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Viscous dissipation and Joule heating effects on an unsteady magnetohydrodynamic flow over a linearly stretching permeable surface with uniform wall temperature

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An analysis has been presented to describe the effects of viscous dissipation and Joule heating on an unsteady laminar two-dimensional flow of a viscous incompressible electrically conducting fluid over a stretching permeable surface in the presence of a uniform transverse magnetic field. Similarity solutions for the problem have been formulated and reduced nonlinear ordinary differential equations have been solved numerically using fourth order Runge-Kutta method with shooting technique. Influences of various parameters, namely, mass transfer parameter, unsteadiness parameter, magnetic parameter, Prandtl number and Eckert number on velocity and temperature distributions have been plotted graphically while skin-friction coefficient and Nusselt number have been shown numerically. A comparison of the obtained numerical results has been made with previously published results for non-magnetic case.

Keywords: Viscous dissipation, Joule heating, Unsteady flow, Magnetohydrodynamic flow, Stretching surface, Permeable surface

1 Introduction

Owing to the numerous applications in industrial manufacturing, modern metallurgical and metal-working processes such as hot rolling, glass blowing, paper production, wire drawing, drawing of plastic films, metal spinning, extrusion of plastic sheets, liquid composite moulding metal and polymer extrusion etc, the study of magnetohydrodynamic (MHD) flow of an electrically conducting fluid past a heated surface has attracted considerable interest of many researchers during the past few decades. Sakiadis[1] was the first who obtained boundary layer flow over a continuous solid surface moving with constant speed. Further, Erickson *et al.*[2] extended this problem and included the wall suction or blowing and investigated its effects on the heat and mass transfer in the boundary layer. Crane[3] studied the steady two dimensional flow caused by a stretching sheet whose velocity varies linearly with the distance from a fixed point on the sheet, and found the exact solution for the flow field. The effects of heat and mass transfer for steady and unsteady flow past a stretching sheet have been presented by several researchers[4-16] in the presence of different physical parameters.

The problem of steady and unsteady laminar flow over a permeable surface has long been a major subject in heat transfer due to its importance from both theoretical and practical viewpoints and has been extensively studied. It also has many applications in engineering and technological processes, such as petroleum industries, ground water flows, extrusion of a polymer sheet from a dye and boundary layer control. Pursuing the pioneering studies of Beavers and Joseph[17], the flow over a permeable surface has been investigated[18-20]. Recently, many authors[21-24] studied the flow and heat transfer over permeable surface in numerous cases. It has also been reviewed in books[25-28].

Fluid properties of various manufacturing processes desired for better outcome mainly depend on two aspects, one is the rate of stretching and other is the cooling liquid used. Sometimes, rapid stretching of the surface results in sudden solidification, which destroys some expected properties of the outcomes, so an extreme care has to given to control the rate of stretching. The use of electrically conducting fluid and applications of magnetic field can control the rate of cooling and the desired properties of the end product. The magnetic field has been used in the process of purification of molten metal from non-metallic inclusions. The study of MHD flow for an electrically conducting fluid past a heated surface has attracted a lot of attention in view of its important applications in many engineering problems such as

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plasma studies, foodstuff processing, solidification of liquid crystals, cooling of nuclear reactors, exotic lubricants and suspension solutions, the boundary layer control in aerodynamics, MHD power generators, MHD flight and in the field of planetary magnetosphere. Andersson[29] presented an exact analytical solution of the MHD flow of Walters liquid B past a stretching sheet. Further, Vajravelu and Nayfeh[30] studied about hydromagnetic flow of a dusty fluid over a stretching surface. Later, several researchers[31-36] have focussed their attention to the various aspects of the problem of heat transfer and hydromagnetic flow. Recently, such problems have been investigated either analytically or numerically by Chaudhary and Kumar[37] and Olajuwon and Oahimire[38].

Although viscous dissipation and Joule heating effects is of utmost importance in the various technological processes especially in nuclear physics and electronics, these effects are neglected in all above studies. Viscous dissipation plays an important role in the natural convection flow when the flow field is of extreme size or in high gravity, and characterized by the Eckert number. On the other hand Joule heating plays a vital role in nuclear engineering in connection with the cooling of reactors and it is characterized by the product of the magnetic parameter and the Eckert number in the energy equation. In electronics and physics, Joule heating is used to increase the temperature of a conductor which opposes the electric current passing through it. Hossain[39] has reported the combine effects of viscous dissipation and Joule heating on free convection flow with variable plate temperature. Later many researchers[40-46] presented the influences of viscous dissipation and Joule heating on heat transfer problems.

A quick review of literature shows that, in spite of numerous studies on the stretching surface and permeable surface, the effects of viscous dissipation and Joule heating on unsteady hydromagnetic flow over stretching permeable surface with uniform wall temperature is not yet available. Therefore, the aim of present paper is to extend the work of Ishak *et al.*[47] for electrically conducting fluid in the presence of a uniform transverse magnetic field.

2 Mathematical Model

Consider an unsteady two dimensional boundary layer flow of an incompressible electrically conducting fluid over a permeable surface coinciding with the plane $y = 0$, the flow being confined to $y > 0$ The x – axis is chosen along the sheet, and a uniform magnetic field B_0 is imposed along y –axis (Fig. 1). The continuous stretching sheet is assumed to have the velocity $U_w = \frac{ax}{1-ct}$, the transpiration velocity through the permeable wall is V_w with injection and suction for $\pm V_w>0$ and the temperature $T_w = T_\infty + \frac{b}{a} U_w$, where a,b and c are constants with $a>0,\ b\geq 0,\ c\geq 0$ and also $c<\frac{1}{t}$, t is the time and T_∞ is the temperature of the fluid far away from the sheet. Under the boundary layer approximations, the unsteady two-dimensional boundary layer equations can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{... (1)}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_e B_0^{\,2}}{\rho} u \qquad \text{... (2)}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma_e B_0^{\,2}}{\rho C_p} u^2 \qquad \text{... (3)}$$

subject to the boundary conditions:

$$\begin{aligned} & y=0:\ \ u=U_w(x,t),\ v=V_w(x,t),\ T=T_w(x,t) \\ & y\to\infty : u\to 0,\ T\to T_\infty \end{aligned} \qquad \text{... (4)}$$

where u and v are the velocity components in the x and y directions, respectively, υ is the kinematic viscosity, σ_e is the electrical conductivity, ρ is the fluid density, T is the temperature of the fluid, α is the thermal diffusivity, μ is the coefficient of viscosity and C_p is the specific heat at constant pressure.

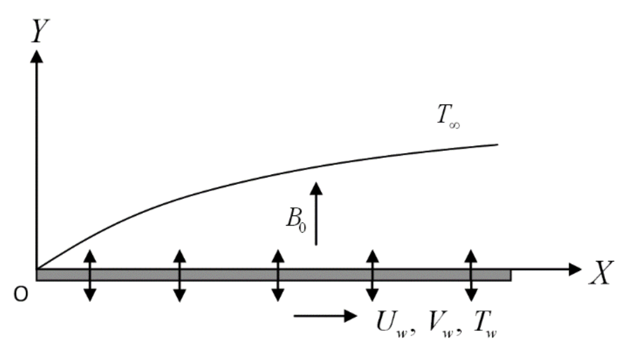

Fig. 1 — Flow geometry and coordinate system.

Moreover, in the energy Eq. (3), the second and third terms on the right hand side signifies the viscous dissipation and the Joule heating, respectively.

3 Similarity Transformations

Using the physical stream function $\psi(x, y, t)$, the continuity Eq. (1) is identically satisfied:

$$u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x} \qquad \text{... (5)}$$

The mathematical analysis of the problem is simplified by introducing the following dimensionless coordinates (Ishak *et al.*[47]):

$$\psi = \sqrt{\upsilon x U_{\mathrm{w}}}\, f(\eta) \qquad \text{... (6)}$$

$$\eta = \sqrt{\frac{U_{\mathrm{w}}}{\upsilon x}}\, y \qquad \text{... (7)}$$

$$T = T_\infty + \frac{b}{a} U_{\mathrm{w}} \theta(\eta) \qquad \text{... (8)}$$

where $f(\eta)$ is the dimensionless stream function, η is the similarity variable, y is the coordinate measured along normal to the stretching surface and $\theta(\eta)$ is the dimensionless temperature. Therefore, using the Eqs (5) to (8), the governing boundary layer Eqs (2) and (3) can be written in a non-dimensional form as:

$$f''' + ff'' - A\left(\frac{1}{2}\eta f'' + f'\right) - f'^2 - Mf' = 0 \qquad \text{... (9)}$$

$$\frac{1}{\Pr}\theta'' + f\theta' - A\left(\frac{1}{2}\eta\theta' + \theta\right) - f\theta + Ec\left(f''^2 + Mf'^2\right) = 0 \qquad \text{... (10)}$$

with the following boundary conditions:

$$\begin{aligned} &\eta = 0: \quad f = f_0,\ f' = 1,\ \theta = 1 \\ &\eta \to \infty: f' \to 0,\ \theta \to 0 \end{aligned} \qquad \text{... (11)}$$

where primes denote differentiation with respect to η. $A = \frac{c}{a}$ is the unsteadiness parameter, $M = \frac{\sigma_{\mathrm{e}} B_0^{\ 2} \upsilon \mathrm{Re}_{\mathrm{x}}}{\rho U_{\mathrm{w}}^2}$ is the magnetic parameter, $Re_{\mathrm{x}} = \frac{U_{\mathrm{w}} x}{\upsilon}$ is the local Reynolds number, $\Pr = \frac{\upsilon}{\alpha}$ is the Prandtl number, $Ec = \frac{U_{\mathrm{w}}^2}{C_{\mathrm{p}}(T_{\mathrm{w}} - T_\infty)}$ is the Eckert number and $f_0 = -\frac{V_{\mathrm{w}}}{U_{\mathrm{w}}}\sqrt{Re_{\mathrm{x}}}$ is the mass transfer parameter.

4 Numerical Method for Solution

The numerical solutions of the Eqs (9) and (10) along with the boundary conditions (Eq. (11)) are solved by converting the boundary value problem (BVP) into initial value problem (IVP). Introducing the new set of dependent variables w_1, w_2, w_3, p_1 and P_2, the following simultaneous linear equations of first order are obtained:

$$w_1' = w_2 \qquad \text{... (12)}$$

$$w_2' = w_3 \qquad \text{... (13)}$$

$$w_3' = -\left[w_1 w_3 - A\left(\frac{1}{2}\eta w_3 + w_2\right) - w_2^2 - M w_2\right] \qquad \text{... (14)}$$

and

$$p_1' = p_2 \qquad \text{... (15)}$$

$$p_2' = -\Pr\left[w_1 p_2 - A\left(\frac{1}{2}\eta p_2 + p_1\right) - w_2 p_1 + Ec\left(w_3^2 + M w_2^2\right)\right] \qquad \text{... (16)}$$

with the boundary conditions:

$$\begin{aligned} &\eta = 0: \quad w_1 = f_0,\ w_2 = 1,\ p_1 = 1 \\ &\eta \to \infty: w_2 \to 0,\ p_1 \to 0 \end{aligned} \qquad \text{... (17)}$$

where $w_1 = f$ and $p_1 = \theta$.

In order to solve Eqs (14) and (16) subject to the boundary conditions Eq. (17) as an IVP, the values for $w_3(0)$ and $p_2(0)$ are required but no such values are given at the boundary. So the suitable estimated values for $w_3(0)$ and $p_2(0)$ are chosen and the fourth order Runge-Kutta method along with shooting technique is applied with step size $\Delta\eta = 0.001$ to obtain the solution. Comparing the calculated values for w_2 and p_1 for various values of different parameters at the far field boundary condition $\eta \to \infty = 6$ (say) with the given boundary conditions $w_2(6) \to 0$ and $p_1(6) \to 0$, the values of $w_3(0)$ and $p_2(0)$ are adjusted from the guess values to give a better approximation for the solution. The process is repeated until the results accuracy of the 10^{-6} as the criterion of convergence.

5 Skin Friction and Nusselt Number

The physical quantities of primary interest are the local skin-friction coefficient C_f and the local Nusselt number Nu_x, which are defined as:

$$C_f = \frac{\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}}{\frac{\rho U_w^{\,2}}{2}} \quad \text{... (18)}$$

$$Nu_x = \frac{-x\left(\frac{\partial T}{\partial y}\right)_{y=0}}{T_w - T_\infty} \quad \text{... (19)}$$

Using the Eqs (5) to (8), Eqs (18) and (19) are converted as:

$$f''(0) = \frac{1}{2} C_f \sqrt{Re_x} \quad \text{... (20)}$$

$$\theta'(0) = -\frac{Nu_x}{\sqrt{Re_x}} \quad \text{... (21)}$$

6 Computational Results and Discussion

In order to get clear insight of the physical problem, numerical results of velocity $f'(\eta)$ and temperature $\theta(\eta)$ profiles for various parameters such as the mass transfer parameter f_0, the unsteadiness parameter A, the magnetic parameter M, the Prandtl number Pr and the Eckert number Ec are illustrated with the help of graphs. Moreover the computations of the functions $f''(0)$ and $\theta'(0)$ which are proportional to local skin friction coefficient C_f and local Nusselt number Nu_x, respectively, have been carried out through tables.

Effects of the mass transfer parameter f_0 on the velocity $f'(\eta)$ and the temperature $\theta(\eta)$ profiles have been plotted in Figs 2 and 3, respectively, while the other parameters are constant. These figures show that the velocity and the temperature decrease with the increasing values of the mass transfer parameter f_0. Practically, applying suction at the boundary surface causes to draw some amount of the fluid into the surface, and consequently momentum and thermal

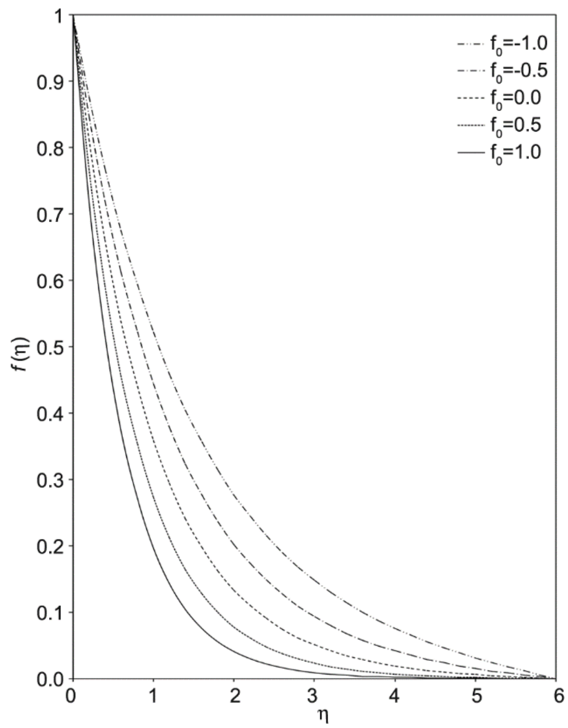

Fig. 2 — Variation of velocity $f'(\eta)$ with η for several values of f_0 when $A = 0.1$ and $M = 0.01$.

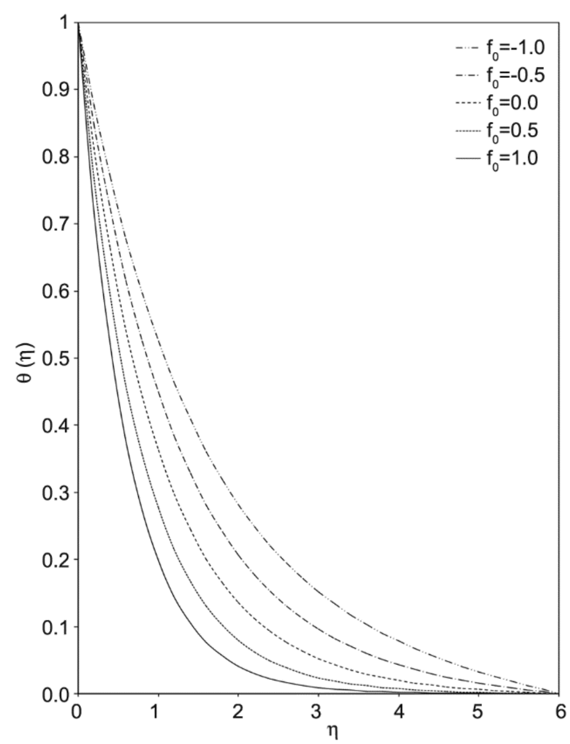

Fig. 3 — Variation of temperature $\theta(\eta)$ with η for several values of f_0 when $A = 0.1$, $M = 0.01$, $Pr = 1.0$ and $Ec = 0.01$.

boundary layer thickness get thinner. Thus the actual effect of the mass transfer parameter is to make the velocity and temperature distribution more uniform within the boundary layer. So, it can be effectively used for the fast cooling of the sheet.

The effects of the unsteadiness parameter A on the fluid flow $f'(\eta)$ and the temperature $\theta(\eta)$ distribution have been studied taking other parameters constant and the results are represented in Figs 4 and 5, respectively. It can be observed that there is a special point near $\eta \approx 2$ called 'crossing over point', and the velocity and the temperature profiles have completely conflicting behaviour before and after that point. Further, it is evident that the velocity and the temperature decrease faster with the increasing values of the unsteadiness parameter A while the reverse phenomenon occurs for $\eta > 2$. This is because of the thermal boundary layer thickness rapidly decreases due to increase in unsteadiness before that point but ultimately it increases the thickness of boundary layer.

Figures 6 and 7 depict the velocity $f'(\eta)$ and temperature $\theta(\eta)$ profiles for different values of the magnetic parameter M, respectively, keeping other parameters constant. From these figures it is evident that the velocity decreases with the increasing values of the magnetic parameter M but the reverse is true for the temperature distribution. This can be explained by the fact that the application of a uniform magnetic field normal to the flow direction gives rise to a force which acts in the negative direction of flow. This

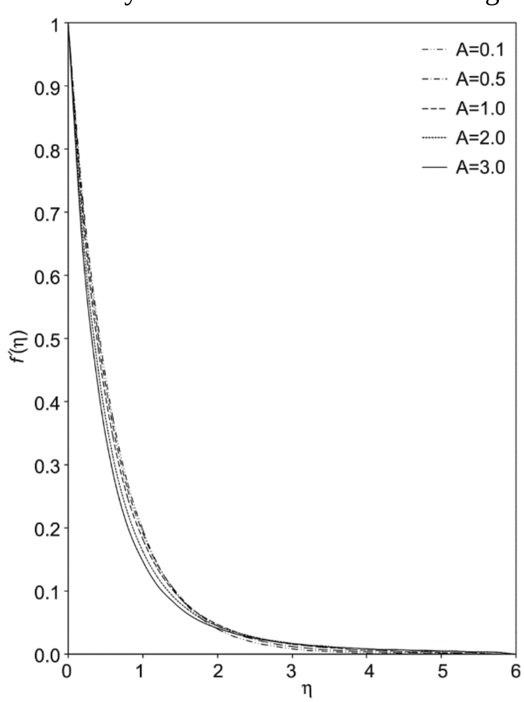

Fig. 4 — Variation of velocity $f'(\eta)$ with η for several values of A when $f_0 = 1.0$ and $M = 0.01$.

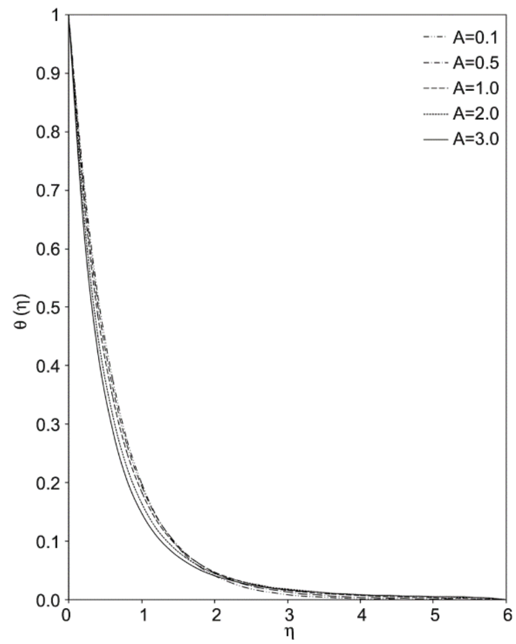

Fig. 5 — Variation of temperature $\theta(\eta)$ with η for several values of A when $f_0 = 1.0$, $M = 0.01$, $Pr = 1.0$ and $Ec = 0.01$.

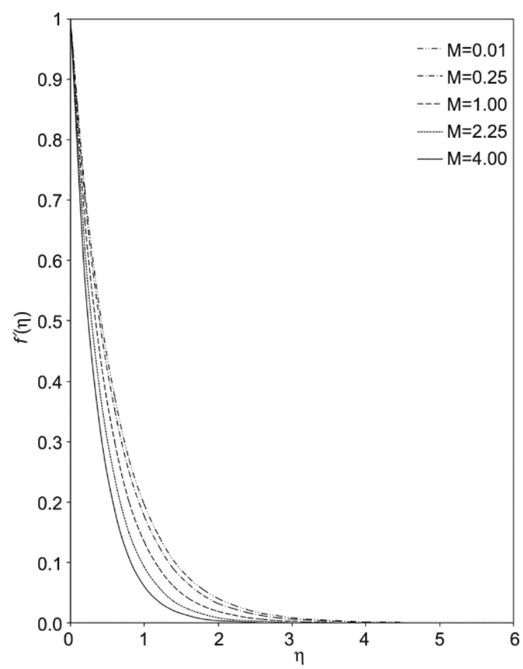

Fig. 6 — Variation of velocity $f'(\eta)$ with η for several values of M when $f_0 = 1.0$, and $A = 0.1$, .

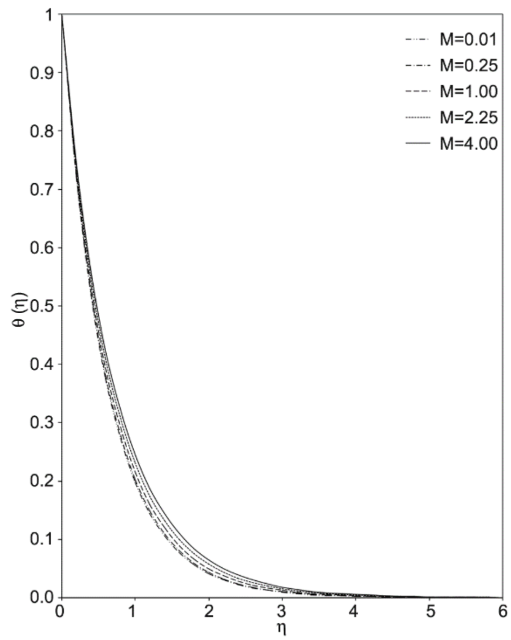

Fig. 7 — Variation of temperature $\theta(\eta)$ with η for several values of M when $f_0 = 1.0$, $A = 0.1$, $Pr = 1.0$ and $Ec = 0.01$.

force is known as Lorentz force and it tends to slow down the movement of the fluid along the surface and increases its temperature.

The influence of several values of the Prandtl number Pr on the temperature $\theta(\eta)$ distribution is displayed in Fig. 8 when the other parameters are kept constant. It can be seen that the increase in the Prandtl number Pr causes the decrease in the temperature profile. From a physical point of view, the fluid with a higher value of the Prandtl number posses a large heat capacity, and hence intensifies the heat transfer while, a smaller Prandtl number increases the thermal conductivity and therefore heat is able to diffuse away from the surface.

In Fig. 9, the consequences of the variation in the Eckert number Ec on the temperature $\theta(\eta)$ profiles are shown taking other parameters constant. It is noticed that the Eckert number Ec has an increasing effect on the temperature profiles. This is a consequence of the fact that for higher values of the Eckert number, there is significant generation of heat due to viscous dissipation near the sheet. Therefore, viscous dissipation in a flow through permeable surface is beneficial for gaining the temperature.

Finally, Table 1 shows the effects of the mass transfer parameter f_0, the unsteadiness parameter A and the magnetic parameter M on the local skin-

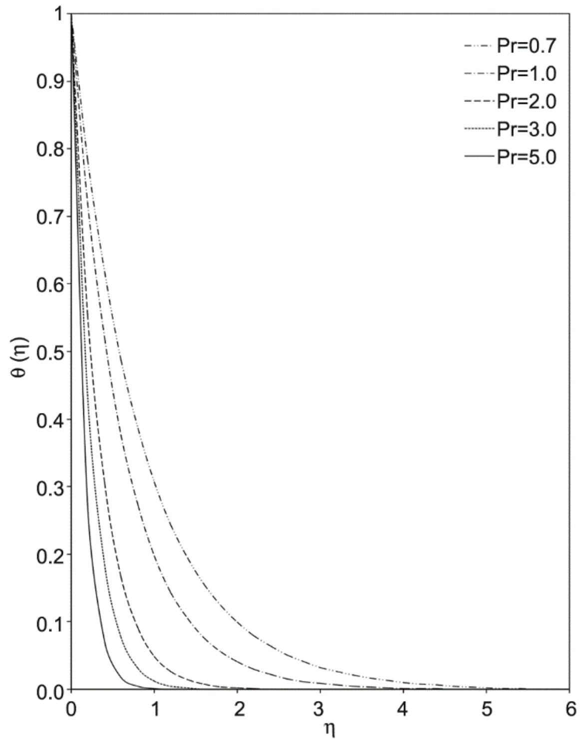

Fig. 8 — Variation of temperature $\theta(\eta)$ with η for several values of Pr when $f_0 = 1.0$, $A = 0.1$, $M = 0.01$, and $Ec = 0.01$.

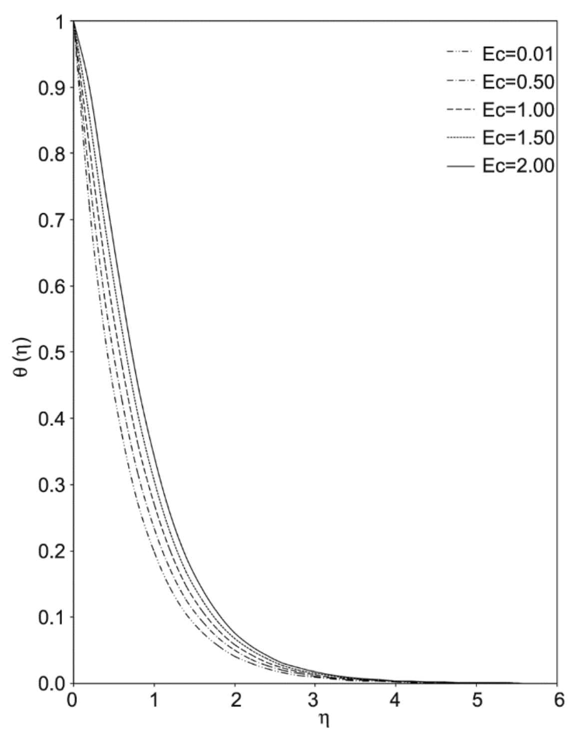

Fig. 9 — Variation of temperature $\theta(\eta)$ with η for several values of Ec when $f_0 = 1.0$, $A = 0.1$, $M = 0.01$, and $Pr = 1.0$.

Table 1 — Computed values of $-f''(0)$ for various values of f_0, A and M.

f_0	A	M	Exact solutions	Present results
-1.0	0.1	0.01	0.6591449	0.65914
-0.5			0.8223909	0.82239
0.0			1.039469	1.03947
0.5			1.316099	1.31610
1.0			1.648648	1.64865
1.0	0.5	0.01	1.75274	1.75274
	1.0		1.88033775	1.88034
	2.0		2.11780217685	2.11780
	3.0		2.330895321488422	2.33090
1.0	0.1	0.25	1.749477	1.74948
		1.00	2.0213458	2.02135
		2.25	2.38866246	2.38866
		4.00	2.806244989	2.80624

friction coefficient $f''(0)$. It is seen that the local skin friction coefficient $f''(0)$ decreases with the increasing values of the mass transfer parameter f_0, the unsteadiness parameter A and the magnetic parameter M, when other parameters kept constant. Moreover, it is found that the values of the local skin friction coefficient $f''(0)$ are always negative for all the values of physical parameters mentioned. From physical point of view, positive sign of skin friction coefficient means the fluid exerts a drag force on the surface and negative sign means the opposite.

The values for the local Nusselt number $\theta'(0)$ are depicted in Table 2 for several values of the mass transfer parameter f_0, the unsteadiness parameter A, the magnetic parameter M, the Prandtl number Pr and the Eckert number Ec. It is noteworthy that the local Nusselt number $\theta'(0)$ decreases with the increasing values of the mass transfer parameter f_0, the unsteadiness parameter A and the Prandtl number Pr but an opposite behaviour is noted in case of the magnetic parameter M and the Eckert number Ec, taking other parameters constant. Further it is quite evident that the values of the local Nusselt number $\theta'(0)$ are always negative for all the values of physical parameters considered. Physically, negative sign of Nusselt number implies that there is a heat flow from the sheet.

From Table 3, the values of the local Nusselt number are compared with some already published works of Ali[7] and Ishak *et al.*[47] in the absence of magnetic and electric fields, which validate the present results. From the table it can be seen that the results are in an excellent agreement with previous researchers.

Table 2 — Computed values of $-\theta'(0)$ for various values of f_0, A, M, Pr and Ec.

f_0	A	M	Pr	Ec	Exact solutions	Present results
-1.0	0.1	0.01	1.0	0.01	0.652202	0.65220
-0.5					0.814011	0.81401
0.0					1.029706	1.02971
0.5					1.305087	1.30509
1.0					1.636402	1.63640
1.0	0.5	0.01	1.0	0.01	1.7408	1.74080
	1.0				1.86872986	1.86873
	2.0				2.10649670829	2.10650
	3.0				2.319587414112824	2.31959
1.0	0.1	0.25	1.0	0.01	1.62098	1.62098
		1.00			1.58185	1.58185
		2.25			1.534318	1.53432
		4.00			1.486899	1.48690
1.0	0.1	0.01	0.7	0.01	1.221658	1.22166
			2.0		2.88804	2.88804
			3.0		4.04265	4.04265
			5.0		6.23198	6.23198
1.0	0.1	0.01	1.0	0.50	1.283	1.28300
				1.00	0.92238	0.92238
				1.50	0.56177	0.56177
				2.00	0.20115	0.20115

Table 3 — Comparison of $-\theta'(0)$ for various values of f_0, A and Pr with $M = Ec = 0.00$.

f_0	A	Pr	Literature[7]	Literature[47]	Present results
-1.5	0.0	0.72		0.4570	0.45880
		1.00		0.5000	0.50027
0.0	0.0	0.01		0.0197	0.17742
		0.72	0.8058	0.8086	0.81207
		1.00	0.9961	1.0000	1.00048
		3.00	1.9144	1.9237	1.92345
1.5	0.0	0.72		1.4944	1.49457
		1.00		2.0000	2.00001
-1.5	1.0	1.00		0.8095	0.80957
0.0				1.3205	1.32064
1.5				2.2224	2.22255

7 Conclusions

In the present investigation, an unsteady MHD flow past a stretching surface with viscous dissipation and Joule heating is analyzed. Governing equations are converted into non-dimensional by introducing similarity transformations and hence solved by Runge-Kutta fourth order method with the help of shooting technique. Further, the effects of the various pertinent parameters on the velocity, temperature, skin friction coefficient and Nusselt number are illustrated and discussed. The main observations of the present study are as follows:

(i) The fluid velocity, the thermal boundary layer thickness, the surface gradient and the rate of heat transfer decrease as the mass transfer parameter and the unsteadiness parameter increase while the reverse behaviour is noted after the 'crossing over point' for velocity as well as thermal boundary layer thickness for the unsteadiness parameter.

(ii) In case of increase in the magnetic parameter the momentum boundary layer thickness as well as the surface gradient decreases while the opposite phenomenon occurs for the thermal boundary layer thickness and the rate of heat transfer.

(iii) The thermal boundary layer thickness and the rate of heat transfer decrease with the increase in the Prandtl number but the effects of the Eckert number are quite opposite.

Nomenclature

a	Positive constant
A	Unsteadiness parameter
b	Non-negative constant
B_0	Uniform magnetic field
c	Stretching rate
C_f	Local skin-friction coefficient
C_p	Specific heat at constant pressure
Ec	Eckert number
f	Dimensionless stream function
f_0	Mass transfer parameter
M	Magnetic parameter
Nu_x	Local Nusselt number
Pr	Prandtl number
Re_x	Local Reynolds number
T	Temperature of the fluid
t	Time
T_w	Surface temperature
T_∞	Free stream temperature
U_w	Stretching velocity
u	Velocity component in the $x-$ direction
v	Velocity component in the $y-$ direction
V_w	Transpiration velocity through the permeable wall
x	Along the stretching surface distance
y	Normal distance

Greek symbols

α	Thermal diffusivity
η	Similarity variable
θ	Dimensionless temperature
μ	Coefficient of viscosity
υ	Kinematic viscosity
ρ	Density
σ_e	Electrical conductivity
ψ	Stream function

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