Indian Journal of Pure & Applied Physics
Vol. 56, December 2018, pp. 941-948

Spectra of distorted quantum ring in external fields

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Received 8 November 2017; accepted 15 May 2018

The spectra and response of a single electron on a distorted quantum ring have been studied in detail. The distortion in the ring has been taken care by a geometrical factor. We take few different cases of distortion. Role of strength of the distortion along with the external static and laser field on spectrum has been studied.

Keywords: Quantum ring, Coupling matrix elements, Finite difference method

1 Introduction

Ring-shaped semiconductor nanostructures, called quantum rings (QRs) or nanorings, have been applied to many revolutionary applications in nanoelectronics, high-density memory and spintronics[1-4]. QRs have been used in technological applications such as storage devices, fluorescent markers and may constitute the core of the laser emitters. These applications lead to a variety of theoretical and experimental studies on these quantum structures[5-18]. In addition, quantum rings with small radius are unique systems as they provide platform to study pure quantum effects such as Ahronov-Bohm (AB) oscillations, persistent currents, etc.[19]

Quantum heterostructures meet with some defects in the process of manufacturing. Fortunately, the defects make these structures more attractive, as controlling these defects in a suitable manner lead to impressive structure and other properties. Geometry of QRs causes interesting applications of its structure in nanoelectronics and spintronics.

Shape effects in the case of one-dimensional quantum rings have shown prominence during the last few years. As shown by Hai-Tao *et al.*[20] that elliptical rings have longer period and larger amplitudes of the AB oscillations. Further, the electronic states in QRs with arbitrary shapes have been studied by Bruno-Alfanso *et al.*[21] Effect of donor impurity dislocation in elliptical quantum rings has been shown by Salehani *et al.*[22] and the energy spectrum of on-axis negatively charged donor in toroidal shaped ring by Bentacur *et al.*[23] The effects of shape on electronic states with an impurity inside a QR have been analysed analytically[24]. In addition, the bound electron states in the presence of magnetic field have been calculated analytically for an impurity centered on a semiconductor QR[25]. Due to many practical applications, quantum rings with distortions, has been subject of theoretical studies[26,27]. The deviation of circular geometry significantly influences the energy spectrum.

As mentioned, the electronic properties of quantum rings in external static and time varying fields have been subject of intense research, because of applications of these quantum rings of different shapes. Also, technological advancement in the field of epitaxial growth techniques, now it is possible to fabricate rings of desired shapes and sizes[28-30]. The electronic properties of circular quantum rings in the presence of external fields have been studied in detail, but distorted quantum rings have been studied scarcely. In particular, the studies have been limited to delta type potentials or a potential barrier of fixed height at particular location[31]. Chwiej and Szafran[32] studied spectra of distorted quantum rings with point defect distortions, with distortions at one, two and three points and have shown the effect of number of locations of defects in the presence of magnetic field. However, in this study, we focus on the spectrum of distorted QRs without any external field. The knowledge of such structures along with the matrix elements will be of great use for many practical applications. The distortion is taken care by a geometrical factor defined later in Eq. (2). Such potentials have been used to study various

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imperfections in many quantum heterostructures[33,34]. The confined rotors like molecules have also been studied by many using potentials like this for confinement[35-38]. These studies have shown that the potentials adopted reproduce reasonable agreements with the earlier studies. In addition, we study the interaction of external static electric field along with the laser field with the distorted ring and study different order parameters defined as $\langle\cos^n\theta\rangle$, where n=1,2,3. The evaluation of these order parameters, help in understanding the response of these distorted rings in external fields.

2 Theory

An electron in a one-dimensional distorted quantum ring can be described by the Hamiltonian:

$$H_0 = -\frac{\hbar^2}{2m_e^* R^2}\frac{d^2}{d\theta^2} + V'(\theta) \qquad \dots (1)$$

where R is the radius of the ring, m_e^* is the mass of the electron in the ring, $V'(\theta)$ is the potential representing the distortion in the ring.

$$V'(\theta) = V_0(\theta) \qquad \dots (2)$$

where $V_0(\theta) = \frac{1}{2}V_d'(1+\cos(2\theta))$ is strength of the distortion.

Three different cases of angular distortion in the ring are taken as:

Case (a): Weak distortion

$$V'(\theta) = V_0(\theta) \text{ for } \pi/4 \le \theta \le \pi/3 \qquad \dots (3)$$

Case (b): Moderate distortion

$$V'(\theta) = V_0(\theta) \text{ for } \pi/4 \le \theta \le \pi/2 \qquad \dots (4)$$

Case (c): Strong distortion

$$V'(\theta) = V_0(\theta) \text{ for } \pi/4 \le \theta \le 3\pi/4 \qquad \dots (5)$$

The Schrodinger equation for the system is:

$$\left[-\frac{\hbar^2}{2m_e^* R^2}\frac{d^2}{d\theta^2} + V'(\theta)\right]\phi = E'\phi \qquad \dots (6)$$

$$\left[-\frac{d^2}{d\theta^2} + \frac{V'(\theta)}{B_e}\right]\phi = \frac{E'}{B_e}\phi \qquad \dots (7)$$

where $B_e = \frac{\hbar^2}{2m_e^* R^2}$

$$\left[-\frac{d^2}{d\theta^2} + V(\theta)\right]\phi = E\phi \qquad \dots (8)$$

Now the potential, $V(\theta)$, V_d and the energy Eigen values, E, are in terms of B_e.

This equation has been solved numerically by accurate nine point finite difference method[38] to obtain the Eigen values and the coupling matrix elements of various orders. It is well known that the electron transitions from ground state to various excited states depend on these coupling elements.

The system is then subjected to the external static electric field and the laser field. The interaction of the system with the static electric field is via '$\cos\theta$' term while with the laser field, it is '$\cos^2\theta$' term.

The material of which the semiconducting one-dimensional quantum ring is composed of, is considered to contain some polarizability parameter through which the laser field may interact. This interaction is averaged to $0.5\,\beta E_L^2\cos^2\theta$, where E_L is the laser field strength, β is the anisotropy parameter and θ is the angle between the laser field and position of the electron.

The Schrodinger equation in the presence of external static electric field and laser field becomes:

$$\left[H_0 + H_{int}^S + H_{int}^L\right] = \varepsilon'\psi \qquad \dots (9)$$

where H_0 is the Hamiltonian defined by (1), $H_{int}^S + H_{int}^L$ is the external perturbation with

$$H_{int}^S = -\mu E_S' \cos\theta \qquad \dots (10)$$

where H_{int}^S is the interaction of static electric field with the system, μ is the dipole moment operator and E_L' is the electric field strength,

and

$$H_{int}^L = -(1/2)\beta E_L'^2 \cos^2\theta \qquad \dots (11)$$

is the interaction of the system with the laser field, E_L' is the strength of the laser field.

In order to define quantities in dimensionless units, let $E_S = \mu E_S'/B_e$ and $E_L = (1/2)\beta E_L'^2/B_e$.

Similar to Eq. (8), we solve Eq. (9) in dimensionless units using E_S and E_L as interaction energies of static and electromagnetic field, respectively. We expand the wave function ψ in terms of the wavefunction of Eq. (8), i.e., $\psi = \sum_n C_n\psi_n$.

3 Results and Discussion

We present our results in units of $B_e = \frac{\hbar^2}{2m_e^* R^2}$ for the sake of simplicity. The energy spectra for one electron distorted QR is presented in Fig. 1. The energy variation of lowest lying few states is shown with the strength of the distortion, V_d. Without distortion (V_d =0), only ground state energy is zero. For higher states, the energy of states increases as 'm^2' in the absence of distortion. For all the states, the Eigen values increase with increase in the strength of distortion. The increase is maximum for the 'strong' case, intermediate for 'moderate' and least for the 'weak' case.

Figure 2 shows the variation of transition matrix elements, $<\psi_n|cos\theta|\psi_m>$, with V_d, for ground state (0,0) and first eight excited states of distorted QR for 'strong', 'moderate' and 'weak' case, given by Eq. (2) in

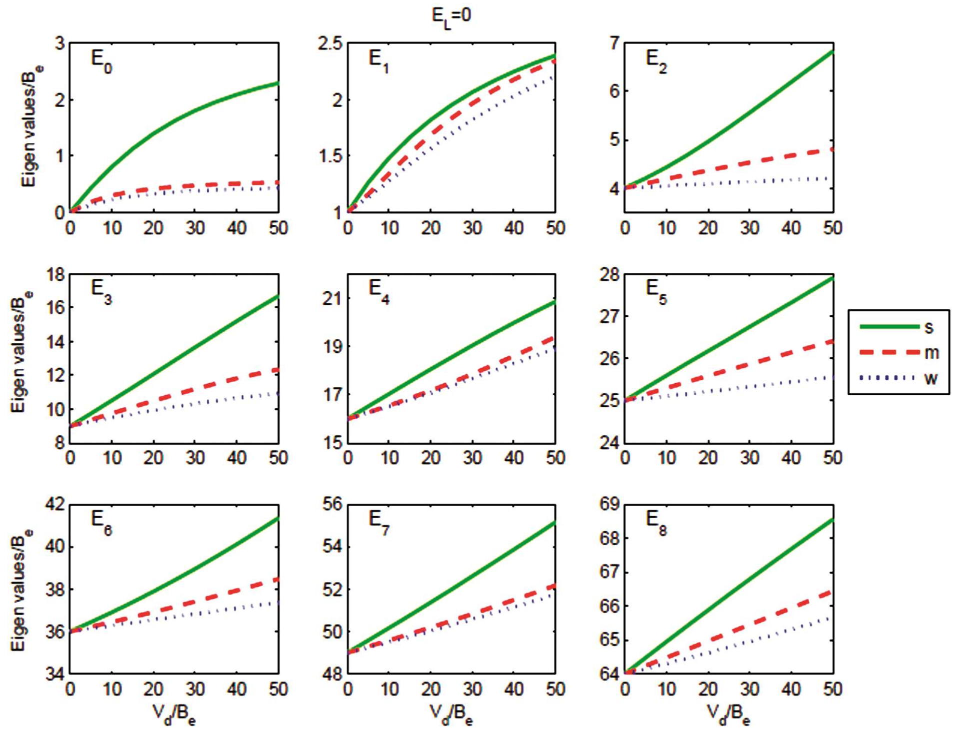

Fig. 1 — Variation of energy Eigen values of the lowest eight states of single electron on the distorted ring with V_d, for three different angular distortions in the ring.

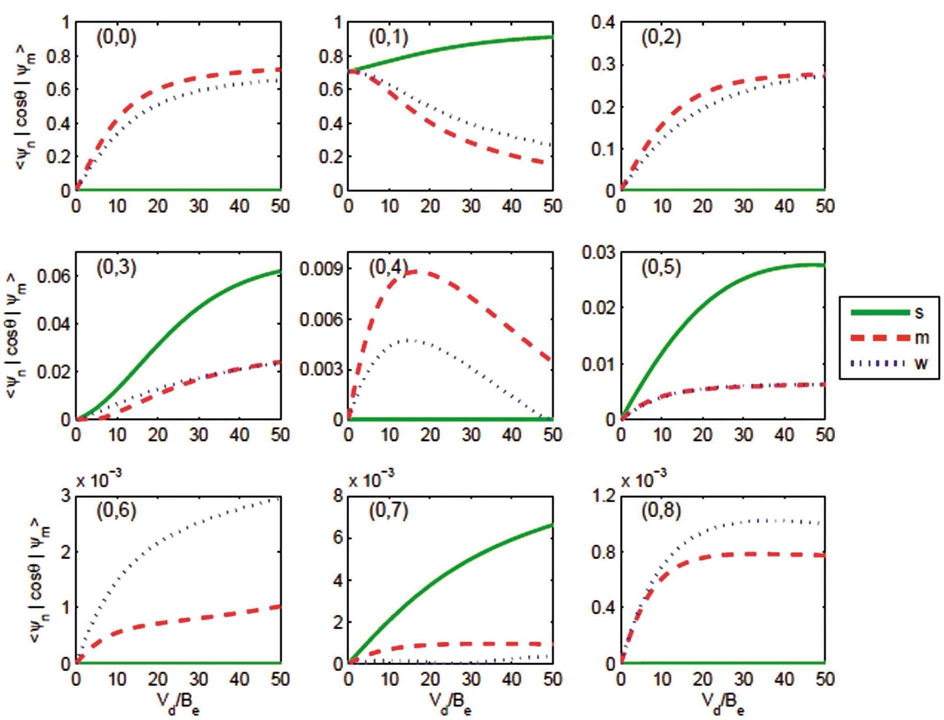

Fig. 2 — Variation of $<\psi_n|cos\theta|\psi_m>$ of single electron on the distorted ring with V_d, for three different angular distortions in the ring.

the theory. For 'strong' case, the value of matrix element is zero for all values of V_d for (0,0) and (0,even) states, its value being maximum for (0,1) state. The values keep on decreasing for states higher than the first excited state.

In Fig. 3 is presented the transition matrix elements, $<\psi_n|cos^2\theta|\psi_m>$, versus V_d for the lowest nine states of distorted QR. The values of transition matrix elements for the 'strong' case are zero for states (n,m)=(0,odd), the values being maximum for the ground state.

Figure 4 shows the variation of transition matrix elements, $<\psi_n|cos^3\theta|\psi_m>$, versus V_d. The values

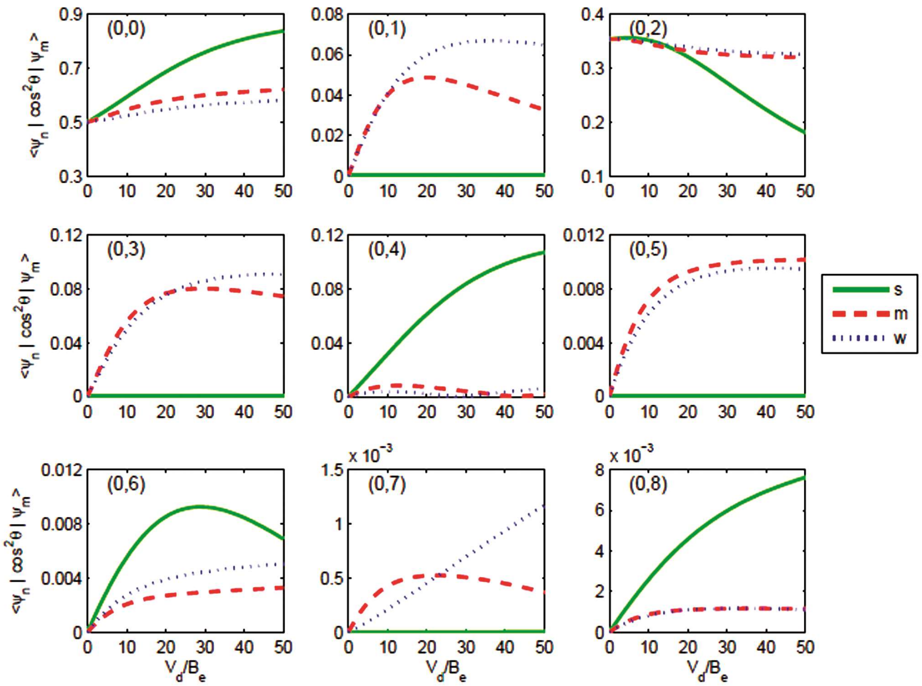

Fig. 3 — Variation of $<\psi_n|cos^2\theta|\psi_m>$ of single electron on the distorted ring with V_d for three different angular distortions in the ring.

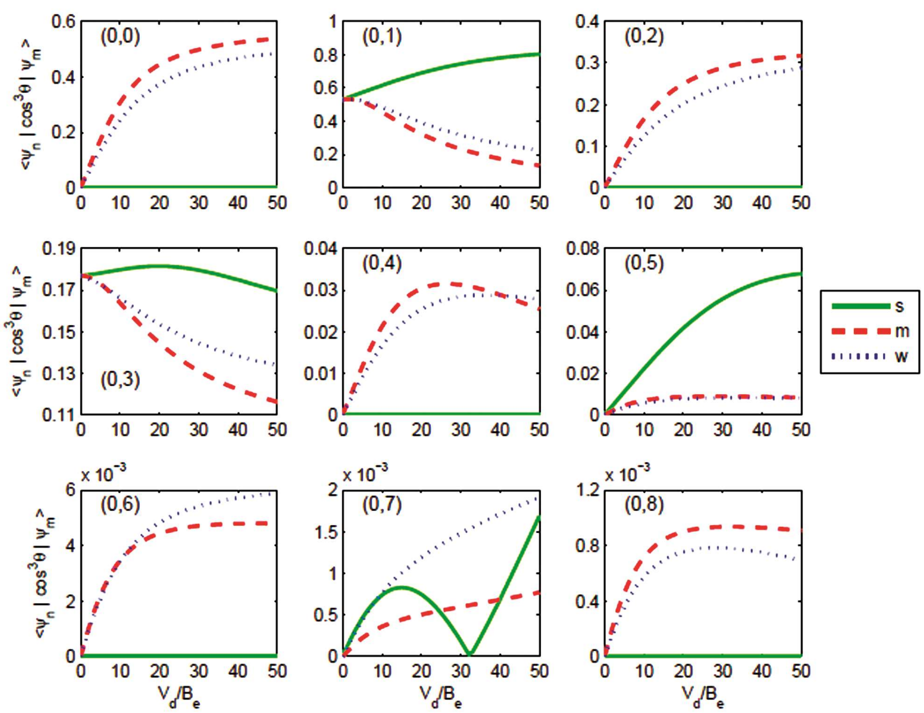

Fig. 4 — Variation of $<\psi_n|cos^3\theta|\psi_m>$ of single electron on the distorted ring with V_d for three different angular distortions in the ring.

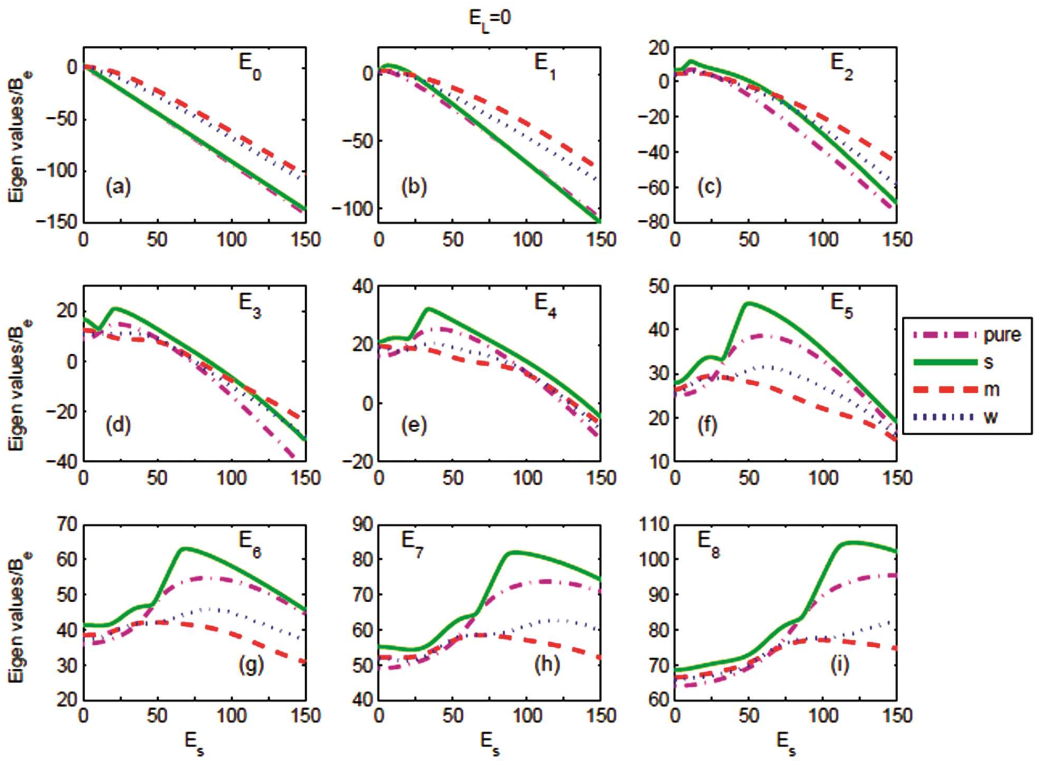

Fig. 5 — Variation of energy Eigen values of the lowest eight states of single electron on the distorted ring with static electric field, E_S, without distortion (pure) and with three different angular distortions in the ring, in the absence of laser field.

of transition matrix elements for the 'strong' case are zero for states m=0, 2, 4, 6 and 8. Its value is maximum for the first excited state and keeps on decreasing as we go to higher excited states. The effects in the Figs (1-4) are due to the fact that without the distortion, the spectrum is that of pure ring, where $E_m=m^2$ and $<\psi_n|cos\theta|\psi_m>$ for n=0, m=1 and 0.5 for $m=n\pm1$. Similarly matrix elements $<\psi_n|cos^2\theta|\psi_m>$ and $<\psi_n\left|\cos^3\theta\right|\psi_m>$ are well known, and our method of numerical diagonalization reproduces it exactly. However, in the presence of distortion, the Schrodinger equation can be solved numerically only. So with the distortion, the wave functions and energies change, as expected.

In Figs 5-9, variation of various physical quantities have been shown with the static electric field, E_S, with V_d =50. Figure 5 shows the variation of energy Eigen values with static electric field without laser field for four cases: (i) without distortion, i.e., V_d =0 (pure), (ii) strong, (iii) moderate and (iv) weak. For all the four cases, there is a change in the Eigen values with increase in the static electric field. For ground state, the Eigen values decrease from zero to negative values as E_S is increased. For E_3, it first decreases, then increases up to a maximum value and then decreases as E_S is increased. This peak value position shifts to higher E_S values as we approach higher states being approximately at E_S =110 for E_8.

Figure 6 is same as Fig. 5, but now in addition to E_S there is presence of laser field with its strength E_L=120. The value of ground state energy becomes-100 for E_S =0, which was zero in the absence of laser field. Similarly with the presence of laser field, the Eigen values of all the states change as shown in the figure. Another feature which arises due to laser field is that the values oscillate as the static electric field is increased.

In Fig. 7 is shown the variation of probability of lowest nine states with the static electric field by taking E_L=20, where $P_{n\text{-}m}$ represents the probability of excitation from n^{th} to m^{th} state. The probability is shown for all the three cases. As can be seen, the probability for the 'strong' case shows large oscillations compared to 'moderate' or 'weak' case. In case of weak distortion, the ground state probability decreases with increase in field. However, the probability of excitation for higher states shows remarkable increase with increase of E_s. As mentioned, this behavior can be explained on the basis of variation of $<\psi_n|cos\theta|\psi_m>$ and effect of higher order interactions.

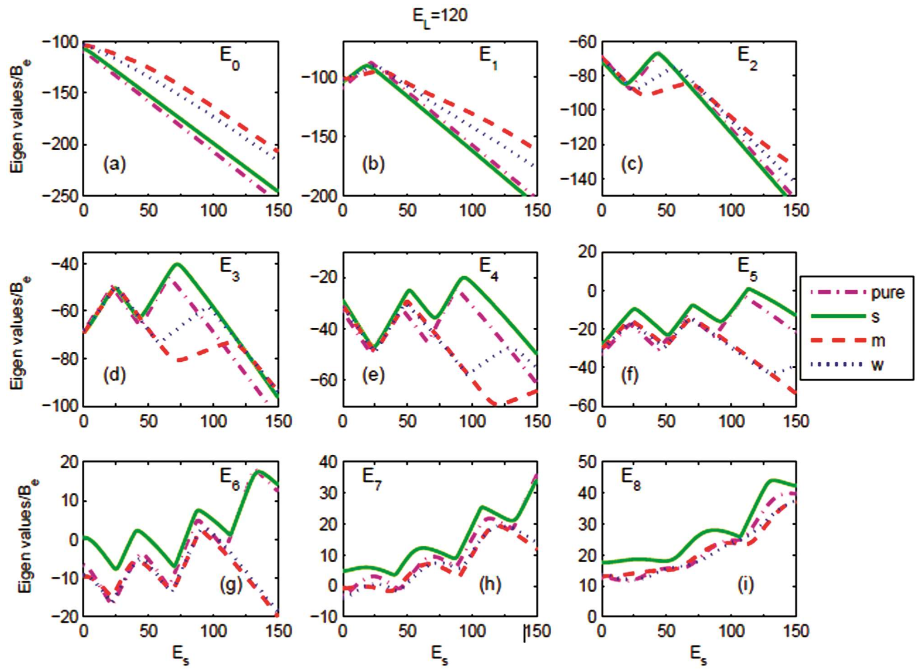

Fig. 6 — Variation of energy eigenvalues of the lowest eight states of single electron on the distorted ring with static electric field, E_S, without distortion (pure) and with three different angular distortions in the ring, in the presence of laser field with field strength 120 in terms of B_e.

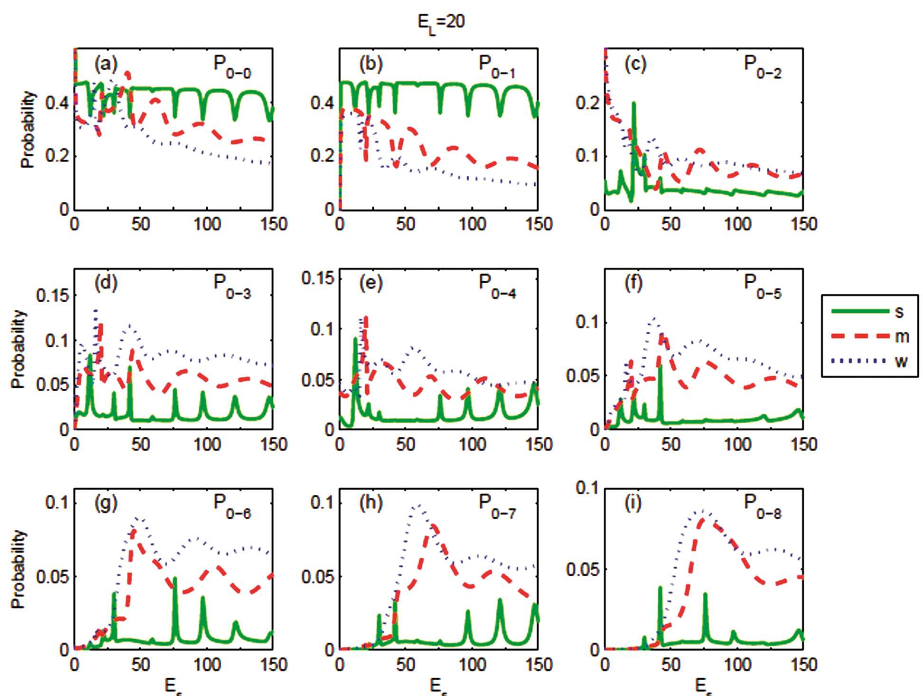

Fig. 7 — Variation of probability of the lowest eight states of single electron on the distorted ring with static electric field E_S for three different angular distortions in the ring, in the presence of laser field.

Figure 8 shows the orientation parameter <cosθ> for ground state and transitions from ground to first eight excited states, with static electric field for E_L=20. For the ground state, the value of <cosθ> for the 'weak and 'moderate' cases rapidly increase to ~1, while it is lower for the 'strong' case. For higher state excitations, there are oscillations for the ' strong' case. There is small difference between the 'weak and 'moderate' cases. For 'strong' case, there is a dip in the value of <cosθ> and the position of dip shifts to higher E_S value when one goes from first excited state (panel b) to the eighth excited state (panel i).

In Fig. 9 is presented the alignment parameter, <$\cos^2\theta$ >, with E_S by taking E_L=20. For the ground state, the value of <$\cos^2 \theta$> increases to ~1 rapidly for the 'weak' case, at E_S =10 for the moderate case, while

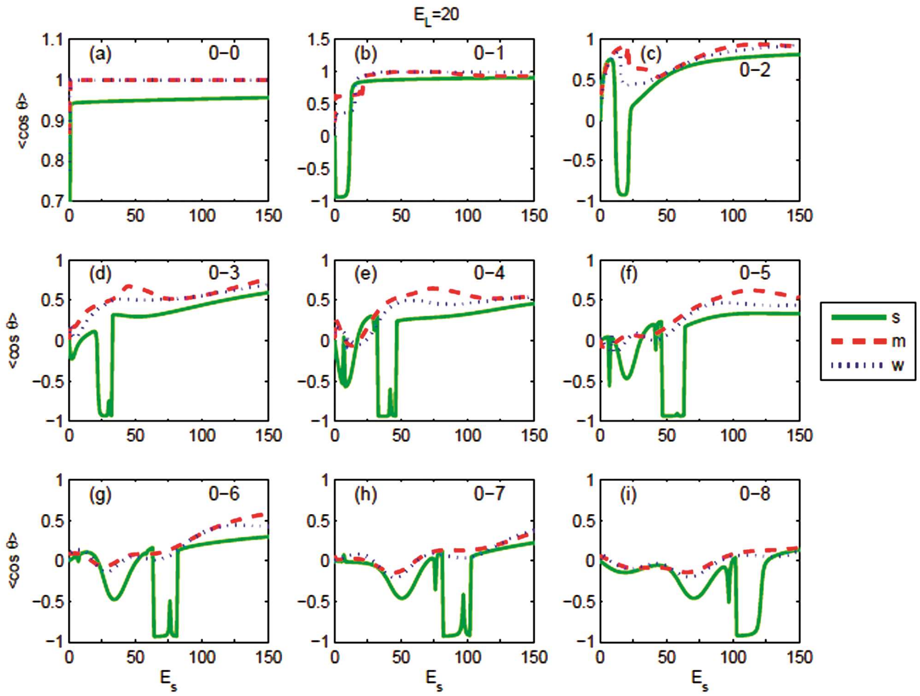

Fig. 8 — Variation of <cosθ> of the lowest eight states of single electron on the distorted ring with static electric field, E_S, for three different angular distortions in the ring, in the presence of laser field.

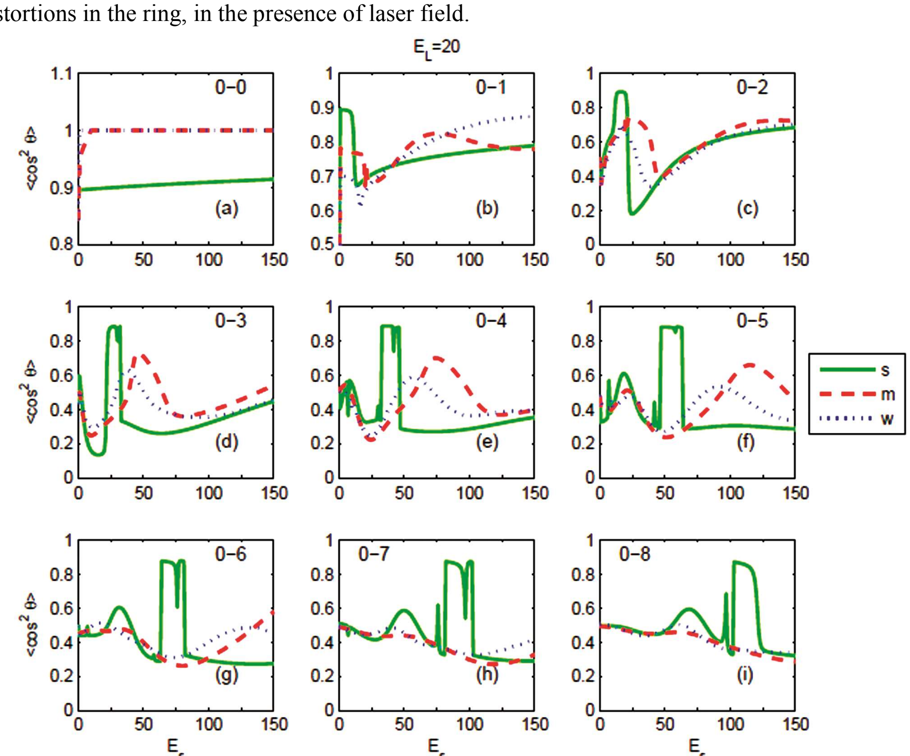

Fig. 9 — Variation of <$\cos^2\theta$> of the lowest eight states of single electron on the distorted ring with static electric field, E_S, for three different angular distortions in the ring, in the presence of laser field.

it is lower for the 'strong' case. For higher states, the value of <$\cos^2 \theta$> approached ~0.84 or more for 'strong' case and the position of this maximum shifts towards higher electric field. For 'moderate' and 'weak' cases, for n=7 and 8 states, it decreases slightly from 0.5 for higher values of E_S. In case of 'weak and 'moderate' distortions, the value of <$\cos^2\theta$> oscillates but remains smaller than the value in 'strong' case.

4 Conclusions

The spectrum and coupling matrix elements of a single electron on a distorted quantum ring vary as the strength of the distortion is increased. A significant change in energies and matrix elements is observed when the 'strong' case is considered. When the laser field is introduced, oscillations are observed in Eigen values, probability and the order parameters of states of electron with the static electric field.

Acknowledgement

One of us (PS) is grateful to University of Delhi for providing the funds under the 'Scheme to strengthen Research and Development'.

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