega_0)}$$

$$\left[\frac{(q+\vec{k}_0)^2}{\Omega+\omega_0}q_z + \frac{k_0}{\omega_0}(\vec{q}+\vec{k}_0).\vec{q} + \frac{q^2}{\Omega}(q_z+k_0)\right] \quad \ldots (13)$$

Using this density perturbation in the Poisson's equation $\nabla^2\phi_{\Omega+\omega_0} = 4\pi e n_{\Omega+\omega_0}$, we get:

$$\varepsilon_{\Omega+\omega_0}\phi_{\Omega+\omega_0} = \frac{\omega_p^2\phi_\Omega v_{0z}.}{2\Omega(\Omega+\omega_0)}$$

$$\left[\frac{q_z}{\Omega+\omega_0} + \frac{k_0}{\omega_0}\frac{(\vec{q}+\vec{k}_0).\vec{q}}{(\vec{q}+\vec{k}_0)^2} + \frac{(k_z+k_0)q^2}{\Omega(\vec{q}+\vec{k}_0)^2}\right] \quad \ldots (14)$$

where $\varepsilon_{\Omega+\omega_0} = 1 - \frac{\omega_p^2}{(\Omega+\omega_0)^2} \approx \frac{3}{4}$.

From the Poisson's equation, one may also write:

$$n_{\Omega+\omega_0} = -\frac{(\vec{q}+\vec{k}_0)^2}{4\pi e}\phi_{\Omega+\omega_0} \quad \ldots (15)$$

From Eqs. (7), (8) and (10), one may write:

$$\phi_{P\Omega} = \frac{(\vec{q}+\vec{k}_0).\vec{v}_0^*}{2(\Omega+\omega_0)}\phi_{\Omega+\omega_0} + \frac{(\vec{q}+\vec{k}_0).\vec{v}_0^*}{4\Omega(\Omega+\omega_0)}\vec{q}.\vec{v}_0\phi_\Omega$$

$$= \phi_\Omega\eta_1, \quad \ldots (16)$$

$$\eta_1 = \frac{(\vec{q}+\vec{k}_0).\vec{v}_0^*\vec{q}.\vec{v}_0}{4\Omega(\Omega+\omega_0)} + \frac{(\vec{q}+\vec{k}_0).\vec{v}_0^*}{2(\Omega+\omega_0)\varepsilon_{\Omega+\omega_0}}\frac{\omega_p^2 v_{0z}}{2\Omega(\Omega+\omega_0)}$$

$$\left[\frac{q_z}{\Omega+\omega_0} + \frac{k_0}{\omega_0}\frac{(\vec{q}+\vec{k}_0).\vec{q}}{(\vec{q}+\vec{k}_0)^2} + \frac{(q_z+k_0)q^2}{\Omega(\vec{q}+\vec{k}_0)^2}\right]$$

From Eq. (9):

$$\vec{v}_\Omega = -\frac{e\vec{k}}{m\Omega}(1+\eta_1)\phi_\Omega \quad \ldots (17)$$

Using Eqs. (9), (15) and (16) in Eq. (12), we obtain:

$$n_\Omega = -\frac{n_0^0 ek^2}{m\Omega^2}\left[1+\eta_1 + \frac{k_0(\vec{q}+\vec{k}_0).\vec{q}q_z}{4\omega_0(\Omega+\omega_0)q^2}|v_0|^2\right]\phi_\Omega -$$

$$\frac{n_0^0 e(\vec{q}+\vec{k}_0)^2}{m\Omega^2}\frac{q_z|v_0|^2\phi_\Omega}{4(\Omega+\omega_0)\varepsilon_{\Omega+\omega_0}}\left(1+\frac{\omega_p^2 k_0/q_z}{\omega_0(\Omega+\omega_0)}\right)$$

$$\cdot\frac{(\vec{q}+\vec{k}_0).\vec{k}}{(\vec{k}+\vec{k}_0)^2}\Bigg)$$

$$\left(\frac{q_z}{\Omega+\omega_0} + \frac{k_0}{\omega_0}\frac{(\vec{q}+\vec{k}_0).\vec{q}}{(\vec{q}+\vec{k}_0)^2} + \frac{(q_z+k_0)q^2}{\Omega(\vec{q}+\vec{k}_0)^2}\right) \quad \ldots (18)$$

The electron susceptibility is:

$$\chi_\Omega = \frac{4\pi e}{k^2}\frac{n_\Omega}{\phi_\Omega} = -\frac{\omega_p^2}{\Omega^2}\left[1+\eta_2\frac{|v_0|^2}{4}\right] \quad \ldots (19)$$

where

$$\eta_2 = \frac{(q_z+k_0)q_z}{\Omega(\Omega+\omega_0)} + \frac{\omega_p^2(q_z+k_0)}{\Omega(\Omega+\omega_0)^2\varepsilon_{\Omega+\omega_0}}$$

$$\left\{\frac{q_z}{\Omega+\omega_0} + \frac{k_0}{\omega_0}\frac{(\vec{q}+\vec{k}_0).\vec{q}}{(\vec{q}+\vec{k}_0)^2} + \frac{(q_z+k_0)q^2}{\Omega(\vec{q}+\vec{k}_0)^2}\right\}$$

$$+ \frac{k_0 q_z}{q^2}\frac{(\vec{q}+\vec{k}_0).\vec{q}}{\omega_0(\Omega+\omega_0)}$$

$$+ \frac{q_z^2}{q^2}\frac{(\vec{q}+\vec{k}_0)^2}{(\Omega+\omega_0)\varepsilon_{\Omega+\omega_0}}$$

$$\left(1+\frac{\omega_p^2 k_0/q_z}{\omega_0(\Omega+\omega_0)}\cdot\frac{(\vec{q}+\vec{k}_0).\vec{q}}{(\vec{q}+\vec{k}_0)^2}\right)$$

$$\left[\frac{1}{\Omega+\omega_0} + \frac{k_0/q_z}{\omega_0}\frac{(\vec{q}+\vec{k}_0).\vec{q}}{(\vec{q}+\vec{k}_0)^2} + \frac{(q_z+k_0)q^2/q_z}{\Omega(\vec{q}+\vec{k}_0)^2}\right]$$

We have solved Eq. (19) numerically for the following parameters: $|v_0|^2/c^2 = 0.2$, $k_{0z}c/\omega_p = 1$, $q_z' = q_z c/\omega_p = 0-1.5$. In Fig. 1, we have plotted the normalized frequency of the plasma wave,

$\Omega / \omega_p$, as a function of normalized wave number, $q'_z$, at $q' = q'_z$, where $q' = qc / \omega_p$. The frequency increases nonlinearly with the wave number such that the group velocity of the plasma wave increases with the wave number. In Fig. 2, we have plotted $\Omega / \omega_p$ vs $q'_z$ for the Langmuir wave
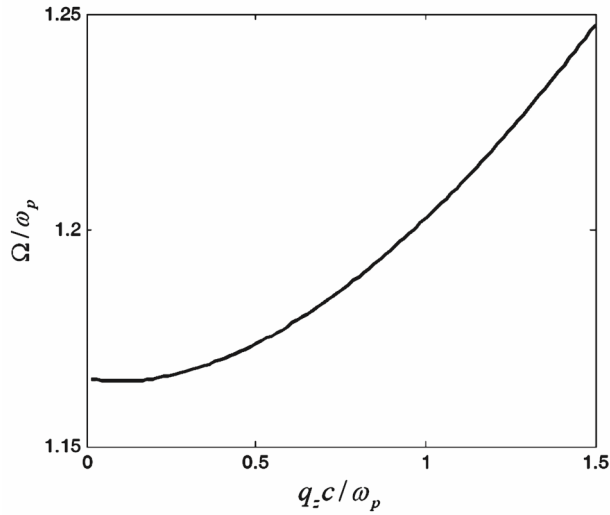


Fig. 1 — Normalized frequency, $\Omega / \omega_p$, of the Langmuir wave as a function of normalized wave number for $|v_0|^2 / c^2 = 0.2$, $k_{0z} c / \omega_p = 1$ at $q' = q'_z$
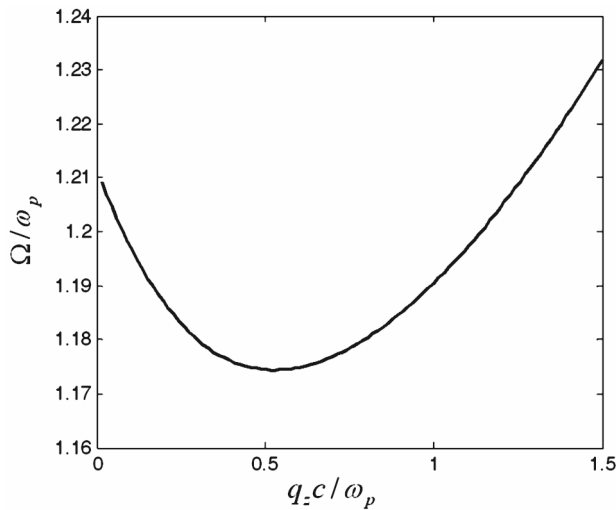


Fig. 2 — Normalized frequency, $\Omega / \omega_p$, of the Langmuir wave as a function of normalized wave number for $|v_0|^2 / c^2 = 0.2$, $k_{0z} c / \omega_p = 1$ at $q' = 2q'_z$

propagating obliquely at $60^0$ to the pump wave ($q' = 2q'_z$), The frequency of the plasma wave decreases up to $q'_z = 0.5$, i.e. $q = 2q_z = k_{0z}$. Beyond this point as the wave number of the Langmuir wave becomes greater than that of the large amplitude Langmuir wave, the plasma wave frequency starts increasing.

For $k_0 << q$, $\eta_2 \cong 2q_z^2 / \omega_p^2$ and $\chi_\Omega$ can be written as:

$$\chi_\Omega = -\frac{\left(\omega_p^2 + q_z^2 |v_0|^2 / 2\right)}{\Omega^2} \qquad \ldots (20)$$

One may note that this susceptibility is similar to the one in a thermal plasma where $q^2 v_{th}^2$ is replaced by $|\vec{q}.\vec{v}_0|^2 / 2$, where $v_{th}$ is the electron thermal speed. This result is the same as one would obtain for $\chi_\Omega$ in a Maxwellian thermal plasma if one replaces, following[24], electron temperature $T_e$ by $T_e[1 + m|\vec{q}.\vec{v}_0|^2 / 2T_e q^2]$.

## 3 Oscillating Two Stream Instability

We consider the four wave coupling of the long wavelength, large amplitude plasma wave $\phi_0 = A_0(r, z, t) e^{-i(\omega_0 t - k_0 z)}$, to a low frequency electrostatic mode of potential

$\phi = A \exp\left[-i(\omega t - \vec{k}.\vec{r})\right]$, and two shorter wavelength Langmuir wave sidebands

$\phi_j = A_j \exp\left[-i(\omega_j t - \vec{k}_j.\vec{r})\right]$,

where $\omega_{1,2} = \omega \mp \omega_0$ and $\vec{k}_{1,2} = \vec{k} \mp \vec{k}_0$ ($\vec{k} > \vec{k}_0$), $\vec{k}_j = k_{jz} \hat{z}$ and j=1,2.

In the case of wakefield excitation by a Gaussian laser pulse of pulse duration comparable to plasma period, the pump plasma wave amplitude is related to laser amplitude $A_{L0}$ and laser frequency $\omega_L$ (in the non-relativistic limit) as:

$$A_0 \approx \frac{e A_{L0}^2 \sqrt{\pi}}{2.3 \, m \omega_L^2}$$

The sidebands give oscillatory velocities to electrons $\vec{v}_j = -e\vec{k}_j\phi_j / m\omega_j$, and in conjunction with the pump, exert a low frequency ponderomotive force on them at $(\omega, \vec{k})$

$$\vec{F}_p = e\nabla\phi_p = -(m/2)\nabla(\vec{v}_0.\vec{v}_1 + \vec{v}_0^*.\vec{v}_2),$$

$$\phi_p = \frac{\vec{k}_1.\vec{v}_0}{2\omega_1}\phi_1 + \frac{\vec{k}_2.\vec{v}_0^*}{2\omega_2}\phi_2,$$

$$\ldots (21)$$

The ponderomotive and self-consistent potentials $\phi_p$ and $\phi$ produce electron and ion density perturbations, $n, n_i$.

$$n = \frac{k^2}{4\pi e}\chi_e(\phi + \phi_p),\qquad\ldots (22)$$

$$n_i = -(k^2/4\pi e)\chi_i\phi,\qquad\ldots (23)$$

where $\chi_e \approx 2\omega_p^2 / k^2 v_{th}^2 = \omega_{pi}^2 / k^2 c_s^2$,
$\chi_i = -\omega_{pi}^2 / \omega^2, c_s = (T_e / m_i)^{1/2}$,
$\omega_{pi} = (4\pi n_0^0 e^2 / m_i)^{1/2}$ and we have taken $\omega < k v_{th}$. Using $n$ and $n_i$ in the Poisson's equation $\nabla^2\phi = 4\pi e(n - n_i)$, we obtain:

$$\varepsilon\phi = -\chi_e\phi_p,\qquad\ldots (24)$$

where $\varepsilon = 1 + \chi_e + \chi_i$.

The nonlinear density perturbations $n_1^{NL}$ at the lower sideband and $n_2^{NL}$ at the upper sideband, on solving the equation of continuity, $\partial n_1^{NL} / \partial t + (1/2)\nabla.(n\vec{v}_0^*) = 0$, can be written as:

$$n_1^{NL} = n\frac{\vec{k}_1.\vec{v}_0^*}{2\omega_1} = \frac{\vec{k}_1.\vec{v}_0^*}{2\omega_1}\frac{k^2}{4\pi e}\frac{(1+\chi_i)\chi_e}{\varepsilon}\phi_p,\qquad\ldots (25)$$

$$n_2^{NL} = n\frac{\vec{k}_2.\vec{v}_0}{2\omega_2} = \frac{\vec{k}_2.\vec{v}_0}{2\omega_2}\frac{k^2}{4\pi e}\frac{(1+\chi_i)\chi_e}{\varepsilon}\phi_p\qquad\ldots (26)$$

The self-consistent potentials $\phi_1, \phi_2$ produce the linear density perturbations at the sidebands:

$$n_j^L = (k_j^2 / 4\pi e)\chi_{ej}\phi_j, \text{j=1, 2}\qquad\ldots (27)$$

where $\chi_{ej}$ are the electron susceptibilities at sidebands $(\omega_j, \vec{k}_j)$ given by Eq. (20), $\chi_{ej} = -(\omega_p^2 + k_z^2|v_0|^2 / 2)/\omega_j^2$. Using Eqs. (25) – (27) in the Poisson's equation $\nabla^2\phi_j = 4\pi e(n_j^L + n_j^{NL})$, we obtain:

$$\phi_1 = -\frac{4\pi e}{k_1^2\varepsilon_1}n_1^{NL} = \frac{k^2}{k_1^2\varepsilon_1}\frac{\vec{k}_1.\vec{v}_0^*}{2\omega_1}(1+\chi_i)\phi,$$

$$\phi_2 = -\frac{4\pi e}{k_2^2\varepsilon_2}n_2^{NL} = \frac{k^2}{k_2^2\varepsilon_2}\frac{\vec{k}_2.\vec{v}_0}{2\omega_2}(1+\chi_i)\phi,\qquad\ldots (28)$$

where $\varepsilon_j = 1 + \chi_{ej}$.

Using Eqs. (21) and (28) in Eq. (24), we obtain (for $k_0 << k$):

$$\varepsilon = -\chi_e(1+\chi_i)\frac{|\vec{k}.\vec{v}_0|^2}{4}\frac{\Delta}{\omega_0(\Delta^2 - \omega^2)}\qquad\ldots (29)$$

where $\Delta = \omega_0 - (\omega_p^2 + k_z^2|v_0|^2 / 2)^{1/2}$ is a frequency mismatch and $\omega_1 = \omega - \omega_0, \omega_2 = \omega + \omega_0$.

In the limit $\omega_{pi} << \omega << kv_{th}$, we can neglect the ion motion $(\chi_i = 0)$ and Eq. (29) turns out to be:

$$\omega^2 = \Delta^2 + \frac{|\vec{k}.\vec{v}_0|^2}{4(1 + k^2c_s^2 / \omega_{pi}^2)}\frac{\Delta}{\omega_0}\qquad\ldots (30)$$

Instability will occur when $\Delta$ is negative and:

$$|\Delta| < \frac{|\vec{k}.\vec{v}_0|^2}{4(1 + k^2c_s^2 / \omega_{pi}^2)\omega_0}$$

This condition implies:

$$\frac{k_z^2 v_0^2}{4} < \frac{k_z^2|v_0|^2}{4(1 + k^2c_s^2 / \omega_{pi}^2)},$$ which is never satisfied,

hence oscillating two stream instability does not occur, when ion motion is ignored.

By including the ion motion and treating the ions as cold, Eq. (29) can be written as:

$$\omega^4 - \omega^2(\Delta^2 + \omega_{ac}^2 + A_1\Delta) + \Delta^2\omega_{ac}^2 + \omega_{pi}^2 A_1\Delta = 0,$$

$$\ldots (31)$$

where

$$A_1 = \frac{\left(\omega_{pi}^2 / k^2 c_s^2\right)\left(\left|\vec{k} \cdot \vec{v}_0\right|^2 / 4\omega_0\right)}{1 + \omega_{pi}^2 / k^2 c_s^2},$$

$$\omega_{ac}^2 = \frac{k^2 c_s^2}{1 + k^2 c_s^2 / \omega_{pi}^2}.$$

Equation (31) gives a root:

$$\omega^2 = \frac{1}{2}\left[\left(\Delta^2 + \omega_{ac}^2 + A_1\Delta\right) - \sqrt{\left(\Delta^2 + \omega_{ac}^2 + A_1\Delta\right)^2 - 4\left(\Delta^2 \omega_{ac}^2 + \omega_{pi}^2 A_1\Delta\right)}\right]$$
$$\dots (32)$$

Using the values of $\omega_{ac}$, $A_1$ and the approximate value of $\Delta \approx -k_z^2 v_0^2 / 4\omega_p$, one obtains:

$$\frac{\gamma}{\omega_{pi}} = \frac{v_0 G_z}{2v_{th}}\left(\frac{m_i}{m}\right)^{1/4}\left[\begin{array}{c}\left\{\frac{1}{64}\left(\frac{v_0}{v_{th}}\right)^4 \frac{m_i}{m} \frac{G_z^8 (k / k_z)^4}{\left(1 + G_z^2 k^2 / k_z^2\right)^2} + \frac{\left(1 - G_z^2 k^2 / k_z^2\right)}{\left(1 + G_z^2 k^2 / k_z^2\right)}\right\}^{1/2} - \\ \frac{v_0^2}{8v_{th}^2}\left(\frac{m_i}{m}\right)^{1/2} \frac{G_z^4 (k / k_z)^2}{\left(1 + G_z^2 k^2 / k_z^2\right)}\end{array}\right]$$
$$\dots (33)$$

where $G_z = k_z c_s / \omega_{pi}$ is the normalized wave number.

Instability occurs when $\Delta$ is negative and

$$|\Delta| < \frac{\left|\vec{k} \cdot \vec{v}_0\right|^2}{4\omega_0} \frac{\omega_{pi}^2}{k^2 c_s^2}.$$

We have solved Eq. (33) numerically for the following parameters: $m_i / m = 2000$, $|v_0|^2 / v_{th}^2 = 10$, $k / k_z = 1, 2$. In Fig. 3, we have plotted the variation of normalized growth rate, $\gamma / \omega_{pi}$, with the normalized wave number, $k_z c_s / \omega_{pi}$ for $k / k_z = 1, 2$. In both cases, the growth rate increases with $k_z c_s / \omega_{pi}$, acquires a maximum and then falls off. The maximum value of growth rate is $2.4\,\omega_{pi}$ in the case of $k / k_z = 1$. However, the growth rate shows a maximum, $\gamma = 1.6\,\omega_{pi}$ at $k_z c_s / \omega_{pi} = 0.22$ for the case of $k / k_z = 2$ (i.e., when $\vec{k}$ is at $60^0$ to the direction of $\vec{E}_0$). We have also plotted the normalized growth rate with normalized parallel wave number in
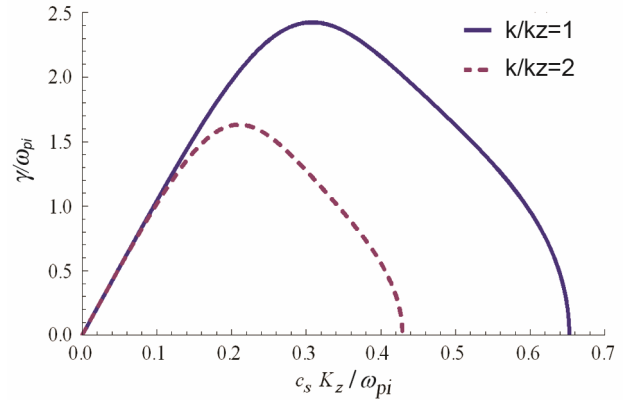


Fig. 3 — Variation in normalized growth rate, $\gamma / \omega_{pi}$, as a function of $k_z c_s / \omega_{pi}$ for $m_i / m = 2000$, $|v_0|^2 / v_{th}^2 = 10$ at $k / k_z = 1, 2$
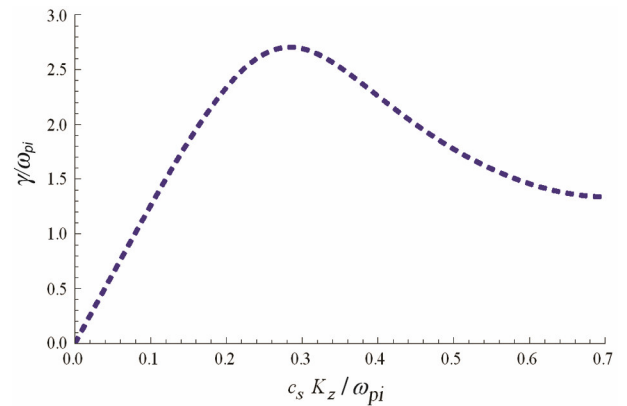


Fig. 4 — Variation in normalized growth rate, $\gamma / \omega_{pi}$, as a function of $k_z c_s / \omega_{pi}$ for $m_i / m = 4000$, $|v_0|^2 / v_{th}^2 = 10$ at $k / k_z = 1$

Fig. 4 for $m_i / m = 4000$ and $k / k_z = 1$. It is obvious from the graph that growth rate attains a maxima at larger value of $2.7\,\omega_{pi}$ as compared to $2.4\,\omega_{pi}$ in the case of $m_i / m = 2000$. It means there is a significant dependence of growth rate on the ratio of ion mass to electron mass.

**4 Conclusions**

A large amplitude long wavelength plasma wave undergoes oscillating two stream instability on time scale of the order of ion plasma period and ion motion is mandatory for its growth. The pump plasma wave

strongly modifies the short wavelength plasma wave eigen modes. As a consequence, the frequency mismatch between the pump and short wavelength sidebands is huge and it suppresses the growth rate of the parametric instability. If one ignores the ion motion, there is no parameter space where OTSI could occur. With the inclusion of ion motion, there exists a narrow parameter regime where OTSI can occur. The growth rate is comparable to ion plasma frequency. For $\left| v_0 \right|^2 / v_{th}^2 = 10$, the growth rate is

$\gamma = 2.4 \omega_{pi}$ for $\vec{k} \| \vec{E}$ and $\gamma \sim 1.6 \omega_{pi}$ for $\vec{k}$ at 60° to the electric field of the pump plasma wave. The growth rate is also dependent on the ratio of mass of ion to electron. As the ratio of ion to electron mass increases, the growth rate increases.

The present treatment is limited to homogenous plasma. The plasma inhomogeneity can localize the region of parametric coupling and lower the growth rate of OTSI.

## Acknowledgment

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