

Higher partial wave energy dependent and independent two-nucleon interactions via supersymmetry formalism

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Within the formalism of supersymmetric quantum mechanics both energy-dependent and independent two-nucleon potentials for the D-wave are constructed from their respective ground state interactions. These potentials in turn are used to compute nucleon-nucleon scattering phase shifts to judge their merits. Energy-dependent potential is found to be more effective than its energy independent counterpart except in the very low energy range.

Keywords: Energy-dependent and independent interactions, Supersymmetric quantum mechanics, Phase function method, *n-p* scattering phase shifts

1 Introduction

The nucleon-nucleon (N-N) system has been studied quite extensively and provided a large number of reliable experimental data. In this context, the phase shifts predicted by several groups¹⁻²⁴ have undergone minor changes although the methods applied by them are different. Thus, one can safely rely on these data. All these phase shift analyses involve a large number of free parameters. In the recent past we have studied N-N scattering²⁵⁻²⁷ within the framework of supersymmetry inspired factorization method by constructing higher partial wave N-N potentials from their S-wave partners. Our phase shifts compare well with the standard data^{3,24} for low and intermediate energies. In our previous paper²⁸ we made a comparative study of the energy-dependent and independent interactions generated from their ground state interactions and analyzed the associated phase shifts for the partial wave state $\ell = 0$ and 1.

However, the construction of D-wave interaction requires the knowledge of wave function for the P-wave. Arnold and MacKellar²⁹ developed a method for constructing phase equivalent local potential from a separable nonlocal interaction. At the same time they have also parameterized the Hulthén potential to fit the deuteron binding energy and S-wave scattering lengths. The S-wave bound state wave functions of

the nonlocal Yamaguchi³⁰ and Hulthén³¹ potentials are identical. Thus, the phase shifts produced by them permit comparison. Both these interactions are strictly applicable for S-wave only. Several groups³²⁻³⁴ have studied the supersymmetric aspects of the N-N scattering in the context of one pion exchange approximation and transformation of deep potential to a shallow one with repulsive core. Both deep and shallow potentials are frequently used in nuclear physics for the description of nucleus-nucleus interactions.

Following the methodology of supersymmetric quantum mechanics we shall first construct the wave function for the partial wave $\ell=1$ for the energy dependent Yamaguchi potential and develop the associated D-wave potential. In this connection we compare its phase shifts with that of energy independent nuclear Hulthén potential for the partial wave $\ell=2$.

2 Energy-dependent and independent potentials for $\ell=2$

Any second order differential operator of the form:

$$H_n = -\frac{\partial^2}{\partial x^2} + V_n(x) \quad \dots (1)$$

can be factorized in terms of the following two first order differential operators:

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$$A_n^{(\pm)}(x) = \pm \frac{d}{dx} + \frac{d}{dx} \ln \psi_n^{(0)}(x) \quad \dots (2)$$

with

$$V_n(x) = V_{n-1}(x) - \frac{d^2}{dx^2} \ln \psi_{n-1}^{(0)}(x); \quad \dots (3)$$

$n = 1, 2, 3, \dots$

Here $A^{(\pm)}(x)$ stand for the lowering and raising operators. These relations are valid not only for discrete energy Eigen values of H_n but also for the continuous part of the spectrum. The Eigen functions of H_n are connected by:

$$\psi_n^{(m)}(x) = (E_n^{(m)} - E_{n-1}^{(0)})^{-1/2} A_{n-1}^- \psi_{n-1}^{(m+1)}(x); \quad \dots (4)$$

$m = 0, 1, 2, \dots$

For positive energy states, however, the normalization constant is not indispensable and the wave functions are related by:

$$\psi_n(x) = A_{n-1}^- \psi_{n-1}(x) \quad \dots (5)$$

The regular and irregular solutions corresponding to local and non-local potentials²⁹ are related by:

$$\phi_N(k, r) = A(k, r) \phi_L(k, r) \quad \dots (6)$$

and

$$f_N(k, r) = A(k, r) f_L(k, r), \quad \dots (7)$$

where $A(k, r)$ is the damping function.

$$V(r, r') = \lambda e^{-\beta(r+r')}, \quad \dots (8)$$

Since its appearance the Yamaguchi potential³⁰, reads as:

becomes a very popular tool in dynamical calculations. The one term separable non-local potential proposed by Yamaguchi³⁰ provides an adequate account of the binding energy of the deuteron and the low energy two nucleon scattering parameters as well. Since many of the two-term separable potentials like two-term Tabakin potential³⁵, Case IV Mongan potential³⁶ etc. which fit the two-nucleon phase shifts use the Yamaguchi potential for the attractive term, it is convenient to study the potential first. For the nonlocal Yamaguchi potential the regular and irregular solutions²⁸ read as:

$$\phi_N(k, r) = \frac{\sin kr}{k} + C \left[e^{-\beta r} + \frac{\beta}{k} \sin kr - \cos kr \right] \quad \dots (9)$$

and

$$f_N(k, r) = e^{ikr} + \frac{\lambda(\beta + ik)}{D(k)(\beta^2 + k^2)^2} e^{-\beta r} \quad \dots (10)$$

with the Fredholm determinant

$$D(k) = 1 - \frac{\lambda}{2\beta(\beta^2 + k^2)} \quad \dots (11)$$

and

$$C = \frac{\lambda}{D(k)(\beta^2 + k^2)^2} \quad \dots (12)$$

The damping function $A(k, r)$ corresponding to Yamaguchi potential^{28,29} is written as:

$$A(k, r) = (1 + k^{-1} C_S e^{-\beta r} \sin kr)^{1/2} \quad \dots (13)$$

with

$$C_S = \frac{\lambda}{(\beta^2 + k^2 - \lambda/2\beta)} \quad \dots (14)$$

Combining Eqs (7), (10), (13) and (14) the bound state solution of the equivalent local Yamaguchi potential can easily be obtained by substituting $k = i\alpha$. As the zero of the irregular solution produces the bound state energy one gets $\lambda = -2\beta(\beta + \alpha)^2$. Therefore, near the origin the bound state solution of equivalent Yamaguchi potential behaves as:

$$\Psi_L^{(0)} = \lim_{r \rightarrow 0} f_L^{(B)}(\alpha, r) = \frac{e^{-\alpha r} - e^{-\beta r}}{(1 + C_B r e^{-\beta r})^{1/2}} \quad \dots (15)$$

with

$$C_B = \frac{\lambda}{(\beta^2 - \alpha^2 - \lambda/2\beta)} \quad \dots (16)$$

In view of Eqs (2), (15) and (16) the raising operator $O^{(-)}$ leads to:

$$O^{(-)} = -\frac{\partial}{\partial r} + \frac{(\beta e^{-\beta r} - \alpha e^{-\alpha r})}{(e^{-\alpha r} - e^{-\beta r})} - \frac{1}{2} \frac{C_B e^{-\beta r} (1 - \beta r)}{(1 + C_B r e^{-\beta r})} \quad \dots (17)$$

From Eqs (5), (6), (9), (11), (13) and (17) the desired P-wave regular solution for the potential $V_1(r)$ is obtained as:

$$\begin{aligned} \varphi_L^p(k, r) = & -\frac{\varphi'_N(k, r)}{A(k, r)} + \frac{\varphi_N(k, r)A'(k, r)}{(A(k, r))^2} \\ & + \frac{(\beta e^{-\beta r} - \alpha e^{-\alpha r})}{(e^{-\alpha r} - e^{-\beta r})} \frac{\varphi_N(k, r)}{A(k, r)} \quad \dots (18) \\ & - \frac{1}{2} \frac{C_B e^{-\beta r} (1 - \beta r)}{(1 + C_B r e^{-\beta r})} \frac{\varphi_N(k, r)}{A(k, r)} \end{aligned}$$

Construction of D-wave potential requires the behavior of $\varphi_L^p(k, r)$ near the origin. As $r \rightarrow 0$, $\varphi_L^p(k, r)$ is written as:

$$\begin{aligned} \varphi_L^{p(0)}(k, r) = & -\frac{\varphi_N^{(0)}(k, r)}{A^{(0)}(k, r)} + \frac{\varphi_N^{(0)}(k, r)A'^{(0)}(k, r)}{(A^{(0)}(k, r))^2} \\ & + \frac{(\beta e^{-\beta r} - \alpha e^{-\alpha r})}{(e^{-\alpha r} - e^{-\beta r})} \frac{\varphi_N^{(0)}(k, r)}{A^{(0)}(k, r)} \\ & - \frac{1}{2} \frac{C_B e^{-\beta r} (1 - \beta r)}{(1 + C_B r e^{-\beta r})} \frac{\varphi_N^{(0)}(k, r)}{A^{(0)}(k, r)} \quad \dots (19) \end{aligned}$$

with

$$\varphi_N^{(0)}(k, r) = r + C(e^{-\beta r} + \beta r - 1), \quad \dots (20)$$

$$\varphi_N^{\prime(0)}(k, r) = 1 + C\beta(1 - e^{-\beta r}), \quad \dots (21)$$

$$A^{(0)}(k, r) = 1 + C_S r e^{-\beta r} \quad \dots (22)$$

and

$$A^{\prime(0)}(k, r) = C_S(1 - \beta r)e^{-\beta r} \quad \dots (23)$$

By following the relation (3) one gets:

$$V_2(k, r) = V_1(k, r) - \frac{\partial^2}{\partial r^2} \ln \varphi_L^{p(0)}(k, r) \quad \dots (24)$$

with

$$\begin{aligned} \frac{\partial^2}{\partial r^2} \ln \varphi_L^{p(0)}(k, r) = & \frac{T_1''(k, r) + T_2''(k, r) + T_3''(k, r) + T_4''(k, r)}{T_1(k, r) + T_2(k, r) + T_3(k, r) + T_4(k, r)} \quad \dots (25) \\ & - \left(\frac{T_1'(k, r) + T_2'(k, r) + T_3'(k, r) + T_4'(k, r)}{T_1(k, r) + T_2(k, r) + T_3(k, r) + T_4(k, r)} \right)^2 \end{aligned}$$

$$T_1(k, r) = -\frac{\varphi_N^{\prime(0)}(k, r)}{A^{(0)}(k, r)} = -\frac{1 + C\beta(1 - e^{-\beta r})}{(1 + C_B r e^{-\beta r})^{1/2}}, \quad \dots (26)$$

$$\begin{aligned} T_2(k, r) = & \left[\frac{\varphi_N^{(0)}(k, r)}{A^{(0)}(k, r)} \right] \left[\frac{A^{\prime(0)}(k, r)}{A^{(0)}(k, r)} \right] \\ = & T_{21}(k, r)T_{22}(k, r), \quad \dots (27) \end{aligned}$$

$$\begin{aligned} T_3(k, r) = & \left[\frac{\varphi_N^{(0)}(k, r)}{A^{(0)}(k, r)} \right] \left[\frac{(\beta e^{-\beta r} - \alpha e^{-\alpha r})}{(e^{-\alpha r} - e^{-\beta r})} \right] \quad \dots (28) \\ = & T_{31}(k, r)T_{32}(k, r), \end{aligned}$$

$$\begin{aligned} T_4(k, r) = & \left[\frac{\varphi_N^{(0)}(k, r)}{A^{(0)}(k, r)} \right] \left[-\frac{1}{2} \frac{C_B e^{-\beta r} (1 - \beta r)}{(1 + C_B r e^{-\beta r})} \right] \\ = & T_{41}(k, r)T_{42}(k, r), \quad \dots (29) \end{aligned}$$

$$\begin{aligned} T_{21}(k, r) = T_{31}(k, r) = T_{41}(k, r) \\ = & \frac{C(e^{-\beta r} - 1)}{(1 + C_B r e^{-\beta r})^{1/2}}, \quad \dots (30) \end{aligned}$$

$$T_{22}(k, r) = \frac{1}{2} \frac{C_B e^{-\beta r} (1 - \beta r)}{(1 + C_B r e^{-\beta r})}, \quad \dots (31)$$

$$T_{32}(k, r) = \frac{(\beta e^{-\beta r} - \alpha e^{-\alpha r})}{(e^{-\alpha r} - e^{-\beta r})}, \quad \dots (32)$$

and

$$T_{42}(k, r) = -\frac{1}{2} \frac{C_B e^{-\beta r} (1 - \beta r)}{(1 + C_B r e^{-\beta r})} \quad \dots (33)$$

Here $T_i'(k, r)$ ($i = 1, 2, 3, 4$) and $T_i''(k, r)$ ($i = 1, 2, 3, 4$) stand for the single and double derivatives of $T_i(k, r)$ ($i = 1, 2, 3, 4$) with respect to r . The other quantity $V_1(k, r)$ in (24) refers to the P-wave potential²⁸ and is given by:

$$V_1(k, r) = V_0(k, r) + \frac{J''(k, r)}{2J(k, r)} - \frac{1}{2} \left[\frac{J'(k, r)}{J(k, r)} \right]^2 + \frac{\beta^2 e^{-\beta r}}{(e^{-\beta r} - 1)^2}, \quad \dots (34)$$

where

$$V_0(k, r) = -\frac{J''(k, r)}{2J(k, r)} + \frac{3}{4} \left[\frac{J'(k, r)}{J(k, r)} \right]^2 + \frac{\lambda e^{-\beta r}}{(\beta^2 + k^2)J(k, r)} \left(1 + \frac{\lambda}{2\beta(\beta^2 + k^2)D(k)} \right) \times (k \sin kr + \beta \cos kr) \quad \dots (35)$$

with

$$J(k, r) = [A(k, r)]^2 \quad \dots (36)$$

The energy independent counterpart²⁵ of $V_2(r)$ in (24) reads as:

$$V_{2H}(r) = V_{0H}(r) + \frac{6(\beta - \alpha)^2}{(e^{-\alpha r} - e^{-\beta r})^2} e^{-(\alpha + \beta)r} \quad \dots (37)$$

with

$$V_{0H}(r) = -(\beta^2 - \alpha^2) \frac{e^{-\beta r}}{(e^{-\alpha r} - e^{-\beta r})} \quad \dots (38)$$

Here the subscript H stands for the nuclear Hulthén potential and $V_{0H}(r)$ and $V_{2H}(r)$ are the S- and D-wave nuclear Hulthén potentials. The N-N scattering phase shifts for the partial wave $\ell = 2$ will be computed both for the energy-dependent local Yamaguchi $V_2(k, r)$ and energy independent nuclear Hulthén $V_{2H}(r)$ potentials by the application of the phase equation³⁷:

$$\delta'_\ell(k, r) = -k^{-1}V(r) [\hat{j}_\ell(kr) \cos \delta_\ell(k, r) - \hat{\eta}_\ell(kr) \sin \delta_\ell(k, r)]^2 \quad \dots (39)$$

The quantities $\hat{j}_\ell(kr)$ and $\hat{\eta}_\ell(kr)$ are the Riccati Bessel functions.

3 Results and Discussion

In Figs 1 and 2 we portray the energy-dependent and independent N-N potentials as a function of

distance for the partial wave $\ell = 2$ with the 3S_1 parameters $\lambda = -3.901 \text{ fm}^{-3}$, $\beta = 1.095 \text{ fm}^{-1}$ and $\alpha = 0.232 \text{ fm}^{-1}$. These potentials, generated via the supersymmetry formalism from 3S_1 parameters, correspond to 3D_1 state. It is noticed that repulsive cores develop in the D-wave potentials. For the energy dependent potentials the strength of the repulsive cores decreases as energy increases ensuring more and more overlap of the particles. In Fig. 3 we have plotted the corresponding phase shifts for the energy-dependent and independent interactions and compared them with those of standard data^{3,24}. Looking closely into phase shift plots it is clearly seen that phase shift values of energy independent potentials compare well with that of Arndt *et al.*³ and Gross-Stadler²⁴ up to 100 MeV and beyond that they differ significantly. On the other hand,

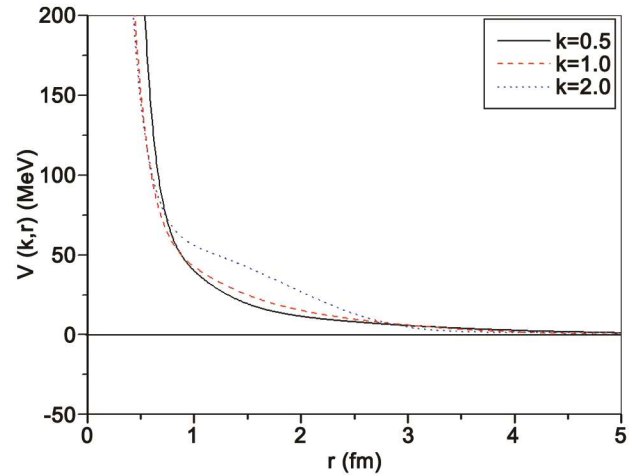


Fig. 1 — 3D_1 Energy-dependent potential as a function of r .

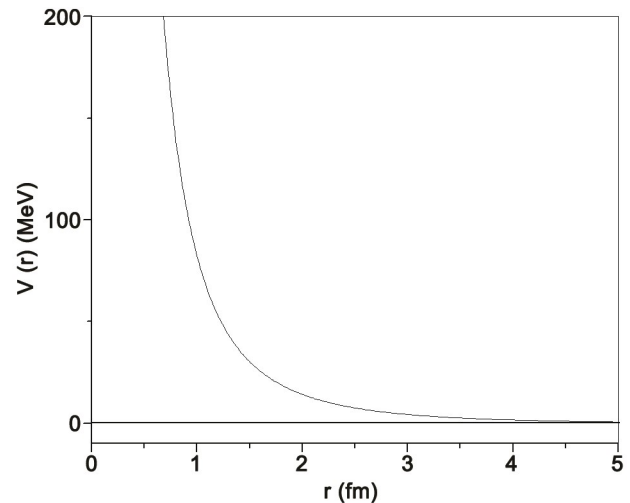


Fig. 2 — 3D_1 Energy-independent potential as a function of r .

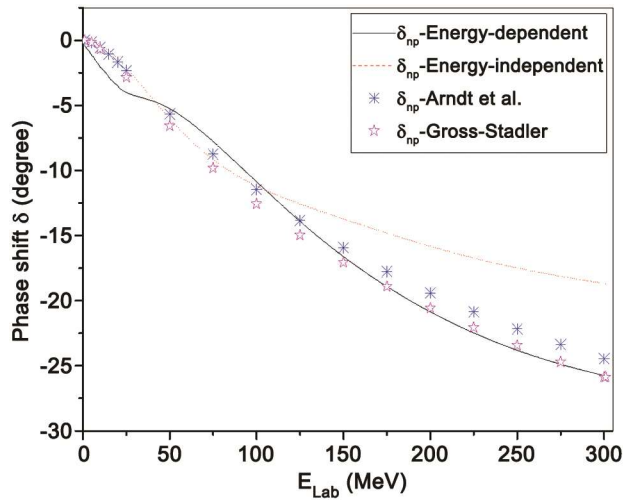


Fig. 3 — 3D_1 Energy-dependent and independent phase shifts as a function of E_{Lab} .

although the phase shifts of energy-dependent potential differ slightly with standard results^{3,24} in the low energy range but are in exact agreement with literature^{3,24} beyond 30 MeV. Therefore, it is of importance to note that our higher partial wave two parameter energy-dependent and independent potentials, generated via supersymmetry formalism, have the ability to describe N-N scattering and deserve some attention. We have also verified that 1S_0 parameters are unable to produce 1D_2 phase shifts.

4 Conclusions

The present text addresses itself to a comparative study of the energy-dependent and independent potentials, developed within the formalism of supersymmetric quantum mechanics, in the partial wave $\ell=2$. The D-wave interactions are generated by the addition of certain terms with their ground state potentials. These additional terms behave as centrifugal barriers in the higher partial wave potentials. The Hamiltonian hierarchy corresponds to the addition of an appropriate centrifugal potential and consequently the higher partial wave potentials are generated fairly accurately in atomic physics. In contrast to atomic case, the nuclear interactions are strongly state dependent in which separate strength and range parameters are indispensable for correct description in various partial waves. One of the reasons for undertaking this calculation is to examine how far the algebra of SQM becomes effective for deducing higher partial wave potentials from its ground state information in subatomic realm. Even though the supersymmetry generated D-wave

potentials, both energy-dependent and independent, are not capable of producing accurate phase shifts over the entire energy range under consideration it can still consider to be useful as a source of rough trial potentials. From our observations we conclude by noting that energy-dependent potential is superior to its energy independent counterpart and an inherent symmetry is embedded in the triplet series of two-nucleon forces that facilitate supersymmetry operation in contrary to singlet series.

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