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Magnetohydrodynamic stagnation point flow past a porous stretching surface with heat generation

Santosh Chaudhary* & Pradeep Kumar

Department of Mathematics, Malaviya National Institute of Technology, Jaipur 302 017, India

E-mail: *d11.santosh@yahoo.com, pradeep17matrix@gmail.com

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A general analysis has been developed to study the two-dimensional, laminar flow of a viscous, incompressible, electrically conducting fluid near a stagnation point of a stretching sheet through a porous medium with heat generation in the presence of a magnetic field. The governing boundary layer equations have been transformed to ordinary differential equations by using suitable similarity variables. The solutions of momentum and energy equations have been obtained independently by a perturbation technique for a small magnetic parameter. The effects of various parameters such as magnetic parameter, porosity parameter, stretching parameter, Prandtl number, Eckert number and heat generation coefficient for velocity and temperature distributions along with local skin friction coefficient and local Nusselt number have been studied in detail through graphical and numerical representations.

Keywords: Magnetohydrodynamic, Stagnation point, Porous stretching surface, Heat generation

1 Introduction

Flow of an incompressible viscous fluid over a stretching surface has important applications in the industry such as the extrusion of polymer in a melt-spinning process, the aerodynamic extrusion of plastic sheets, manufacturing plastic films, artificial fibers etc. Further, glass blowing, continuous casting of metals and spinning of fibers involve the flow due to a stretching surface. Crane[1] probably was first who studied the flow at a stretching sheet and produced a similarity solution in closed analytical form for the steady two-dimensional problem. Gupta and Gupta[2], Dutta *et al*[3]., Chiam[4], Mahapatra and Gupta[5], Andersson[6], Elbashbeshy and Bazid[7], Miklavcic and Wang[8] and Jat and Chaudhary[9] studied the heat transfer to steady the two-dimensional stagnation point flow over a stretching surface taking into account different aspects of the problem.

Recently, boundary layer flow through porous media is a subject of great interest due to its various applications such as oil recovery, composite manufacturing processes, filtration processes, paper and textile coating, geothermal engineering. Its engineering and geophysical applications are flow of groundwater, geothermal energy utilization, insulation of buildings, energy storage, recovery and chemical reactor engineering. Attia[10], Jat and Chaudhary[11,12], Pal and Hiremath[13], Bhattacharyya and Layek[14], Rosali *et al*[15]., Singh and Pathak[16], Mukhopadhyay and Layek[17] and Ram *et al*[18]. studied the boundary layer flow near the stagnation point of a stretching sheet through porous and non-porous boundaries under different physical situations. Very recently Mahapatra and Nandy[19] analyzed a stability of dual solutions in stagnation-point flow and heat transfer over a porous shrinking sheet with thermal radiation.

In the present paper, steady two-dimensional stagnation point flow has been investigated in a porous medium with heat generation of an electrically conducting fluid over a stretching surface in the presence of magnetic field. The results of velocity and temperature distribution, skin friction and surface heat transfer for different parameters such as the magnetic parameter, the porosity parameter, the stretching parameter, the Prandtl number, the Eckert number and the heat generation coefficient have been obtained.

2 Formulation of the Problem

Consider the steady two-dimensional stagnation point flow $(u,v,0)$ in a porous medium with heat generation of a viscous incompressible electrically conducting fluid near a stagnation point at a surface placed in the plane y=0 of a Cartesian coordinates system with the x-axis along the surface, such that the surface is stretched in its own plane with velocity proportional to the distance from the stagnation point in the presence of an externally applied normal magnetic field of constant strength $(0,H_0,0)$. The stretching surface has velocity u_w and temperature T_w, while the velocity of the flow external to the boundary layer is u_e and temperature T_∞. The system of boundary layer equations (refer to Fig. 1) is given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad ... (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{du_e}{dx}+\upsilon\frac{\partial^2 u}{\partial y^2}+\frac{\upsilon}{K}\left(u_e-u\right)-\frac{\sigma_e\mu_e^2H_0^2u}{\rho} \quad ... (2)$$

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\kappa\frac{\partial^2 T}{\partial y^2}+Q\left(T-T_\infty\right)+\mu\left(\frac{\partial u}{\partial y}\right)^2+\sigma_e\mu_e^2H_0^2u^2 \quad ... (3)$$

where υ is the kinematic viscosity, K the Darcy permeability, σ_e the electrical conductivity, μ_e the magnetic permeability, ρ the density, C_p the specific heat at constant pressure, κ the thermal conductivity, Q the volumetric rate of heat generation and μ is the coefficient of viscosity of the fluid under consideration. The other symbols have their usual meanings.

The boundary conditions are:

$$\begin{aligned} &y=0:\ u=u_w=cx,\ v=0;\ T=T_w\\ &y=\infty:\ u=u_e=ax;\ T=T_\infty \end{aligned} \quad ... (4)$$

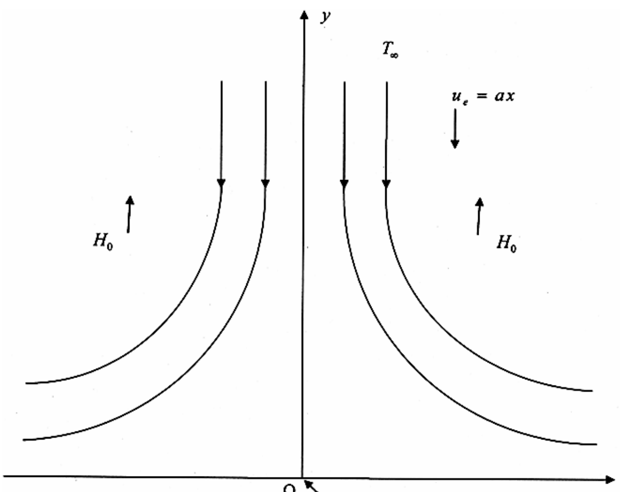

Fig. 1 — Physical model and coordinate system

where c is a proportionality constant of the velocity of the stretching sheet and a is a constant proportional to the free stream velocity far away from the stretching sheet.

3 Analysis

The continuity Eq. (1) is identically satisfied by stream function $\Psi(x,yx,y)$, defined as:

$$u=\frac{\partial\psi}{\partial y},\qquad v=-\frac{\partial\psi}{\partial x} \quad ... (5)$$

For the solution of the momentum and the energy Eqs (2) and (3), the following dimensionless variables are defined:

$$\psi(x,y)=\sqrt{c\,\upsilon}\ x\,f(\eta) \quad ... (6)$$

$$\eta=\sqrt{\frac{c}{\upsilon}}\ y \quad ... (7)$$

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \quad ... (8)$$

Eqs (5) to (8), transform Eqs (2) and (3) into:

$$f'''+f\,f''-f'^2-\mathrm{Re}_m^2\,f'-M\left(f'-C\right)+C^2=0 \quad ... (9)$$

$$\theta''+\Pr f\,\theta'+\Pr B\,\theta+\Pr Ec\,f''^2+\Pr Ec\,\mathrm{Re}_m^2\,f'^2=0 \quad ... (10)$$

where the prime (′) denotes differentiation with respect to η, $\mathrm{Re}_m=\mu_e H_0\sqrt{\frac{\sigma_e}{\rho c}}$ the magnetic parameter, $M=\frac{\upsilon}{K c}$ the porosity parameter, $C=\frac{a}{c}$ the stretching parameter, $\Pr=\frac{\mu C_p}{\kappa}$ the Prandtl number, $B=\frac{Q}{c\rho C_p}$ the heat generation coefficient and $Ec=\frac{u_w^2}{C_p\left(T_w-T_\infty\right)}$ the Eckert number.

The corresponding boundary conditions are:

$$\begin{aligned} &\eta=0:\ f=0,\ f'=1;\ \theta=1\\ &\eta=\infty:\ f'=C;\ \theta=0 \end{aligned} \quad ... (11)$$

It may be noted that Chiam[4] assumed $\mathrm{Re}_m = M = 0$ and $a = c$ without any justification and derived the solution of the Eq. (9), satisfying the Eq. (11), as $f(\eta) = \eta$ leading to $u = ax$, $v = -ay$. From this he inferred that no boundary layer is formed near the stretching surface.

For numerical solution of the Eqs (9) and (10), we apply a perturbation technique as:

$$f(\eta) = \sum_{i=0}^{\infty} \left(\mathrm{Re}_m^2\right)^i f_i(\eta) \qquad \text{... (12)}$$

$$\theta(\eta) = \sum_{j=0}^{\infty} \left(\mathrm{Re}_m^2\right)^j \theta_j(\eta) \qquad \text{... (13)}$$

Substituting Eqs (12) and (13) and its derivatives in Eqs (9) and (10) and then equating the coefficients of like powers of Re_m^2, we get the following set of differential equations:

$$f_0''' + f_0 f_0'' - f_0'^2 - M\left(f_0' - C\right) + C^2 = 0 \qquad \text{... (14)}$$

$$\theta_0'' + \Pr f_0 \theta_0' + \Pr B \theta_0 = -\Pr Ec\, f_0''^2 \qquad \text{... (15)}$$

$$f_1''' + f_0 f_1'' - \left(M + 2f_0'\right) f_1' + f_0'' f_1 = f_0' \qquad \text{... (16)}$$

$$\theta_1'' + \Pr f_0 \theta_1' + \Pr B \theta_1 = -\Pr f_1 \theta_0' - \Pr Ec\left(2 f_0'' f_1'' + f_0'^2\right) \qquad \text{... (17)}$$

$$f_2''' + f_0 f_2'' - \left(M + 2f_0'\right) f_2' + f_0'' f_2 = -f_1 f_1'' + \left(f_1' + 1\right) f_1' \qquad \text{... (18)}$$

$$\theta_2'' + \Pr f_0 \theta_2' + \Pr B \theta_2 = -\Pr\left(f_1 \theta_1' + f_2 \theta_0'\right) - \Pr Ec\left(2 f_0'' f_2'' + f_1''^2 + 2 f_0' f_1'\right) \qquad \text{... (19)}$$

with the boundary conditions:

$$\begin{aligned} &\eta = 0: \; f_i = 0,\; f_0' = 1,\; f_j' = 0;\; \theta_0 = 1,\; \theta_j = 0 \\ &\eta = \infty: \; f_0' = C,\; f_j' = 0;\; \theta_i = 0 \qquad i \ge 0,\; j > 0 \end{aligned} \qquad \text{... (20)}$$

Eq. (14) is obtained by Attia[10] for the non-magnetic case and the remaining equations are ordinary linear differential equations and have been solved numerically by standard techniques. The velocity and temperature distributions for various values of the parameters are shown in Fig 2 and Figs 3 and 5, respectively.

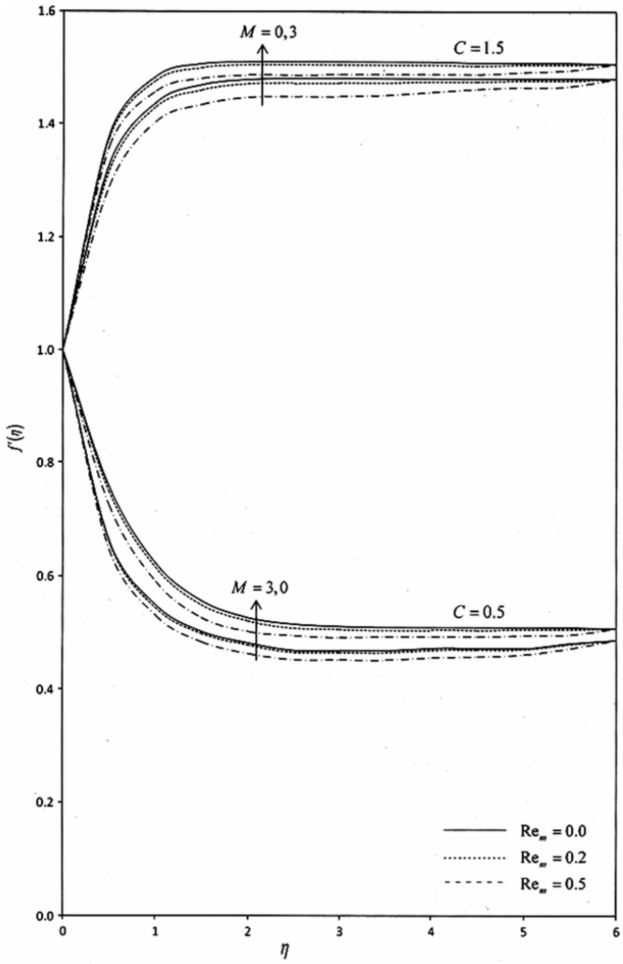

Fig. 2 — Velocity distribution against η for various values of Re_m, M and C

4 Skin Friction and Surface Heat Transfer

The physical quantities of interest, the local skin friction coefficient C_f and the local Nusselt number Nu i.e. surface heat transfer are given by:

$$C_f = \frac{\tau_w}{\rho u_w^2 / 2} = \frac{\mu\left(\dfrac{\partial u}{\partial y}\right)_{y=0}}{\rho u_w^2 / 2} \qquad \text{... (21)}$$

and

$$Nu = -\frac{x\left(\dfrac{\partial T}{\partial y}\right)_{y=0}}{\left(T_w - T_\infty\right)} \qquad \text{... (22)}$$

which in the present case, can be expressed in the following forms:

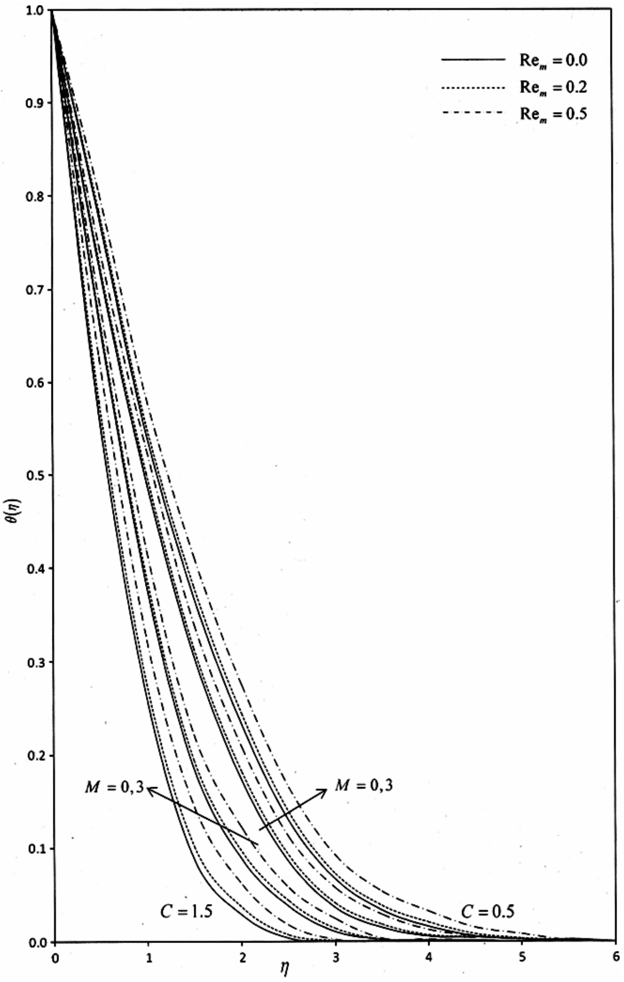

Fig. 3 — Temperature distribution against η for various values of Re_m, M and C with Pr=0.7, Ec=0.0 and B=0.1

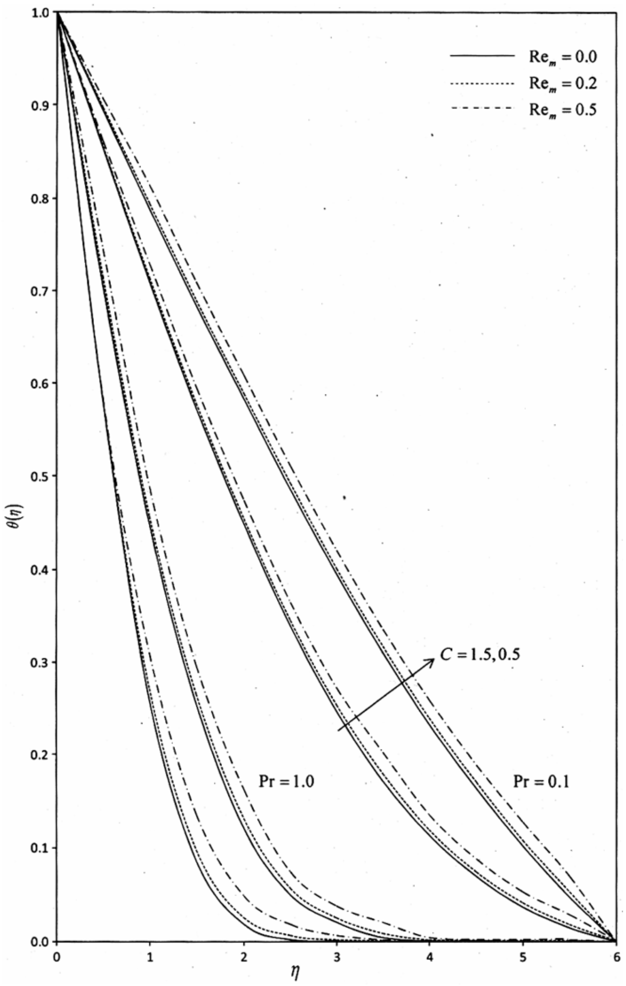

Fig. 4 — Temperature distribution against η for various values of Re_m, C and Pr with M=3, Ec=0.0 and B=0.1

$$C_f = \frac{2}{\sqrt{\text{Re}}} f''(0) = \frac{2}{\sqrt{\text{Re}}} \sum_{i=0}^{\infty} \left(\text{Re}_m^2\right)^i f_i''(0) \quad \text{... (23)}$$

and

$$Nu = -\sqrt{\text{Re}}\, \theta'(0) = -\sqrt{\text{Re}} \sum_{j=0}^{\infty} \left(\text{Re}_m^2\right)^j \theta_j'(0) \quad \text{... (24)}$$

where $\text{Re} = \frac{u_w x}{\upsilon}$ is the local Reynolds number.

Numerical values of the functions $f''(0)$ and $\theta'(0)$, which are proportional to local skin friction and local heat transfer rate at the surface, respectively for various values of the parameters are presented in Tables 1, 2 and 3, respectively.

5 Results and Discussion

Figure 2 shows the variation of velocity distribution against η for various values of the parameters, namely, the magnetic parameter Re_m, the porosity parameter M and the stretching parameter C. It may be observed that the velocity increases as the stretching parameter C increases, whereas it decreases as the magnetic parameter Re_m, increases for a fixed η. Also, it can be seen that the velocity increases as the porosity parameter M decreases for C<1 and when C>1, the opposite phenomenon occurs.

Figures 3 to 5 show the variation of the temperature distribution against η for various values of the

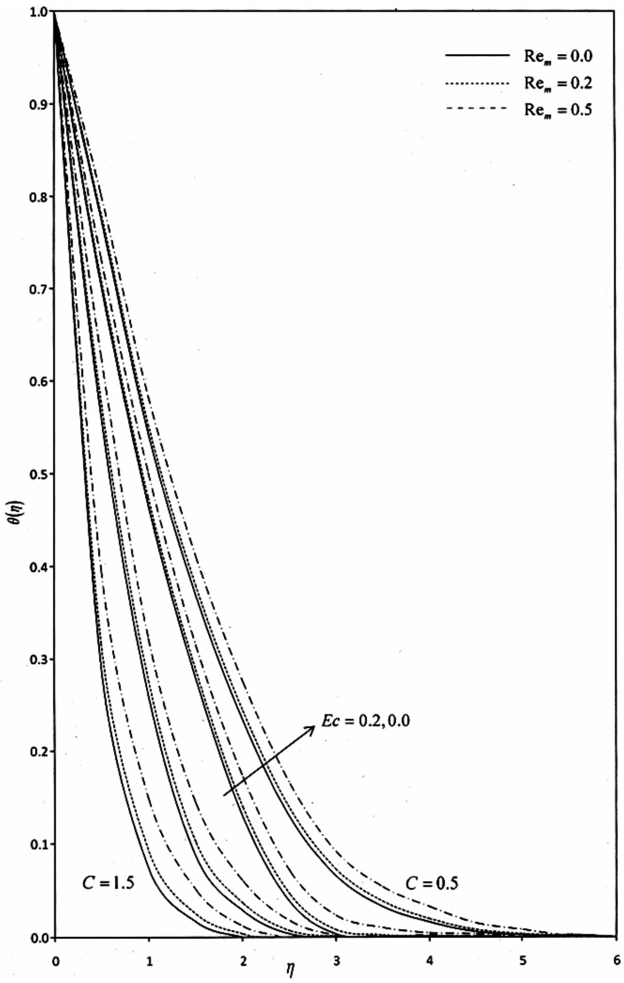

Fig. 5 — Temperature distribution against η for various values of Re_m, C and Ec with M=3, Pr=0.7 and B=0.1

parameters, namely, the magnetic parameter Re_m, the porosity parameter M, the stretching parameter C, the Prandtl number Pr, the Eckert number Ec and the heat generation coefficient B. From Figs 3 to 5, it may be observed that the temperature distribution increases with the increasing value of the magnetic parameter Re_m. It is also seen that for fixed Prandtl number Pr, temperature distribution decreases with increasing value of the stretching parameter C and same phenomenon occurs for the Eckert number Ec. In Fig. 3, it is seen that temperature distribution increases with the increasing value of the porosity parameter M for C<1 and when C>1, the opposite phenomenon occurs. In Fig. 4, it is observed that the temperature distribution decreases with the increasing value of the Prandtl number Pr.

In Tables 1-3, the numerical values of the functions $-f''(0)$ and $-\theta'(0)$ for various values of the magnetic parameter Re_m, the porosity parameter M, the stretching parameter C, the Prandtl number Pr and the Eckert number Ec with the heat generation coefficient B=0.1 are given, respectively. It may be observed from the Tables 1-3 that the boundary values of $f''(0)$ and $\theta'(0)$ for the non-magnetic flow are the same as those obtained by Attia[10]. Further, it may be observed from the Tables 1-3 that for C<1, the value of $-f''(0)$ increases with the increasing values of the porosity parameter M and the magnetic parameter Re_m, and when C>1 same phenomenon occurs for the magnetic parameter Re_m, while opposite phenomenon occurs for the porosity parameter M. It may also be observed that when the stretching parameter C increases, the

Table 1 — Numerical values of $-f''(0)$ for various values of the parameters Re_m, M and C

M	C=0.5			C=1.5		
	Re_m=0.0	Re_m=0.2	Re_m=0.5	Re_m=0.0	Re_m=0.2	Re_m=0.5
0	0.6673	0.6939	0.7707	-0.9095	-0.8752	-0.7746
3	1.0910	1.1056	1.1485	-1.2533	-1.2308	-1.1642

Table 2 — Numerical values of $-\theta'(0)$ for various values of the parameters Re_m, M, Pr and Ec with C=0.5 and B=0.1

M	Pr	Ec=0.0			Ec=0.0		
		Re_m=0.0	Re_m=0.2	Re_m=0.5	Re_m=0.0	Re_m=0.2	Re_m=0.5
	0.1	0.2148	0.2148	0.2140	0.2148	0.2148	0.2133
0	0.7	0.5089	0.5089	0.5058	0.5089	0.5089	0.5010
	1.0	0.6220	0.6220	0.6186	0.6220	0.6220	0.6221
	0.1	0.2103	0.2102	0.2100	0.2103	0.2102	0.2094
3	0.7	0.4821	0.4821	0.4809	0.4821	0.4821	0.4769
	1.0	0.5888	0.5888	0.5874	0.5888	0.5888	0.5820

Table 3 — Numerical values of $-\theta'(0)$ for various values of the parameters parameters Re_m, M, Pr and Ec with C=1.5 and B=0.1

M	Pr	Ec=0.0			Ec=0.2		
		Re_m=0.0	Re_m=0.2	Re_m=0.5	Re_m=0.0	Re_m=0.2	Re_m=0.5
	0.1	0.2898	0.2898	0.2890	0.2898	0.2898	0.2862
0	0.7	0.7131	0.7131	0.7111	0.7131	0.7130	0.6985
	1.0	0.8437	0.8437	0.8415	0.8437	0.8436	0.8255
	0.1	0.2921	0.2921	0.2916	0.2921	0.2920	0.2887
3	0.7	0.7235	0.7235	0.7224	0.7235	0.7234	0.7089
	1.0	0.8566	0.8566	0.8553	0.8566	0.8565	0.8383

value of $-f''(0)$ decreases. Moreover, for the fixed value of the Prandtl number Pr, value of the function $-\theta'(0)$ decreases with the increasing values of the porosity parameter M, the Eckert number Ec and the magnetic parameter Re_m when C<1 while the opposite phenomenon occurs for the porosity parameter M when C>1. Again the function $-\theta'(0)$ increases with the increase of the Prandtl number Pr and the stretching parameter C for the fixed value of the porosity parameter M. Also, when the stretching parameter C increases, the value of $-\theta'(0)$ increases.

6 Conclusions

In the present paper, the two-dimensional stagnation point flow in a porous medium with heat generation of an electrically conducting fluid over a stretching surface in the presence of magnetic field has been studied. Similarity equations are derived and solved numerically. It is found that the velocity boundary layer thickness increases with the increasing value of the stretching parameter and decreases with the increasing value of the magnetic parameter. It is further concluded that velocity boundary layer thickness increases with the increasing value of the porosity parameter when the stretching parameter is greater than one while it decreases when the stretching parameter is less than one, but the reverse phenomenon occurs for the thermal boundary layer thickness. Further, the thermal boundary layer thickness decreases with the increasing value of the Prandtl number and the Eckert number but for fixed Prandtl number the thermal boundary layer thickness decreases with the increasing value of the stretching parameter. From the results, it can be concluded that skin friction and Nusselt number vary in reverse phenomenon as compared to velocity boundary layer thickness and thermal boundary layer thickness, respectively with different parameters.

Nomenclature

u	Component of velocity in x direction
v	Component of velocity in y direction
x	Along the surface distance
y	Normal distance
u_e	Velocity of the flow external to the boundary layer
K	Darcy permeability
H_0	Externally applied normal magnetic field of constant strength
C_p	Specific heat at constant pressure
T	Temperature
Q	Volumetric rate of heat generation
T_∞	Temperature of the flow external to the boundary layer
u_w	Velocity of the stretching surface
c	Proportionality constant of the velocity of the stretching sheet
T_w	Temperature of the stretching surface
a	Constant proportional to the free stream velocity far away from the stretching sheet
f	Dimensionless stream function
f'	First order derivative with respect to η
f''	Second order derivative with respect to η
f'''	Third order derivative with respect to η
Re_m	Magnetic parameter
M	Porosity parameter
C	Stretching parameter
Pr	Prandtl number
B	Heat generation coefficient
Ec	Eckert number
C_f	Local skin friction coefficient
Nu	Local Nusselt number
Re	Local Reynolds number

Greek symbols

υ	Kinematic viscosity
σ_e	Electrical conductivity
μ_e	Magnetic permeability
ρ	Density
κ	Thermal conductivity
μ	Coefficient of viscosity
Ψ	Stream function

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