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Monte carlo method for evaluation of uncertainty of measurement in brinell hardness scale

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Though method based on law of propagation of uncertainty (LPU), described in GUM, is widely used as an international method for estimation of the uncertainty associated with measurements, JCGM through supplement JCGM 101:2008, which deals with the propagation of distributions, recommends the application of Monte Carlo method (MCM) to evaluate the uncertainty of measurement as an alternative method to LPU. In MCM approach, probability distribution function of measurand has been determined by assigning appropriate probability distribution of the input quantities. In the present paper, effort has been made to discuss the procedure for the application of MCM for computing the uncertainty of measurement of hardness blocks, which have been calibrated by Brinell hardness machine and observations have been recorded. A comparison of the findings of the LPU and MCM has been made, which shows good agreement between the two methodologies adopted. The paper attempts to highlights the MCM for uncertainty of measurement evalution and its implications in this regard. The paper thrusts upon the practical viability of MCM in similar applications and reliability of method have been discussed and presented.

Keywords: Uncertainty of measurement, Law of propagation of uncertainty (LPU), Monte Carlo Method (MCM), Brinell hardness measurement, Hardness block

1 Introduction

The concept of uncertainty is a perceptible trait in the history of measurement. The uncertainty associated with any measurement is a quantitative expression of its quality and is accepted as the extent of reliability of measured value[1]. Although error and its analysis have already been a part of metrological practices, but now it is widely recognized that even after applying appropriate corrections to all the identified sources of error, still there remains an uncertainty about the correctness of the stated measurement result[2]. Evaluation of measurement uncertainty has also been very essential in quality assurance also, because of the increasing tolerances in production[3]. Uncertainty of measurement is the doubt that exists about the result of any measurement. Still, after correction for recognized systematic effects, the result of measurement is only an approximation of the value of measurand, because there are some random effects and inappropriate corrections applied which cause the uncertainty in measurement results[4,5].

Some possible sources of uncertainty in a measurement are as following:

(i) Definition of the measurand: Incomplete and imperfect.
(ii) Samples: Taken for measurement of any physical parameter may not represent the defined measurand.
(iii) Effects of environmental conditions: Effect of such conditions over the measurement is unknown or imperfectly measured.
(iv) Limited or Finite resolution of the measuring instruments or discrimination threshold.
(v) Values of measurement standards and reference materials are not known properly or precisely.
(vi) Approximations and or assumptions adopted for the measurement method and the procedure.
(vii) Variable results in repeated observations of the measurand under identical conditions.
(viii) Personal bias in reading analogue instruments.

2 Measurement Uncertainty Evalauation: GUM Approach

According to GUM evaluation for measurement uncertainty becomes important because it allows the evaluated measurement uncertainty of a measured quantity to be used in a new measurement where this quantity is to be used as input quantity[6]. GUM

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document is fundamentally based upon the supposition that systematic errors are rectified initially while making a measurement. GUM describes the standard uncertainty associated with the results of a measurement as a parameter 'that characterizes the dispersion of the values that could reasonably be attributed to the measurand'. The input quantities are put in the model equation and uncertainty is obtained by taking into relative uncertainty component of each input quantity into consideration. If the X_1, X_2, ….X_N are the input quantities and output Y is a function of X, then the uncertainty of measure and, Y is also dependent on the input quantities, method adopted and procedure adopted. The relative uncertainty components of the different factors will lead to the overall uncertainty[7-9] of Y.

The uncertainty of measurement is a collective impact of the individual uncertainty component. This leads to a general expression for propagation of uncertainties as follows:

$$u_y^2 = \sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 u_{x_i}^2 + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\left(\frac{\partial f}{\partial x_i}\right)\left(\frac{\partial f}{\partial x_j}\right)\mathrm{cov}(x_i, x_j) \quad \text{... (1)}$$

where u_y is the combined standard uncertainty for the measurand y and u_{xi} is the uncertainty for the i^{th} input quantity. The second term of equation is related to the correlation between the input quantities. If there is no supposed correlation between them, equation is further simplified as:

$$u_y^2 = \sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 . u_{x_i}^2 \quad \text{... (2)}$$

The steps to be followed for evaluating and expressing the uncertainty of measurement result are described as follows:

(i) Express mathematically the relationship between the measure and, Y and the input quantities X_i on which Y depends: $Y= f\ (X_1, X_2,..., X_N)$. The function f should contain every quantity, including all corrections and correction factors that can contribute a significant component of uncertainty to the result of the measurement.

(ii) Determine x_i, the estimated value of input quantity X_i, either on the basis of the statistical analysis of series of observations or by other means.

(iii) Evaluate the standard uncertainty $u(x_i)$ of each input estimate x_i. For an input estimate obtained from the statistical analysis of series of observations.

(iv) Evaluate the covariance associated with any input estimates that are correlated.

(v) Calculate the result of the measurement, that is, the estimated, y of the measure and, Y, from the functional relationship f for input quantities X_i as estimated x_i, obtained in step (ii).

(vi) Determine the combined standard uncertainty u_c(y) of the measurement result y from the standard uncertainties and covariance associated with the input estimates. If the measurement determines simultaneously more than one output quantity, calculate their covariance.

(vii) If it is necessary to give an expanded uncertainty U, whose purpose is to provide an interval $y – U$ to $y + U$ that is expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measure and, Y multiply the combined standard uncertainty, $u_c(y)$ by a coverage factor k, typically in the range 2 to 3, to obtain $U = k.u_c(y)$. Select k on the basis of the level of confidence required of the interval.

(viii) Report the result of the measurement as an estimate y of the measure and along with its associated expanded uncertainty U with coverage factor k.

Figure 1 summarizes the outlay of the procedure for evaluation of uncertainty of measurement according to GUM.

Farrance and Frenkel[6] have published a study, pinpointing of some of the limitations of the GUM approach. The measure and model must not have significant non-linearity. If there is substantial non-linearity, GUM's Taylor series are truncated to the first term which may not estimate the uncertainty properly. GUM approach relies on central limit theorem, according to which the convolution of a large number of distributions, which may be of different types results in normal distribution. Hence, the resulting output distribution is normal and can be represented by a t-distribution but in contrary the resulting distribution may be of different types.

For calculating the expanded uncertainty, effective degree of freedom is essential which is calculated by Welch-Satterthwaite formula. The evaluation of effective degrees of freedom by analytical method is still not fully solved and hence not adequate for all the cases[6].

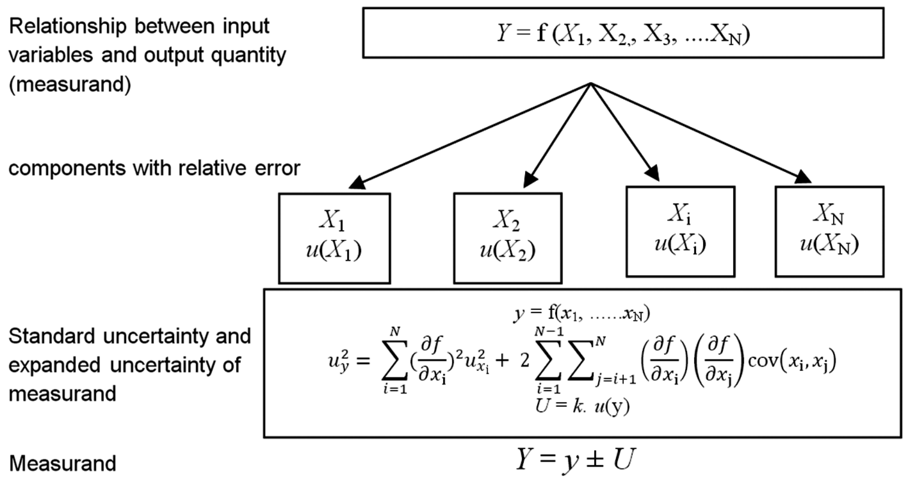

Fig. 1 – Flow chart for law of propagation of uncertainty[9]

3 Measurement Uncertainty Evalauation: MCM Approach

Monte Carlo Method has been emerged as a practical alternative to the GUM approach for evaluating measurement uncertainty. MCM can be addressed as an experimental and statistical technique to validate theoretical statistical results obtained by GUM modeling approach for evaluation of measurement uncertainty. The Monte Carlo method as discussed by the GUM Supplement[10] involves the propagation of the distributions of the input sources of uncertainty by using the model to provide the distribution of the output, whereas as per GUM, the uncertainties are to be propagated according to their distributions. In MCM approach pseudo random numbers are generated and they follow specified probability distribution functions.

In case of LPU, uncertainties due to error sources are propagated. Here x_1, x_2 and x_3 are input quantities for any measurement parameter (the output) and $u(x_1)$, $u(x_2)$ and $u(x_3)$ are their respective uncertainties (some of them may be relative uncertainties also). y and $u(y)$ are the measure and or output quantity (output as a function of the input quantities) and its associated uncertainty on the other hand in case of MCM, propagation of distributions is taking place. Here $g(x_1)$, $g(x_2)$ and $g(x_3)$ are the distribution functions of input quantities or parameters, and $g(y)$ is the distribution function of measure and or the output quantity[11]. By following GUM approach we obtain the mean and standard uncertainty of the measure and as final results while, the MCM approach provides us, the actual PDF of the measured quantity which imparts much more information. While the Gum modeling approach is not able to clearly determine the PDF for output data, the extra information gathered from MCM can be utilized to plot graph for probability distribution of output data and further to check the coverage interval. MCM approach has numerous advantages over gum modeling approach such as it considers the nonlinearities of functional relationships, it provides joint simulation of a bivariate distribution provided the correlation coefficient, and provides explicit graph for output data distribution function[12].

The procedure adopted as per Monte Carlo method approach for measurement uncertainty evaluation consists of the steps including; defining the measurand (output) and input quantities, modeling the equation for uncertainty evaluation, estimating the probability density functions for all input quantities, setup and run Monte Carlo method, summarizing and expressing the results.

Figure 2 depicts the outlay of procedure for evaluation of uncertainty of measurement according to MCM. The steps (i) and (ii) are same as done in the conventional method. Step (iii) involves the selection of the most appropriate probability density functions (PDFs) for each of the input quantities depending upon information provided or using own experience. Selecting appropriate probability density functions for input quantities contributes significantly to the measured output quantity. After defining PDFs for input quantities depending upon the nature and relative contribution of the input quantity, the number of trails of Monte Carlo method should be selected. Greater number of trials is helpful to have better the convergence of results. The GUM Supplement 1

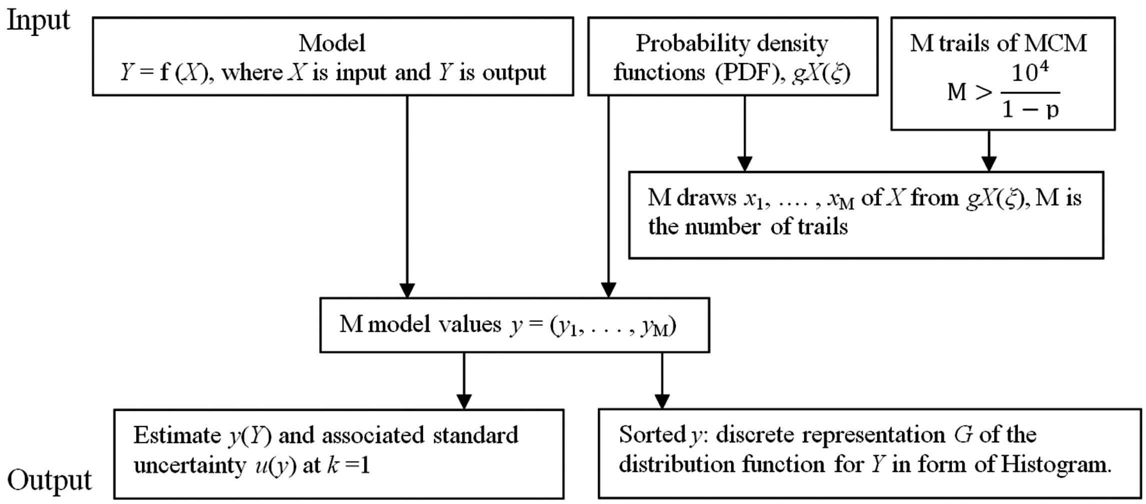

Fig. 2 – Flow chart for Monte Carlo method (MCM)[12]

recommends the selection of a number, M of trials, according to the following general rule:

$$M > \frac{10^4}{1-p} \quad \ldots (3)$$

where, p is the selected coverage probability.

The results of interest in Monte Carlo method are the mean value, the standard deviation of the output quantity and the endpoints of the chosen interval. Now, the Monte Carlo method is run for the defined number of trials. At last, the results are summarized as estimate of the output quantity, standard uncertainty, chosen coverage factor and points of the chosen interval.

Some advantages of the MCM approach over LPU are MCM is based on propagation of distribution of input quantities, not on input's associated uncertainty, MCM gives actual PDF of the output not necessarily Gaussian distribution as in case of GUM, MCM give better estimate of output in case of non-linear models, no need to calculate sensitivity coefficients and hence partial derivatives with respect to input quantity not required, less mathematical skills required, output PDF can be easily seen from the graphical representation of distribution of measure and earlier studies[10].

4 Hardness Measurement

Hardness is a very vital property of materials and plays an important role in materials testing, quality control and acceptance of components in various production houses. It is prerequisite for certain applications that the material possesses desired hardness. It may be defined as the resistance of a material to permanent plastic deformation, penetration, indentation, and scratching. Measurement of hardness includes indentation of the indentor of specific shape and dimensions as mentioned in the standard procedures to the surface of the material under test for specific duration of time. The indentation is measured and hardness is measured for the given scale[13]. There are different scales of hardness for specifying the hardness of the materials, namely Rockwell, Vickers and Brinell hardness. In turn, the each type of hardness (Rockwell, Vickers or Brinell) has different scales like HRA, HRC etc. in case of Rockwell hardness, 2.5/187.5, 5/750 etc. in case of Brinell hardness. Different scales of hardness are realized on the basis of different standards and the hardness blocks (serving as the specimen under test) are prepared according to the guidelines of the standards. The standard provides comprehensive guidelines for procedure to adopt along with prerequisite pertaining to specimen, measuring instruments and measurement procedure. ISO 6508, ISO 6506 and ISO 6507 provide guidelines pertaining to Rockwell hardness, Brinell hardness and Vickers hardness, respectively[14-16].

4.1 Primary Brinell hardness machine

CSIR-National Physical Laboratory, India (NPLI) is designated as the National Metrology Institute (NMI) of the country and maintains national standards of different hardness scales. Brinell hardness is used for various applications and over many applications, it is mentioned that the specimen has specific hardness over Brinell hardness scale. Brinell hardness is evaluated by applying the specific force through the specific indentor of spherical shape made of tungsten carbide to the specimen for given duration. The diameter of indentation on the surface is measured by precision microscope with resolution of 1 μm or better. There are indentor of different diameter like 1 mm, 2.5 mm and 10 mm and the load applied ranges from 1 kgf to 3000 kgf. Precision Brinell hardness machines serving as primary standards employs dead weights and the secondary machines amplifies the force applied by means of lever. NPLI has both primary and secondary Brinell hardness machine. Softer the material, lower is the hardness and tougher or brittle the material is, higher is the hardness value. The hardness is calculated by dividing the load by the area of the curved surface of the indention:

$$\text{BHN} = \frac{F}{\frac{\pi}{2}.D\,(D-\sqrt{D^2-d^2}} \quad \text{... (4)}$$

where, BHN is the Brinell hardness value of the hardness block ; F is the force applied in kgf; D is the diameter of the indenter in mm; d is the mean diameter of indentation in mm.

The primary Brinell hardness machine has been established at NPLI and has been designed, developed by M/s Foundrax, United Kingdom (Fig. 3). The primary Brinell hardness machine employs precision dead weight of nominal capacities and the force applied may range from 1 kgf to 3000 kgf depending upon the scale. The hardness machine employs tungsten carbide indentor for indentation on surface and for measurement of indentation, it uses a precision microscope in conjunction with CCD camera. The indentation with resolution of 0.1 μm can be measured with the arrangement. UKAS certified reference hardness blocks have been used for affirmation of the metrological capabilities of the primary Brinell hardness machine. The uncertainty of measurement of the primary Brinell hardness for HBW 1/1 to 1/30 is 1.5 % and for rest scales is 1.0 %. The hardness machine has been discussed elsewhere in detail earlier[17].

Fig. 3 – Primary Brinell hardness machine at CSIR-NPL[15]

4.2 Case studies: Metrological characterization of hardness blocks

As discussed earlier, Monte Carlo method (MCM) has now been recommended as an alternative method for evaluation of uncertainty of measurement of any measure and. Hence, there is a need of rigorous investigations to see the suitability of MCM as a suitable technique for evaluation of uncertainty of measurement. In addition, there is also a need to compare the outcomes of the MCM with the outcomes of conventional technique to evaluate uncertainty of measurement using law of propagation of uncertainty (LPU). Different factors taken into consideration are mentioned in Table 1 and Fig. 4.

The present paper discusses an in-depth analysis of the MCM and LPU for Brinell hardness. The investigation involves calibration of two hardness blocks of different scales (2.5/187.5 and 10/3000) using the primary Brinell hardness machine at NPLI. The hardness blocks are calibrated according to the calibration procedure based on the standard ISO 6506. The uncertainty of measurement of the hardness blocks has been evaluated according to LPU by taking the suitable factors into account with their contribution to the uncertainty of measurement[5]. Typical observations are recoreded as shown in Table 2. Similarly, procedure as discussed earlier for MCM has been adopted and number of trials is defined. The procedure comprises generation of random numbers for different factors and computation of hardness accordingly, Microsoft excel has been proved helpful for MCM and has been used by different researchers earlier for investigations for evaluation of uncertainty of measurement[6] using

Table 1 – Different uncertainty components and their distribution

S. No.	Parameter	Distribution
1	Force (F)	Gaussian
2	Indentor diameter (D)	Gaussian
3	Diameter of indentation (d)	Gaussin

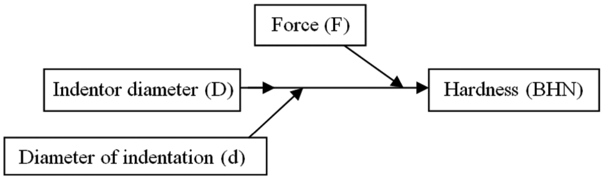

Fig. 4 – Model for Brinell hardness measurement

Table 2 – Typical observations for Brinell hardness block

S. No.	Force (kgf) F	Diameter of indentor (mm) D	Diameter (mm) d_1	Diameter (mm) d_2	Mean Diameter (mm) d
1	750	5	1.151	1.097	1.124
2	750	5	1.153	1.086	1.1195
3	750	5	1.142	1.088	1.115
4	750	5	1.072	1.173	1.1225
5	750	5	1.154	1.117	1.1355

MCM. A comparison between the outcomes of both has been made and discussed accordingly.

5 Results and Discussion

As discussed above, LPU is the most widely accepted method for evaluation of uncertainty of measurement in different measurement related problems and widely used in NMIs and calibration laboratories. However, with the advent of JCGM 101: 2008, a new valuable approach has come, alternative to the traditional LPU method. JCGM has issued a supplement in 2008, giving new dimension to the evolution of uncertainty of measurement by allowing the use of MCM in evaluation of the uncertainty of measurement. Since then a lot of efforts are done to implicate MCM in different measurement related problems. The MCM approach is particularly useful in complex models.

5.1 Hardness block HBW 2.5/187.5

A hardness block for Brinell hardness scale HBW 2.5/187.5 has been investigated. The hardness block has been calibrated according to the standard procedure and observations are given in Table 3. The results obtained by MCM for the above case is given in Table 4 and a histogram is plotted showing probability distribution function (PDF) of Brinell hardness. For convenient graphical representation two vertical lines are drawn representing low and high endpoints covering 95 % interval for Brinell hardness value (Fig. 5).

Table 3 – Observations for block I HBW 2.5/187.5

S. No.	d_1 (mm)	d_2 (mm)	D (mm)	Hardness of block
1	1.0253	1.0250	2.5	217.28
2	1.0248	1.0251	2.5	217.37
3	1.0239	1.0241	2.5	217.80
4	1.0245	1.0246	2.5	217.55
5	1.0251	1.0255	2.5	217.22
6	1.0249	1.0250	2.5	217.37
7	1.0258	1.0261	2.5	216.93
8	1.0238	1.0242	2.5	217.80
9	1.0245	1.0248	2.5	217.51
10	1.0250	1.0247	2.5	217.42
Nominal force applied				187.5 kgf
Uncertainty of force transducer				0.1 % (k = 2)
Diameter of the indentor				2.5 mm
Uncertainty of indentor				0.003 mm (k = 2)
Mean indentation diameter of the hardness block surface				1.0249 mm
Standard deviation of indentation diameter of hardness block				0.000568
Mean Brinell hardness value of block				217.44
Standard deviation of Brinell hardness value of hardness block				0.27
Uncertainty of measurement of hardness block according to LPU				0.28

Table 4 – Summary of MCM for Brinell hardness block I

Parameter	BHN
Estimate of Brinell hardness	217.44
Standard uncertainty	0.28
Low end point for 2.5 %	216.89
High end point for 95 %	217.99
Coverage factor (k)	1

5.2 Hardness block HBW 10/3000

A hardness block for Brinell hardness scale HBW 10/3000 has been investigated. Observations are given in Table 5. The results obtained by MCM for the above case is given in Table 6 and a histogram is plotted showing probability distribution function (PDF) of Brinell hardness. For convenient graphical representation two vertical lines are drawn representing low and high endpoints covering 95% interval for Brinell hardness value (Fig. 6).

5.3 Comparison of LPU and MCM

Table 7 describes the comparison of the uncertainty of measurement of two hardness blocks (HBW 2.5/187.5 and HBW 10/3000) evaluated by LPU and MCM techniques. It is found that the uncertainty of measurement of hardness blocks by both the means is

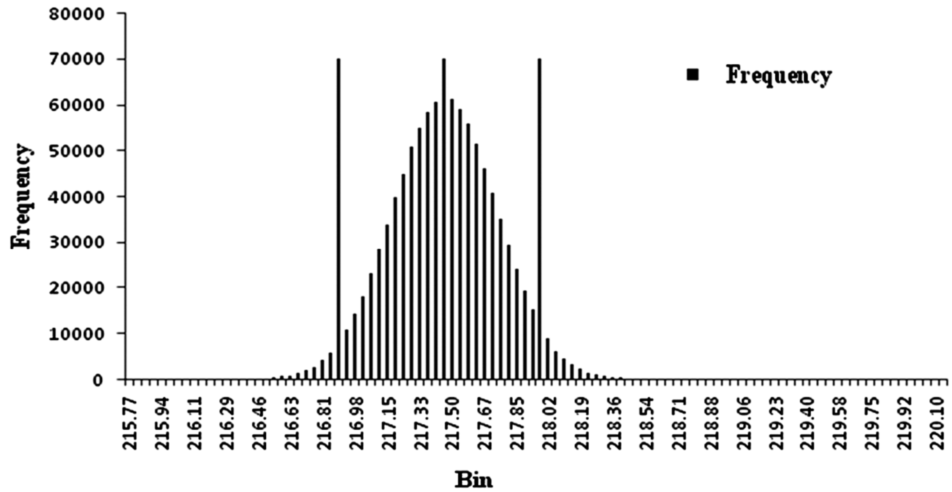

Fig. 5 – Histogram showing Brinell hardness PDF estimated by MCM (Hardness block HBW 2.5/187.5)

Table 5 – Observations for block I HBW 10/3000

S. No.	d_1 (mm)	d_2 (mm)	D (mm)	Hardness of block
1	3.3815	3.3818	10	324.35
2	3.3825	3.3823	10	324.20
3	3.3832	3.3832	10	324.04
4	3.3828	3.3825	10	324.15
5	3.3842	3.3840	10	323.86
6	3.3836	3.3835	10	323.97
7	3.3830	3.3832	10	324.06
8	3.3854	3.3858	10	323.57
9	3.3826	3.3829	10	324.13
10	3.3845	3.3848	10	323.75
Nominal force applied				3000 kgf
Uncertainty of force transducer				0.1 % (k = 2)
Diameter of the indentor				10 mm
Uncertainty of indentor				0.005 mm (k = 2)
Mean indentation diameter of the hardness block surface				3.3834 mm
Standard deviation of indentation diameter of hardness block				0.000586
Mean Brinell hardness value of block				324.00
Standard deviation of Brinell hardness value of hardness block				0.26
Uncertainty of measurement of hardness block according to LPU				0.27

quite close and if, very minor deviation is there, could be ignored due to the statistical procedure adopted, while doing simulation using MCM. The uncertainty of measurement of hardness block in close proximity by both techniques proves that MCM has been supplementary technique to LPU and may be very useful for the problems, where it is difficult to evaluate uncertainty of measurement by LPU.

Table 6 – Summary of MCM for Brinell hardness block II

Parameter	BHN
Estimate of Brinell hardness	324.00
Standard uncertainty	0.28
Low end point for 2.5 %	323.44
High end point for 95 %	324.56
Coverage factor (k)	1

It is evident from Table 7 that uncertainty of measurement has been very close for both of the approaches adopted. The later techniue (MCM) seems to be well close to the earlier one (LPU). The observations are based on two different hardness blocks for Brinell scale using primary Brinell hardness scale at NPL, India in accordance to widely accepted calibration procedure. MCM has been now expected to be applied over the similar problems on larger basis for uncertainty of measurement evalaution for different metrological applications. Though, in some of the metrological applications, there may be dissent among the uncertainty of measurement evaluted across both approaches, the uncertainty of measurement evalauted using MCM to be considered as relibale and correct one.

5.4 JCGM supplement 101: 2008

Since the domain of validity for MCM is broader than that for the GUM uncertainty framework, it is recommended that both the GUM uncertainty framework and MCM be applied and the results compared. Should the comparison be favourable, the GUM uncertainty framework could be used on this occasion and for sufficiently similar problems in the future. Otherwise, consideration should be given to

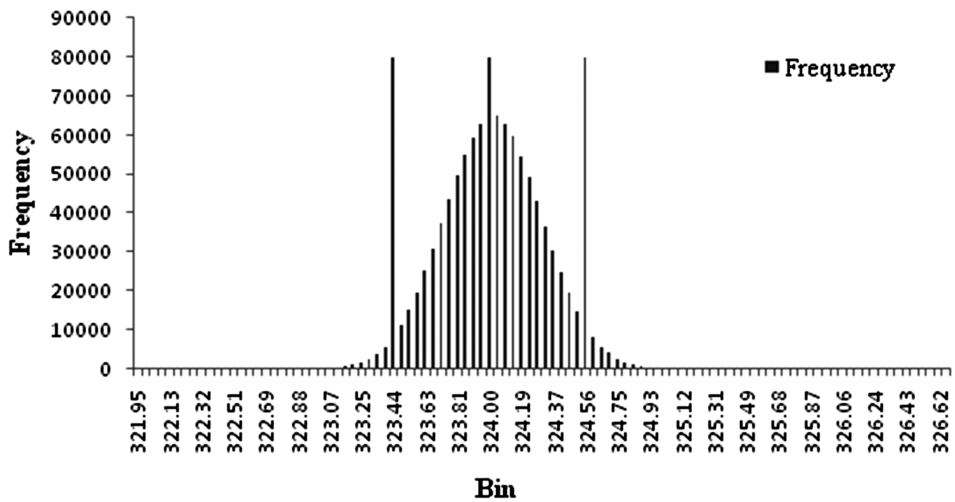

Fig. 6 – Histogram showing Brinell hardness PDF estimated by MCM (Hardness block HBW 10/3000)

Table 7 – Comparison of uncertainty of measurement evaluated by LPU and MCM

S. No.	Brinell hardness scale	Standard uncertainty (k=1) BHN	
		LPU	MCM
1	HBW 2.5/187.5	0.28	0.28
2	HBW 10/3000	0.27	0.28

using MCM or another appropriate approach instead described elsewhere[9].

6 Conclusions

The following conclusions are drawn from the studies:

(i) An introduction to the adoption of LPU/GUM and MCM discussed during the present work reported. The paper discusses the approaches mentioned above to evaluate uncertainty of measurement of hardness blocks for Brinell harness scale.

(ii) Practical viability of approaches mentioned above is proved vital regarding metrological characterization of hardness blocks. Uncertainty of measurement of the hardness blocks has been computed using different approaches with same factors contributing to uncertainty of measurement in either case.

(iii) The uncertainty of measurement of hardness block is found to be very close and thus indicates the agreement among the both methods. This clearly suggests that the efforts are need to be made to apply MCM to other measurement related applications. It further thrust upon the implementation of MCM for different physical parameters as the case may be.

(iv) After undergoing through a lot of literature and research articles, it is learnt that the software and its capability are vital for implementing MCM. The efficiency of the software mostly depends upon the random numbers to be generated and affects greatly, the random number generated exceeds 2 lacs in this particular case.

(v) The procedure presented herewith is adopted in compliance to the JCGM supplement 101: 2008.

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Symbols

F Force applied
D Indentor diameter
d_1 & d_2 Measured diameters of indentation
d mean diameter of indentation
BHN Brinell hardness
u Standard uncertainty
U Expanded uncertainty
k Coverage factor
M Number of trails
p Probability

References

1 JCGM 200: 2012, *International vocabulary of metrology - basic and general concepts and associated terms (VIM)*, 2012.

2 Werickson F C, Rocha & Nogueira R, *J Braz Chem Soc*, 23 (2012) 385.

3 Lee S W, Chung B D, Bang Y S & Sung W B, *Nucl Eng Technol*, 46 (2014) 481.
4 Wubbeler G, Krystek M & Elster C, *Meas Sci Technol*, 19 (2008).
5 ISO GUM, *Guide of the expression of uncertainty in measurement*, 1995.
6 Farrance I & Frenkel R, *Clin Biochem Rev*, 35 (2014) 37.
7 Cox M, Harris P & Siebert B R L, *Meas Technol*, 46 (2003) 824.
8 Couto P R G, Damascena J C & Oliverira S P, *Monte Carlo method applied to uncertainty in measurement*, INTECH, 2013.
9 Muste M, *Guidelines for the assessment of uncertainty of hydrometric measurement*, World Meteorological Organization, Iowa, 2009.
10 JCGM 101:2008, *Evaluation of measurement data — Supplement 1 to the Guide to the expression of uncertainty in measurement — Propagation of distributions using a Monte Carlo method*, 2008.
11 Nara M, Abbe M & Takamasu K, *Key Eng Mater*, 381-382 (2008) 587.
12 Nachtigalová I & Suchánek M, *Measurement uncertainty evaluation using Monte Carlo method*, Technical note, 2010.
13 Dhawan J K, *Uncertainty in hardness measurement*, 2nd Workshop on Force Metrology, New Delhi, India, 2003.
14 ISO 6508-1: 2015, *Metallic materials – Rockwell hardness test – Part 1: Test method*, 2015.
15 ISO 6506-1: 2014, *Metallic materials – Brinell hardness test – Part 1: Test method*, 2014.
16 ISO 6507-1: 2005, *Metallic materials – Vickers hardness test – Part 1: Test method*, 2005.
17 Kumar R, Titus S S K & Jain S K, *Mapan J Metrol Soc I*, 27 (2012) 123.