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Modulational instability of bright envelope soliton and rogue waves in ultra-relativistic degenerate dense electron-ion-positron plasma

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Modulational instability and bright envelope solitons of ion acoustic waves in dense plasma consisting of ultra-relativistic degenerate electrons and positrons, cold and mobile inertial ions, and negatively charged static dust particles have been investigated using Fried and Ichikawa method. Nonlinear Schrodinger equation has been derived and the growth rate of modulationally unstable ion acoustic wave in such plasma is discussed. It has been found that ion acoustic wave will be always modulationally unstable for all possible values of density of positrons, electrons and charged dust particle but there is no instability of the wave for any value of plasma parameters in such plasma. The solutions of ion acoustic envelope- solitons and rogue waves are obtained from the Nonlinear Schrodinger equation. The theoretical results have been analyzed numerically for different values of plasma parameters and the results are presented graphically. It is found that only bright envelope soliton would be excited in the ultra-relativistic plasma. Our results are new and may be applicable for the study of nonlinear wave processes in relativistic degenerate dense plasmas of astrophysical objects, namely, in white dwarfs and neutron stars.

Keywords: Ultra-Relativistic degenerate, Nonlinear schrodinger equation, Modulational instability, Envelope soliton, White dwarfs, Neutron stars

1 Introduction

Propagation of waves in relativistic plasma gives some fascinating results which are helpful to understand various phenomena observed in laser-plasma experiments and in the plasma of astrophysical objects[1-5]. Nonlinear interaction of waves and particles in plasma waves give rise to frequency shift, wave precession, solitary waves, modulational instability etc. For the studies of solitary waves and modulational instability of electrostatic waves in relativistic plasma Das-Paul model[6] have been used by various authors[7-12] incorporating different parameters in non-degenerate plasma. On the other hand, in degenerate plasma considering relativistic effect, nonlinear propagation waves have been studied by a number of authors. Sahu[13] have studied small amplitude quantum ion-acoustic solitary waves in an unmagnetized two-species relativistic quantum plasma system, comprised of electrons and ions using reductive perturbation method. The properties of quantum ion-acoustic solitary waves, obtained from the deformed KdV equation, are studied taking into account the quantum mechanical effects in the weak relativistic limit. It is found that relativistic effects significantly modify the properties of quantum ion-acoustic waves. Later, Ghosh *et al.*[14] have investigated relativistic effects on the linear and nonlinear properties of electron plasma waves following Das-Paul[6] and using one-dimensional quantum hydrodynamic (QHD) model for two-component electron–ion dense quantum plasma. Using standard perturbation technique, nonlinear Schrodinger (NLS) equation containing both relativistic and quantum effects has been derived and the modulational instability of wave has been discussed. Through numerical calculations they have also shown that relativistic effects significantly change the linear dispersion character of the wave. Subsequently, Chandra and Ghosh[15] have investigated modulational instability of electron plasma waves by using QHD model for quantum plasma at finite temperature by deriving NLS equation with relativistic effects. It has been shown that the electron degeneracy parameter and streaming velocity significantly affect the linear and non-linear properties of electron plasma waves in finite temperature quantum plasma. Recently, McKerr *et al.*[16] have considered a self-consistent relativistic two-fluid model for one-dimensional electron-ion

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plasma dynamics and employed a multiple scales perturbation technique, leading to an evolution equation for the wave envelope, in the form of NLS-type equation. The inclusion of relativistic effects is shown to introduce density-dependent factors, not present in the non-relativistic case in the conditions for modulational instability of wave. They have discussed the role of relativistic effects on the linear dispersion laws and on the envelope soliton solutions of the NLS equation.

But, in relativistically-degenerate dense plasma, propagation of waves must be studied following the works of Chandrashekhar[17,18] instead of Das-Paul[6]. The equation of state for degenerate electrons in the interstellar compact objects has been mathematically explained by Chandrasekhar[17,18] for two limits, namely, non-relativistic and ultra-relativistic limits. The degenerate electron equation of state suggested by Chandrasekhar is $P_e \propto n_e^{5/3}$ for the nonrelativistic limit, and $P_e \propto n_e^{4/3}$ for the ultrarelativistic limit, where P_e is the degenerate electron pressure and n_e is the degenerate electron number density. Shah *et al.*[19] have undertaken investigation on the effect of trapping on the formation of solitary structures in relativistic degenerate (RD) plasmas. They have used the relativistic Fermi-Dirac distribution to describe the dynamics of the degenerate trapped electrons by solving the kinetic equation. The Sagdeev potential approach has been employed to obtain the arbitrary amplitude solitary structures both when the plasma has been considered cold and when small temperature effects have been taken into account. The theoretical results obtained have been analyzed numerically for different parameter values, and the results have been presented graphically. Masood and Eliasson[20] have studied electrostatic solitary waves in quantum plasma with relativistically degenerate electrons and cold ions. The inertia is given by the ion mass while the restoring force is provided by the relativistic electron degeneracy pressure, and the dispersion is due to the deviation from charge neutrality. A nonlinear Korteweg–de Vries (K-dV) equation is derived for small but finite amplitude waves and is used to study the properties of localized ion acoustic solitons for parameters relevant for dense astrophysical objects such as white dwarf stars. Later, Chandra *et al.*[21] have studied electron-acoustic solitary waves in relativistically degenerate quantum plasma with two-temperature electrons using the QHD model and degenerate electron pressure $P_e \propto n_e^{4/3}$. They have shown that degeneracy parameter significantly influences the conditions of formation and properties of solitary structures. Recently, Chandra *et al.*[22] have investigated the nonplanar ion-acoustic waves in relativistically degenerate quantum plasma using QHD model and deriving a nonlinear Spherical Kadomtsev–Petviashvili (SKP) equation using the standard reductive perturbation method. It has been found that the electron degeneracy parameter significantly affects the linear and nonlinear properties of ion-acoustic waves in quantum plasma.

It is important to be mentioned that for the study of nonlinear waves in compact astrophysical objects Chandrasekhar Model is more applicable in ultra-relativistic degenerated (URD) dense plasma. For this reason, planner and nonplanner electrostatic solitary waves have been studied by Mamun and Shukla[23] using reductive perturbation method in a ultra-relativistic degenerate dense plasma, which is relevant to interstellar spherical compact objects like white dwarfs. The degenerate dense plasma sphere has found to support spherical solitary structures whose basic features (amplitude, width, speed, *etc.*) depend only on the plasma number density. It has been shown here that the amplitude, width, and speed increase with the increase of the plasma number density. Later, Mamun and Shukla[24] have investigated arbitrary amplitude solitary waves and double layers in a ultra-relativistic degenerate dense dusty plasma containing URD ultra-cold electron fluid, inertial ultra-cold ion fluid, and negatively charged static dust by the pseudo-potential approach. Later, Esfandyari-Kalejahi *et al.*[25] have studied arbitrary amplitude ion-acoustic solitary waves using Sagdeev-Potential approach in electron-positron-ion plasma with ultra-relativistic or non-relativistic degenerate electrons and positrons and numerically investigated the matching criteria of existence of such solitary waves. They have shown that the relativistic degeneracy of electrons and positrons has significant effects on the amplitude and the Mach-number range of solitary waves. Recently, Roy *et al*.[26] investigated the nonlinear propagation of waves (specially solitary waves) in an ultra-relativistic degenerate dense plasma containing ultra-relativistic degenerate electrons and positrons, cold, mobile, inertial ions, and negatively charged static dust by the reductive perturbation method. They have derived the linear dispersion relation and the K-dV equation whose numerical solutions are analyzed to identify the basic features of electrostatic solitary structures that may form in such degenerate dense plasma. The existence of solitary structures has been also verified by employing the pseudo-potential method. Considering relativistic and degenerate warm dense electron-positron-ion plasma, Rahman *et al.*[27] have investigated the linear and nonlinear properties of ion acoustic excitations. Adopting a reductive perturbation method, the K-dV equation is derived, which admits a localized wave solution in the form of a small-amplitude weakly super-acoustic pulse-shaped soliton. The analysis is extended to account for arbitrary amplitude solitary waves, by deriving a pseudoenergy-balance like equation, involving a Sagdeev-type pseudo-potential. They have shown that the two approaches agree exactly in the small-amplitude weakly super-acoustic limit. The range of allowed values of the pulse soliton speed (Mach number), wherein solitary waves may exist, is determined. The effects of plasma parameters, namely, the electron relativistic degeneracy parameter, the ion (thermal)-to-the electron (Fermi) temperature ratio, and the positron-to-electron density ratio, on the soliton characteristics and existence domain, have been studied in detail.

On the other hand, in recent years, a new nonlinear phenomena called as rogon or rogue waves has been studied in plasma. The rogue waves are a singular, rare, high-energy event with very high amplitude that carries dramatic impact. It appears in seemingly unconnected systems in the form of oceanic rogue waves, communication systems, stock market crashes, superfluid helium, Bose-Einstein condensates, opposing currents flows, propagation of acoustic-gravity waves in the atmosphere, atmospheric physics, and plasma physics etc. It is to be pointed out that originally this phenomenon has been studied in water waves in sea[28] Rogue waves is the name given by oceanographers to the isolated large amplitude waves, that occur more frequently than expected for normal, Gaussian distributed statistical events. Experiments in a multicomponent plasma with negative ions have, recently, reported the evidence of Peregrine solitons of ion-acoustic waves[29]. Recently, Wang *et al.* [30] have investigated the solitary waves and rogue waves in a plasma featuring Tsallis distribution plasma. Within the region of modulational instability, a random perturbation of the amplitude grows and thus creates rouge waves. The rogue wave (rational solution) of the NLS equation in the unstable region can be obtained following the works of Ankievicz[31].

Since, no investigation has yet been reported on the modulation instability of electrostatic waves in URD dense plasma based on the degenerate electron equation of state $P_j \propto n_j^{\frac{4}{3}}$, we have, in this communication, have studied the modulational instability of ion acoustic wave in an URD dense plasma consisting of cold ions and ultra-relativistic degenerate electrons and positrons with static negatively charged dust particles. For our present study on the modulational instability, we have followed famous work of Fried and Ichikawa[32]. Recently, this method has been used by Paul and Roychowdhury[33] for the study of modulational instability of ion acoustic wave in electron-ion-positron plasma with kappa distributed electrons. From the nonlinear Schrodinger they have obtained two kinds of soliton solution, i) envelope soliton and ii) rational soliton. The stability criteria are studied by varying the positron density, positron temperature and wave number. Moreover, they have found both dark- envelope-soliton and bright-envelope-soliton in such plasma.

2 Governing Equations

We consider four-component plasma consisting of inertialess URD electrons and positrons, cold- mobile-inertial ion fluid, and negatively charged static dust. The degenerate pressure of electron-positron fluid has been expressed in terms of density by using the ultra-relativistic limit of Chandrasekhar[17,18]. The nonlinear dynamics of the electrostatic wave in such URD dense plasma is described by the following equations[23-26]:

$$\frac{\partial n_s}{\partial t} + \frac{\partial}{\partial x}(n_s u_s) = 0 \qquad \text{... (1)}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{\partial \varphi}{\partial x} \qquad \text{... (2)}$$

$$n_e \frac{\partial \varphi}{\partial x} - \frac{3}{4}\beta_e \frac{\partial n_e^{4/3}}{\partial x} = 0 \qquad \text{... (3)}$$

$$n_p \frac{\partial \varphi}{\partial x} + \frac{3}{4}\beta_p \frac{\partial n_p^{4/3}}{\partial x} = 0 \qquad \text{... (4)}$$

$$\frac{\partial^2\varphi}{\partial x^2} = \alpha_e n_e - \alpha_p n_p - n_i + \alpha_d \quad \text{... (5)}$$

where, s = e, p, i denote for electron, positron and ion; velocity u_s is normalized by ion-acoustic speed $C_i = (m_e c^2 / e)^{1/2}$, density n_s is normalized by the equilibrium ion density, the electrostatic potential ϕ is normalized by $m_e c^2/e$, space variable x and time variable t are normalized by $(m_e c^2/4\pi n_{i0} e^2)^{1/2}$ and ion plasma period $(w_{pi})^{-1}$; $w_{pi} = (4\pi e n_{i0}/m_i)^{1/2}$;

$$\begin{aligned}
&\alpha_1 = \lambda_c n_{e0}^{1/3}, \alpha_2 = \lambda_c n_{p0}^{1/3}, \lambda_c = hc/m_e,\\
&\beta_e = K_0\alpha_1, \beta_p = K_0\alpha_2, K_0 = (\hbar / m_e c),\\
&\beta_p = \gamma\beta_e, \gamma = (n_{p0}/n_{e0})^{1/3},\\
&\alpha_e = (n_{e0}/n_{i0}), \alpha_p = (n_{p0}/n_{i0}), \alpha_d = (Z_d n_{d0}/n_{i0}).
\end{aligned} \quad \text{... (6)}$$

n_{e0}, n_{p0}, n_{i0} are the equilibrium densities of electrons, positrons and ions; Z_d is number of negative charges of dust particles. Other parameters have their usual meanings.

The charge neutrality condition is

$\alpha_e + \alpha_d = 1 + \alpha_p$, i.e. $i)\alpha_e = 1 + \alpha_p - \alpha_d$, or $ii)\alpha_p = \alpha_e + \alpha_d - 1.$ ... (7)

3 Linear Dispersion Relation

Now, assuming the usual wave like behaviour of all physical variable ~ exp (i k x – i ω t) we get the linear dispersion relation

$$\omega = k\left[\frac{\beta_e\beta_p}{\beta_e\beta_p k^2 + \alpha_e\beta_p + \alpha_p\beta_e}\right]^{1/2} \quad \text{... (8)}$$

The corresponding group velocity (v_g) is given by

$$V_g = \frac{(\alpha_e\beta_p + \alpha_p\beta_e)}{(\beta_e\beta_p k^2 + \alpha_e\beta_p + \alpha_p\beta)^{3/2}} \quad \text{... (9)}$$

From (7)- (9) it is seen that the dispersion of ion acoustic wave and the phase velocity and the group velocity in ultra-relativistic quantum plasma mainly depends on the positron density.

4 Derivation of Non-Linear Schrodinger equation

To start with we assume the existence of a suitable nonlinear dispersion relation

$$\varepsilon(k,\omega,A) = 0 \quad \text{... (10)}$$

where A is the amplitude. If the initial wave amplitude for the electrostatic potential ϕ is ;

$$\varphi(x,0) = \int dk\varphi_k e^{ikx} + c.c. \quad \text{... (11)}$$

with ϕ_k having a maximum around k = k_0 , then the long time behaviour is given as

$$\varphi(x,t) = \int dk\varphi_k \exp[i(kx - \omega t)] + c.c. \quad \text{... (12)}$$

Fried and Ichikawa [28] assumed that we can continue to use (12) even in the nonlinear dispersion case and we can write

$$\varepsilon(k,\omega,A) = \varepsilon(k,\omega,0) + A^2\frac{\partial\varepsilon}{\partial A^2} + \cdots\cdots = 0 \quad \text{... (13)}$$

For ω we then get

$$\omega = \Omega(k) + MA^2 = \omega_0 + \Gamma \quad \text{... 14(a)}$$

where

$$\Gamma = V_g\hat{k} + V_g'\frac{\hat{k}^2}{2} + MA^2 \quad \text{... 14(b)}$$

and $\Omega(k)$ being the solution of the linear dispersion relation

$$\varepsilon[k,\Omega(k),0] = 0, \quad \text{... 15(a)}$$

and

$$\omega_0 = \Omega(k_0), \quad V_g = \left.\frac{\partial\Omega}{\partial k}\right|_{k=k_0}, \quad \text{... 15(b)}$$

$$V_g' = \left.\frac{\partial^2\Omega}{\partial k^2}\right|_{k=k_0}, \quad M = \left.\frac{\partial\omega}{\partial A^2}\right|_{k=k_0} \quad \text{... 15(c)}$$

with $\hat{k} = k - k_0$

Equations (13) and (14) were used by Fried and Ichikawa[28] for deducing the NLS equation

$$i(\frac{\partial\varphi}{\partial t} + V_g\frac{\partial\varphi}{\partial x}) + P\frac{\partial^2\varphi}{\partial x^2} + Q|\varphi|^2\varphi = 0 \quad \text{... (16)}$$

where,

$$P = \frac{1}{2}\frac{\partial V_g}{\partial k}, \quad \text{... (17)}$$

$$Q = -M = -\frac{\partial \omega}{\partial A^2} \quad \text{... (18)}$$

Now, moving to a frame of reference with velocity V and integrating Eqs.(1)-(3) we get

$$\frac{\partial^2 \varphi}{\partial x^2} = F_1\varphi + F_2\varphi^2 + F_3\varphi^3 + \cdots\cdots \quad \text{... (19)}$$

where

$$F_1 = [\frac{\alpha_e}{\beta_e} + \frac{\alpha_p}{\beta_p} - \frac{1}{V^2}] \quad \text{...20(a)}$$

$$F_2 = [\frac{\alpha_e}{3\beta_e^2} - \frac{\alpha_p}{3\beta_p^2} - \frac{3}{2V^4}] \quad \text{... 20(b)}$$

$$F_3 = [\frac{\alpha_e}{27\beta_e^3} + \frac{\alpha_p}{27\beta_p^3} - \frac{5}{2V^6}] \quad \text{... 20(c)}$$

For solving (19) we now use the Fourier decomposition technique and we set;

$$\varphi = \sum_{-\infty}^{+\infty} \varphi_n e^{inkx} \quad \text{... (21)}$$

Along with $\varphi_n = \varphi_{-n}$.

Substituting (21) in (19) and then equating the coefficient of e^{inkx} we obtain

$$-n^2k^2\varphi_n = F_1\varphi_n + F_2\sum \varphi_{n'}\varphi_{n''} + F_3\sum \varphi_{n'}\varphi_{n''}\varphi_{n'''} + \cdots\cdots\cdots \quad \text{... (22)}$$

Now, we make a perturbation in the amplitude φ_n ;

$$\varphi_n = \sum_{1}^{\infty} \varphi_n^{(p)} \varepsilon^p \quad \text{...(23)}$$

and in the wave number

$$\lambda = -k^2 = \sum_{0}^{\infty} \varepsilon^p \lambda_p \text{ with } \varphi'_{\pm 0} = 1 \quad \text{... (24)}$$

Equating different power of ε , we get

$$\varphi_0^{(2)} = -\frac{[2A_2 - 3(k^2+\alpha_0)^2]}{[A_1 - (k^2+\alpha_0]} \quad \text{... (25)}$$

$$\varphi_{\pm 2}^{(2)} = \frac{1}{6}\frac{[2A_2 - 3(k^2+\alpha_0)^2]}{[A_1 - (k^2+\alpha_0]} \quad \text{... (26)}$$

where,

$$A_1 = \frac{(\alpha_e\beta_p + \alpha_p\beta_e)}{\beta_e\beta_p} \quad \text{...27(a)}$$

$$A_2 = \frac{(\alpha_e\beta_p^2 - \alpha_p\beta_e^2)}{3\beta_e^2\beta_p^2} \quad \text{...27(b)}$$

$$A_3 = \frac{(\alpha_e\beta_p^3 + \alpha_p\beta_e^3)}{27\beta_e^3\beta_p^2} \quad \text{... 27(c)}$$

$$\alpha_0 = (\alpha_e/\beta_e + \alpha_p/\beta_p) \quad \text{... 27(d)}$$

Therefore, the nonlinearity Q in NLS equation (16) becomes

$$Q = \frac{5}{54}\frac{[2(\alpha_e\beta_p{}^2 - \alpha_p\beta_e^2) - 9(k^2\beta_e\beta_p + \alpha_e\beta_p + \alpha_p\beta_e)^2]^2}{k^2\beta_e{}^4\beta_p^4} + \frac{[2(\alpha_e\beta_e{}^3 + \alpha_p\beta_p^3) - 135(k^2\beta_e\beta_p + \alpha_e\beta_p + \alpha_p\beta_e)^3]}{18\beta_e{}^3\beta_p^3} \quad \text{... (28)}$$

Now using $\beta_p = \gamma\beta_e, \gamma = (n_{p0}/n_{e0})^{1/3}$ in the expression of Q we obtain after simplification

$$Q = \frac{5}{54}\frac{[2(\gamma^2\alpha_e - \alpha_p) - 9(\gamma k^2 + \gamma\alpha_e + \alpha_p)^2]^2}{\gamma^4\beta_e{}^4k^2} + \frac{[2(\gamma^3\alpha_e - \alpha_p) - 135(\gamma k^2 + \gamma\alpha_e + \alpha_p)^3]}{18\gamma^3\beta_e^3} \quad \text{... (29)}$$

Again after using (9) in (17) we obtain the wave dispersion ***P*** as

$$P = \frac{1}{2}\frac{\partial V_g}{\partial k} = -\frac{3}{2}\frac{k(\gamma\alpha_e + \alpha_p)(\gamma\beta_e)^{3/2}}{(\gamma\beta_e k^2 + \gamma\alpha_e + \alpha_p)^{5/2}} \quad \text{... (30)}$$

5 Modulational Instability

The amplitude modulation of ion acoustic waves in quantum plasma consisting of URD electrons and positrons, cold ions and stationary negatively charged dust particles can be studied by using the NLS Eq. (16). It has been employed widely in connection with the nonlinear propagation of various types of wave modes. It is found that under certain conditions, conversion of an initially uniform wave train into a spatially modulated wave is energetically more favourable and wave train becomes modulationally unstable. In order to investigate the modulational instability against linear perturbations we set

$$\alpha = \alpha_0 \left(1+\alpha\right) \exp\left[i\left(-\Delta\omega\tau+\theta\right)\right] \quad \text{... (31)}$$

In which α, θ are real functions of ξ, τ and represents the perturbations in amplitude and phase, α_0 is a real constant and $\Delta\omega = Q\alpha_0^2$ is the amplitude dependent frequency shift.

The growth rate attains a maximum value

$$\gamma_m = |Q|\alpha_0^2 \quad \text{... (32)}$$

In fact, sign of the product *PQ* determines the stability/instability of the electro-acoustic wave. If the product *PQ* is negative (i.e, PQ < 0) the ion acoustic wave will be modulationally unstable. But, the wave will be modulationally stable if *PQ* (*i.e. PQ* > 0) is positive.

6 Envelope Soliton

From the NLS Eq. (16) the solitary wave solutions of ion acoustic wave may be obtained. The product *PQ* may be positive or negative which give two types localized. solitary wave solutions. For *PQ* < 0 , the wave is modulationally unstable and the bright-envelope-soliton (or bright soliton) is excited. Whereas for *PQ* > 0 the wave in modulationally stable and gives a dark-envelope-soliton (or dark soliton). In fact, Bright solitons are localized large-amplitude excitations on the envelope of certain carrier waves. Their formation requires an attractive or focusing nonlinearity. Dark solitons are dips or holes in a large-amplitude wave background. Their formation requires a repulsive or defocusing nonlinearity. Two classes of dark solitons, temporal and spatial, can propagate in systems with a repulsive nonlinearity. A temporal dark soliton is a dip on a temporal continuous wave. When the dip goes to zero, one has a black soliton. When the amplitude at the dip is nonzero, one has a gray soliton. A spatial dark soliton is a low-intensity hole in a high-intensity background. Like temporal dark solitons, spatial dark solitons can also be black or gray, which depends on the intensity at the soliton regions.

To solve the NLS equation we set

$$\varphi(x,t) = \psi(x,t)\exp[i\theta(x,t)] \quad \text{...(33)}$$

Using (33) in (16) and separating real and imaginary parts we can have two nonlinear equations. The solutions of these equations are:

i) For Q / P > 0, Bright envelope soliton,

$$\psi = \rho_0 \operatorname{sec} h\left(\frac{x-vt}{L}\right)\exp\left[\frac{i}{2P}\{vx-(v_0+\frac{v^2}{2})t\}\right] \quad \text{... (34)}$$

ii) For Q/P < 0 , Dark (Black or Gray) envelope soliton,

$$\psi = \rho_0 \tan h\left(\frac{x-vt}{L}\right)\exp\left[\frac{i}{2P}\{vx-(\frac{v^2}{2}-PQ\rho_0)t\}\right] \quad \text{...(35)}$$

where

$$\rho_0 = \left|\frac{2P}{Q}\right|^{1/2},$$

ρ_0 is the amplitude, L and v_0 are constants.

The equations (34) and (35) are the solutions of the ion acoustic wave for Bright Envelope-Soliton and Dark Envelope-Soliton respectively.

7 Rogue Waves

It may further be noted that another kind of solution of the present NLS equation can be generated if we proceed in a slightly different manner. Consider the equation;

$$i\frac{\partial\varphi}{\partial\tau} + +P\frac{\partial^2\varphi}{\partial x^2} + Q|\varphi|^2\varphi = 0 \quad \text{... (36)}$$

And set

$$\varphi = (\varphi_1 + i\varphi_2)e^{i\psi}, \quad \text{... (37)}$$

So that we get

$$-\frac{\partial\varphi_2}{\partial\tau} - \frac{\partial\psi}{\partial\tau} + P[\frac{\partial^2\varphi}{\partial x^2} - \varphi_1 - 2\frac{\partial\varphi_2}{\partial x}\frac{\partial\psi}{\partial x} - \varphi_2\frac{\partial^2\psi}{\partial x^2} - \varphi_1(\frac{\partial\psi}{\partial x})^2] + Q(\varphi_1^2+\varphi_2^2)\varphi_1 = 0 \quad \text{... (38)}$$

$$\frac{\partial\varphi_1}{\partial\tau} - \frac{\partial\psi}{\partial\tau} + P[\frac{\partial^2\varphi_2}{\partial x^2} + 2\frac{\partial\varphi_2}{\partial x}\frac{\partial\psi}{\partial x} - \varphi_1\frac{\partial^2\psi}{\partial x^2} - \varphi_2(\frac{\partial\psi}{\partial x})^2] + Q(\varphi_1^2+\varphi_2^2)\varphi_2 = 0 \quad \text{...(39)}$$

Let us now introduce new variables; K(x), T(t) and the transformation ;

$$\varphi_1(x,t)=\nu_1+\nu_2 S(K,T) \quad \text{...40(a)}$$

$$\varphi_2(x,t)=\nu_3+W(K,T) \quad \text{...40(b)}$$

$$\psi(x,t)=\chi+T \quad \text{...40(c)}$$

where ν_1,ν_2,ν_3,K,T are real functions to be determined .

We then get χ = 0 , K = x; ν_1,ν_2,ν_3 are arbitrary constants .

Setting $\nu=\sqrt{\frac{2R+P}{Q}};T=(2R+P)T$ leading to the reduced equations

$$(W^2+S^2+2S)W+S_\tau+W_{xx}=0 \quad \text{... (41)}$$

$$(1+S)[W^2+(1+S)^2-1]-W_\tau+S_{xx}=0 \quad \text{... (42)}$$

Finally after elaborate simplification we get the Rogues Wave as ;

$$\varphi=\sqrt{\frac{2R+P}{Q}}\exp[i\tau(2R+P)(\frac{N}{D})] \quad \text{... (43)}$$

Where

$$N=-3+2x^2+4\tau^2(2R+P)^2-8i\tau(2R+P) \quad \text{...44(a)}$$

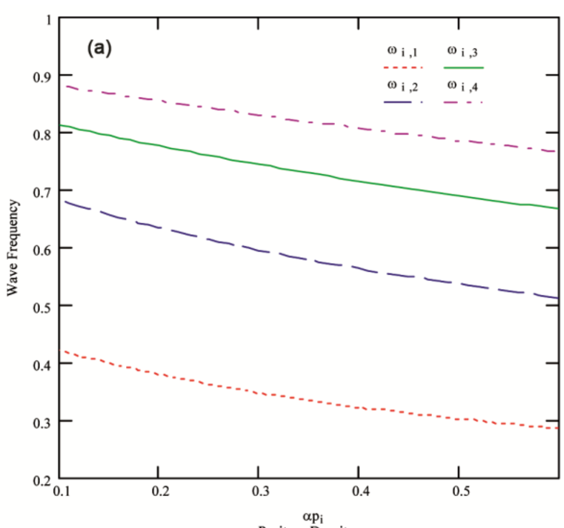
$$D=1+2x^2+4\tau^2(2R+P)^2 \quad \text{...44(b)}$$

The solution (43) is for the rogue waves which indicate that the energy of ion acoustic wave is concentrated into a small region due to the nonlinear properties of the plasma. We can see that the solution is a function of the plasma parameters *i.e.* positron density and positron temperature in the ultra-relativistic degenerate dense plasma consisting of electrons, ions and positrons.

8 Results and Discussion

From the above expressions for modulational instability and envelope soliton, we see that the plasma parameter e.g. positron density, electron density and also dust density have significant influence on the nonlinear propagation of ion acoustic wave in URD plasma. To ascertain the actual nature of the nonlinear wave and its stability we have numerically evaluated the various parameters and discussed graphically. To start with we have plotted the variation of dispersion and group velocity of wave with the plasma parameters shown in Fig.1 & 2.

Fig.1 (a) shows the dispersion of ion acoustic wave for different values of positron density (α_p) and wave number (k) in an URD e-i-p plasma for the fixed



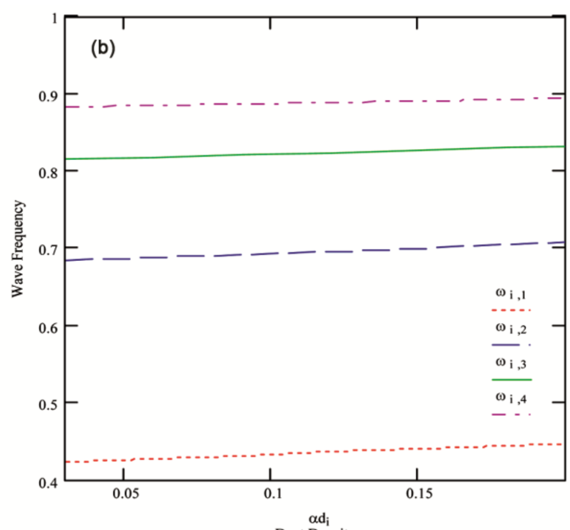


Fig. 1(a) — Dispersion curve of ion acoustic wave in ultra-relativistic degenerate plasma for different values of positron density and wave number .The dotted , dashed, solid and dadotted graphs represent for wave number k=1,2,3,4 respectively; βe = 0.3, βp = 0.1, αd=0.01. ; (b) — Dispersion curve of ion acoustic wave in ultra-relativistic degenerate plasma for different values of dust density and wave number .The dotted, dashed , solid and dadotted graphs represent for wave number k=1,2,3,4 respectively ; βe = 0.3, βp = 0.1, αp =0.1.

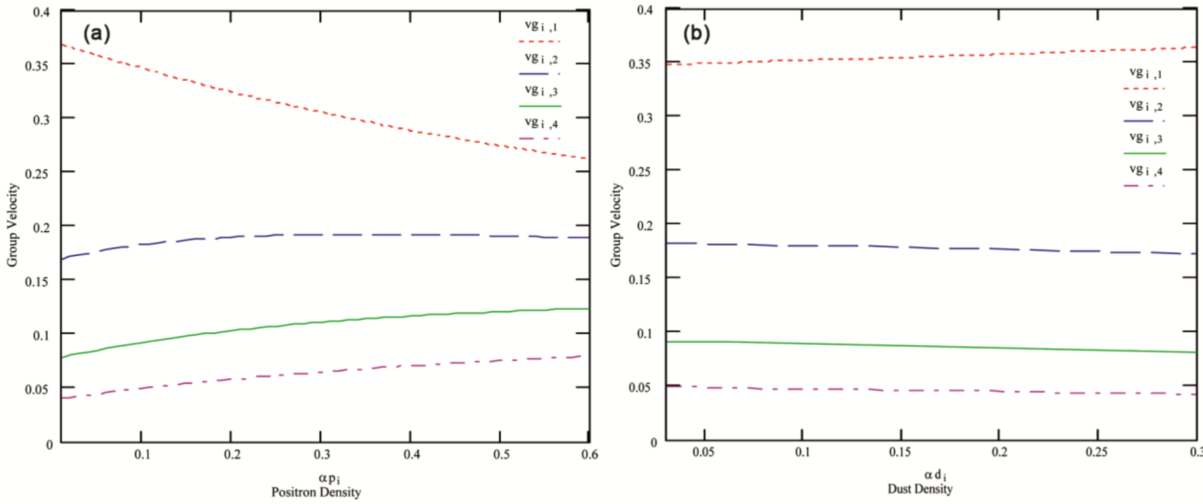


Fig. 2(a) — Group velocity of ion-acoustic wave in ultra-relativistic degenerate plasma for different values of positron density and wave number. The dotted, dashed, solid and dadotted graphs represent for wave number k = 1,2,3,4 respectively; βe = 0.3, βp = 0.1, αd =0.01. ; (b) — Group velocity of ion-acoustic wave in ultra-relativistic degenerate plasma for different values of dust density and wave number. The dotted, dashed, solid, dadotted graphs represent for wave number k = 1,2,3,4 respectively; βe = 0.3, βp = 0.1, αp =0.1.

values of other parameters α_e = 0.1, βe = 0.3, βp = 0.1 (*i.e.* βe > βp) . It is seen that wave frequency decreases with the increase of positron density. Moreover, wave frequency increases with the increase of wave number in such plasma. The effects of dust density and wave number on the dispersion of wave in such degenerate plasma with fixed values of positron density αp = 0.1 when ($\beta_e > \beta_p$) are shown in Fig.1 (b). It is observed that wave frequency increases with the increase of dust density and wave number. Similarly, for the case when $\beta_e < \beta_p$, dispersion curves of the wave in degenerate plasma for different values of positron density and dust density can be drawn and it would be seen that nature of the graphs are almost same as Fig.1 (a) and Fig.1 (b).

The variation of group velocity of ion acoustic wave with positron density and wave number in the URD plasma, when $\beta_e > \beta_p$ and other parameters have fixed values, is shown in Fig. 2 (a) . It is seen that group velocity increases with the increase of positron density when k >1. In the case for k = 1, group velocity decreases with the increase positron density. Moreover, the group velocity decreases with the increase of wave number for any value of positron density. In Fig. 2 (b), variation of group velocity with dust density and wave number has been shown for the same plasma with fixed value of positron density αp = 0.1. It is seen that group velocity slowly increases with the increase of positron density when k=1, but it decreases slowly with increase of positron density when k >1. Moreover, group velocity decrease with increase of wave number for any value of positron density.

It is known that stability of nonlinear wave is actually governed by the sign of the product of P and Q. Numerical computations of PQ in an URD e-i-p plasma are made and the variation of PQ with respect to positron density are graphically shown in Fig.3 (a). It is observed that PQ is negative and it decreases with the increase of positron density and wave number in URD plasma having fixed values of βe = 0.3, βp = 0.1 (βe > βp) and α_d =0.01. The negative value of PQ indicates that IAW will be modulationally unstable in URD e-i-p plasma. To see the effect of dust density on PQ in such URD plasma with fixed value of α_p = 0.1 are shown in Fig.3 (b) from which it observed that PQ is also negative and it increases with the increase of dust density. From Fig.3 (a) and Fig.3 (b) it is evident that the IAW will be modulationally unstable in URD plasma. Drawing the graphs of PQ for βe = 0.1, βp = 0.8 (β_e > βp) different values of positron density and dust density it may be seen that values of PQ are always negative and slightly different from Fig.3 (a) and Fig.3 (b). Since PQ is always negative and there is no positive values of PQ, the IAW will not have no stable modes.

To find growth rate of unstable IAW wave give by Eq.(32), numerically estimations are made for the URD plasma for different values of positron density and dust density. in the cases when $\beta_e > \beta_p$ the growth rate for different values of positron density is shown in Fig.4 (a) and for $\beta_e < \beta_p$ variation of growth rate

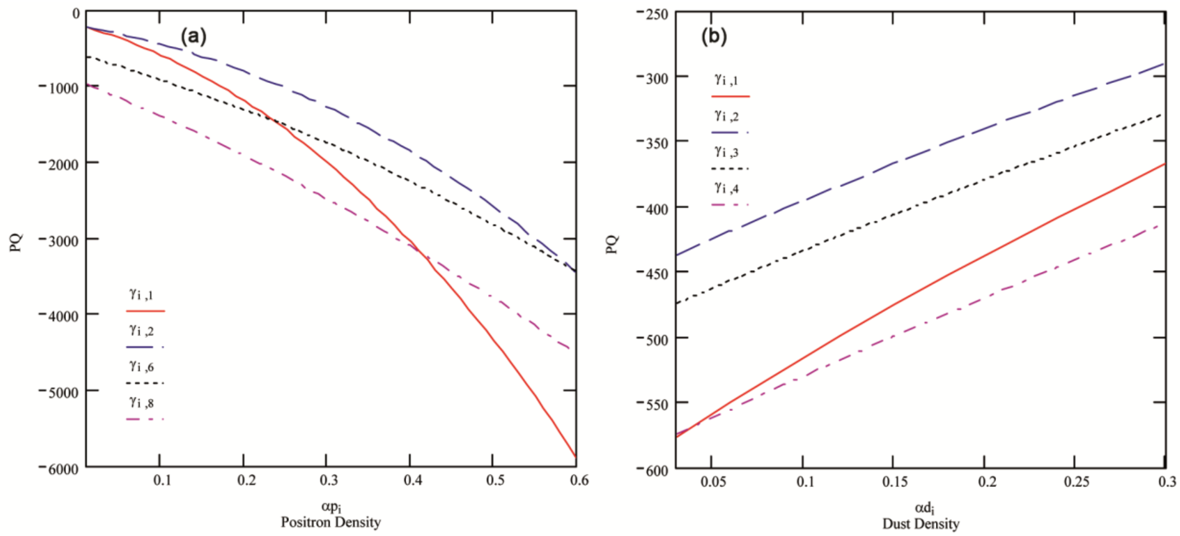


Fig. 3(a) — Variation of (PQ) in ultra-relativistic degenerate plasma for different values of positron density and wave number. The solid, dashed, dotted and dadotted graphs represent for wave number k = 1,2,6,8 respectively; βe = 0.3, βp = 0.1,αd=0.01; (b) — Variation of(PQ) in ultra-relativistic degenerate plasma for different values of dust density and wave number. The solid, dashed, dotted and dadotted graphs represent for wave number k = 1,2,3,4 respectively; βe = 0.3, βp = 0.1,αp=0.1.

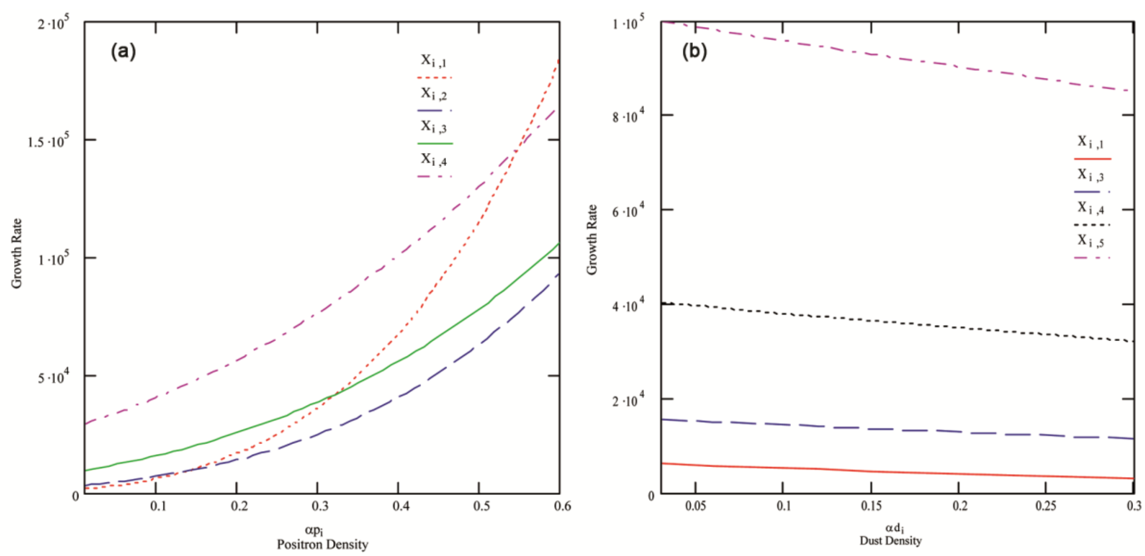


Fig. 4(a) — Growth rate of ion acoustic wave in ultra-relativistic degenerate plasma for different values of positron density and wave number . The dotted, dashed, solid, and dadotted graphs correspond to wave number k = 1,2,3,4, βe = 0.3, βp = 0.1,αd=0.01; (b) — Growth rate of ion acoustic wave in ultra-relativistic degenerate plasma for different values of dust density and wave number . The solid, dashed, dotted and dadotted graphs to wave number k = 1,3,4,5 respectively; βe = 0.3, βp = 0.1,αp =0.1.

with dust density is shown in Fig.4 (b) .It is observed that growth rate of IAW in URD e-i-p plasma increase with positron density. But the growth slowly decreases with increase of dust density.

We know that when $PQ < 0$ the wave becomes unstable for which bright- envelope-soliton will be excited, where as $PQ > 0$ gives a stable wave and a dark envelope-soliton excited. From numerical estimation for the URD plasma considered in this t investigation, it is found that PQ is always negative so only the bright envelope-soliton will be excited .The profiles of bright soliton when $\beta_e > \beta p$ are shown in Figs. 5 (a) for different values of positron density and in Fig.5 (b) for different dust density. It is observed from Fig. 5 (a) that amplitude of bright soliton decreases with the increase of positron density when

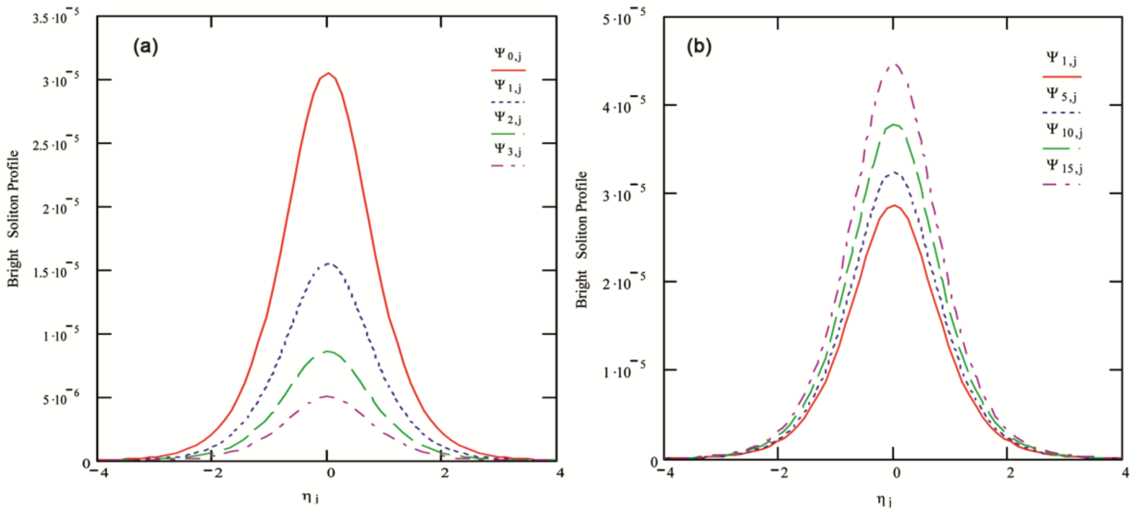


Fig. 5(a) — Profiles of bright soliton for different values of positron density. The dotted, dashed, solid and dadotted lines correspond to αp=0.1,0.15,0.2 and 0.25; k =1, βe = 0.3, βp =0.1, αd = 0.01; (b) — Profiles of bright soliton for different values of dust density. The dotted, dashed, solid and dadotted lines correspond to αd = 0.01,0.05,0.1 and 0.15; k =1, βε =0.3, βπ= 0.1.απ=0.1.

other plasma parameters are constant. But Fig.5 (b) shows that amplitude of bright soliton increases with the increase of dust density.

9 Conclusions

In the present investigation we have studied modulational instability along with the possible generation of both dark- and bright- envelope soliton in a plasma consisting of ultra-relativistic degenerate electrons and positrons, cold and mobile inertial ions, and negatively charged static dust particles using Fried and Ichikawa method. It is important to be mentioned that the conventional reductive perturbation method utilises the scaling in space and time to extract the particular form of nonlinear equation where as the Fried and Ichikawa approach depends on the nonlinear dispersion relation depending on the amplitude. One of the important feature of this method is that one can study modulational instability in the critical case for plasma parameter (*i.e.*, when nonlinear term vanishes) in a much simpler way from the higher order nonlinear Schrodinger equation.

To the best of our knowledge, no work on the modulational instability in ultra-relativistic degenerate dense dusty plasma in presence of positrons and envelope soliton in such plasma has been reported till now. In our analysis, explicit form of envelope soliton is derived from the deduced NLS equation and structure of solitary wave is graphically discussed. Condition of modulational instability are analysed diagrammatically for various plasma parameters. Our main findings are:

(i) The wave frequency decreases with increase of positron density, but it increases with increase of dust density.
(ii) The group velocity increases with increase of positron density but it decreases with the increase of dust density.
(iii) There is no stable ion acoustic mode in ultra-relativistic degenerate plasma, so dark soliton will not be excited in our URD plasma.
(iv) Ion acoustic wave will be always modulationally unstable for all possible values of density of positrons, electrons and charged dust particle. So, bright soliton will be excited. The amplitude of bright soliton decreases with increase of positron density but it is increased with the increase of dust density.
(v) The growth rate of modulational instability increases with the increase of positron density but it decreases with the increase of dust density.

Finally, we would like to point out that our present study might be important for understanding the fundamental processes involved in the nonlinear propagation of waves, (for modulational instability, envelope soliton *etc.*), in relativistic degenerate plasmas, *e.g.*, in dense astrophysical objects. There are many interstellar compact objects *e.g.*, white dwarfs, neutron stars, etc., where matters exist in extreme conditions[17,18] which are not found in terrestrial environments. One of these extreme conditions is the high density of degenerate matter in these compact objects which have ceased burning thermonuclear fuel, and thereby no longer generate thermal pressure. These interstellar compact objects are contracted significantly, and as a result the density of their interiors becomes extremely high to provide nonthermal pressure *via* degenerate Fermion/electron pressure and particle-particle interactions. The degenerate electron number density in such a compact object is very high e.g., in white dwarfs, the degenerate electron number density can be of the order of 10^{30} cm^{-3} even more. The electron Fermi energy is comparable to the electron mass energy and the electron speed is comparable to the speed of light in vacuum. The equation of state for degenerate electrons in such interstellar compact objects is explained by Chandrasekhar[17,18] for two limits, namely, nonrelativistic and ultrarelativistic limits. The degenerate electron pressure depends only on the electron number density, but not on the electron temperature. These interstellar compact objects, therefore, provide us cosmic laboratories for studying the properties of the medium matter as well as waves and instabilities in such a medium at extremely high densities degenerate state for which quantum as well as relativistic effects become important. In recent years, nonlinear propagation of waves in plasma with electrons, ions and positrons are being studied with much attention. Electron-positron-ion plasma exists in places such as active galactic nuclei[34], pulsar magnetospheres[35] and in many dense astronomical environments, namely, neutron stars and white dwarfs[36] and may play a key role in understanding the beginning and evolution of our entire universe[37].

References

1 Kaplan S A & Tsytovich V N, *Plasma Astrophysics* (Chapter- Three and Four) Translated and Edited by D terHarr, Pergamon Press, Oxford (1973).

2 Carlqvist P, *Astrophys Space Sci,* 87 (1982) 21.

3 Chakraborty B, Paul S N, Khan M & Bhattacharyya B, *Phys Rep*, 114 (1982) 182.

4 Punsly B, *Relativistic Plasma Physics*, Astrophys Space Sci Library, (2008) 34.

5 Bulanov S V, *High Intensity Laser Plasma Interaction and laboratory Astrophysics,* Kansai Photon Science Institute, Japan Atomic Energy Agency, Kyoto, 615 0215 Japan ELISS (2014) PRAGUE.

6 Das G C & Paul S N, *Phys Fluids*, 28 **(**1985) 823

7 Roychowdhury A, Pakira G & Paul S N, *IEEE Trans Plasma Sci*, 17 (1989) 1989804.

8 Saeed R, Shah A & Haq M, *Phys Plasmas*, 17 (2010) 102301.

9 Kalita B C, Das R & Sarmah H K, *Phys Plasmas*, 18 (2011) 012304.

10 Pakzad H, *Astrophys Space Sci*, 332 (2011) 269.

11 Lee N C, *Phys Plasmas*, 19 (2012) 082303.

12 Ghosh B & Banerjee S, *J Plasma Phys*, 81 (2015) 905810308.

13 Sahu B, *Pramana*, 76 (2011) 933.

14 Ghosh B, Chandra S & Paul S N, *Pramana*, 78 (2012) 779.

15 Chandra S & Ghosh B, *Indian J Pure Appl Phys*, 51 (2013) 627.

16 McKerr M, Haas F & Kourakis I, *Phys Plasmas*, 23 (2016) 052120.

17 Chandrasekhar S, *Philosophi Mag*, 11 (1931) 592.

18 Chandrasekhar S, *Mon Not R Astron Soc*, 170 (1935) 405.

19 Shah H A, Masood W, Qureshi M N S & Tsintsadze N L, *Phys Plasmas*, 18 (2011) 102306.

20 Masood W & Eliasson B, *Phys Plasmas,* 18 (2011) 034503.

21 Chandra S, Ghosh B & Paul S N, *Astrophys Space Sci*, 343 **(**2013) 213.

22 Chandra S, Das S, Chandra A, Ghosh B & Jash A, *Int J Math Comp Phys Elect Comp Eng,* **9** (2015).

23 Mamun A A & Shukla P K, *Phys Plasmas*, 17 **(**2010) 104504.

24 Mamun A A & Shukla P K , *Phys Lett A*, 374 **(**2010) 4238.

25 Esfandyari-Kalejahi A, Akbari-Moghanjoughi M & Saberian E, *Plasma Fusion Res*, 5 (2010) 045.

26 Roy N, Zobaer M S & Mamun A A, *J Mod Phys*, 3 **(**2012) 850.

27 Rahman A, Kourakis I & Qamar A, *IEEE Trans Plasma Sci*, 43 (2015) 974.

28 Onorato M, Residori S , Bortolozzo U, Montina A & Arecchi F T, *Phys Rep*, (2013) http://dx.doi.org/10.1016/j.physrep.2013.03.001.

29 Bailung H, Sharma S K & Nakamura Y, *Phys Rev Lett*, 107 **(**2011) 255005.

30 Wang Y Y, Li J T, Dai C Q & Zang J F, *Phys Lett A,* 337 (2013) 2097.

31 Ankievicz A, Devine N & Akhmediev N, *Phys Lett* A, 373 (2009) 3997.

32 Fried D E & Ichikawa Y H, *J Phys Soc Jpn*, 34 (1973) 1073.

33 Paul S N & Roychowdhury A, *Chaos Fractals Solitons*, 91 (2016) 406.

34 Miller H R & Witta P J, *Active Galactic Nuclei* (Berlin: Springer, (1987) 202.

35 Michel F C, *Rev Mod Phys*, 54 (1982) 1.

36 Ali S, Moslem W M, Shukla P K & Schlickeiser R, *Phys Plasmas*, 14 (2007) 082307.

37 Rees M J, *The Very Early Universe* (Cambridge: Cambridge University Press, (1983).