Indian Journal of Pure & Applied Physics
Vol. 52, June 2014, pp. 369-373

# Radiative decays of ground state light vector mesons with non-relativistic and relativistic phase spaces and other decay properties

Antony Prakash Monteiro, K B Vijaya Kumar* & Bhaghyesh
Department of Physics, Mangalore University, Mangalagangotri, Mangalore 574 199, India
*E-mail: kbvijayakumar@yahoo.com

*Received 11 October 2012; revised 24 September 2013; accepted 12 March 2014*

The comprehensive study of some of the decay properties of ground state light vector mesons within the framework of non relativistic potential model has been made in the present paper. Some of the decay properties and radiative decay widths of light vector mesons are calculated using spectroscopic parameters. The E1 and M1 transition widths have been studied both in case of non-relativistic and relativistic phase spaces. A good agreement between the calculated and experimental data is obtained in case of relativistic phase space calculations.

**Keywords:** Quark model, Light vector meson states, Decay constants, Radiative decays, E1 and M1 transitions

## 1 Introduction

The experimental facilities such as BES, E835, CLEO, BaBar, Belle, CDF, DO, NA60 etc. are capable of discovering new hadrons, new production mechanisms, new decays and transitions and in general, will be providing high precision data sample with higher confidence level. Hence, the study of mass spectroscopy and decay rates of various mesons becomes significant. Mesons exhibit several different decay modes such as leptonic, semi-leptonic, hadronic and radiative decays and are classified based on the decay products. Goal of elementary particle theory is to calculate the masses, lifetimes and the branching ratios of the mesons. There are different models in literature which describe the light meson mass spectra[1-4]. The prediction of mass spectrum in accordance with the experimental data does not guarantee the validity of the model for describing hadronic interactions. Hence, one needs other observables which rely essentially on the same dynamical operators like transitions between various states, in order to test the model under study. Hence, one of the tests for the success of any theoretical model for mesons is the correct prediction of their decay rates. The radiative decay of pseudo scalar and vector mesons particularly have been advocated as one of the observables most suited to predict the nature of the mesons[5]. There have been different studies on the decay of vector mesons[6-9], scalar mesons[10-12] and axial vector mesons[13-16]. Radiative decays of $\rho$, $\omega$ and $\phi$ mesons have been studied in the SND experiment at the VEPP-2M $e^+.e^-$ collider[17].

In our previous work, we had obtained the masses of light mesons in the frame work of non-relativistic quark models (NRQM) and relativistic quark models and as a natural extension, in the present work, we have calculated the radiative decay widths of light vector mesons[18-20] in the frame work of NRQM. Explicitly, we have estimated the M1 transition widths. The electric dipole term is responsible for the transition between the S and P states with the same spin S of the quark pair, while the M1 term describes the transitions between S=1 and S=0 states with the same orbital angular momentum L.

## 2 Theoretical Background

In the constituent quark model, conventional mesons are bound states of a spin ½ quark and spin ½ anti-quark bound by a phenomenological potential. Essentially, in all phenomenological QCD based quark models, the Hamiltonian for the quark system consists of the kinetic energy, the two body confinement potential and OGEP.

At very short distances, a non-relativistic Coulomb potential with a strength proportional to the strong coupling constant, $\alpha_s$, is derived from the one-gluon-exchange interaction in QCD. At long distances, non-perturbative effects such as confinement have to be considered. Lattice calculations in the quenched approximation suggest a linearly increasing potential[21].

In NRQM, the full Hamiltonian is:

$$H = M + \frac{p^2}{2\mu} + V(r) \qquad (1)$$

where $p$ is the relative momentum, $\mu$ the reduced mass of the $q\bar{q}$ system and $V(r)$ is the interaction potential. $M=m_q+m_{\bar{q}}$ and $\mu=\frac{m_q m_{\bar{q}}}{m_q+m_{\bar{q}}}$ where $m_q$ and $m_{\bar{q}}$ are the masses of the individual quark and antiquark, respectively. The potential $V(r)$ is of the following form:

$$V(r)=V_{conf}(r)+V_{ogep}(r) \qquad \ldots(2)$$

where $V_{conf}$ is the confinement potential and $V_{ogep}$ is the one gluon exchange potential. The confinement term represents the non-perturbative effect of QCD that confines quarks within the colour singlet system, and is taken to be linear. The $r$ is the relative distance between the quarks.

$$V_{conf}(r)=-a_c r(\lambda_i.\lambda_j) \qquad \ldots(3)$$

Where $-a_c$ is the confinement strength. The confinement potential is entirely central in nature. Here, $\lambda_i$ and $\lambda_j$ are the generators of the colour SU(3) group for the $i^{th}$ and $j^{th}$ quark. The following central part of two-body potential due to OGEP is usually employed[22]:

$$V_{ogep}^{cent}(r)=\frac{\alpha_s}{4}\lambda_i.\lambda_j\left[\frac{1}{r}-\frac{\pi}{M_i M_j}(1+\frac{2}{3}\sigma_i.\sigma_j)\delta(r)\right] \ldots(4)$$

where the first term represents the residual Coulomb energy and the second term the chromo-magnetic interaction leading to the hyperfine splitting.
For ground state, mesons spin orbit and tensor part of OGEP do not contribute.

In our calculations, we have used the three dimensional harmonic oscillator wave function as the trial wave function for obtaining the $q\bar{q}$ mass spectra and decay properties which is given by:

$$\psi_{nlm}(r,\theta,\phi)=N\left(\frac{r}{b}\right)^{\iota}L_n^{\iota+1/2}\left(\frac{r^2}{b^2}\right)\exp\left(\frac{-r^2}{b^2}\right)Y_{lm}(\theta,\phi) \qquad \ldots(5)$$

where $N$ is the normalization constant:

$$\left|N^2\right|=\frac{2n!}{b^3\pi^{1/2}}\frac{(2(n+\iota)+1}{2n+2\iota+1)!}(n+\iota)!, \qquad \ldots(6)$$

and $L_n^{\iota+1/2}(r)$ are the associated Laguerre polynomials. In our previous work[18,20,23], the harmonic oscillator wave function was used for the prediction of light meson spectra successfully.

The parameters of our model are the mass of up and down quarks $M_{u,d}$, the mass of strange quark $M_s$, the confinement strength $a_c$, the harmonic oscillator size parameter $b$ and the quark-gluon coupling constant $\alpha_s$. We use the following set of parameter values[18].

$M_{u,d}$=352 MeV; $M_s$=545 MeV; $a_c$=20 MeV fm$^{-1}$; $b$=0.9 fm; $\alpha_s$=0.6

From this set of parameter values, we had successfully obtained the mass spectrum for light vector mesons[18].

## 3 Some Decay Properties of Vector Mesons

### 3.1 Wave function at the origin

Within the context of the non-relativistic potential models, the wave function at the origin for the S-wave bound state of a quark-antiquark system is very important quantity for calculating spin state hyperfine splitting and in evaluating the production and decay amplitude of the meson system. The wave function at the origin for $L$=0 states are obtained using the standard relation[24].

$$\left|\psi_v(0)\right|^2=\frac{\mu}{2\pi}\left\langle\frac{dV(r)}{dr}\right\rangle \qquad \ldots(7)$$

where $\mu$ is the reduced mass of the $q\bar{q}$ system. The expectation value is calculated using the trial harmonic oscillator wave function. The calculated values of wave function at the origin are given in the Table 1.

### 3.2 Leptonic decay constant

The vector decay constant ($f_v$ =) is defined through the matrix element[25]:

$$m_v f_v^{\mu}=\left\langle 0\left|\bar{\psi}\gamma^{\mu}\psi\right|v\right\rangle \qquad \ldots(8)$$

where $m_v$, $\varepsilon^{\mu}$ and $|v\rangle$ are the mass, polarization and the state vector of the vector meson, respectively. The calculation of the above matrix elements using the quark model wave function in the momentum space gives[25]:

Table 1 — Wave function at the origin and the leptonic decay constants of light vector mesons

| Vector Meson | $\left|\psi_v(0)\right|^2$ (MeV$^3$) | $f_v$ (MeV$^2$) |
|---|---|---|
| $\rho$ | 2.97×10$^6$ | 214.86 |
| $\omega$ | 2.97×10$^6$ | 199.89 |
| $\phi$ | 4.39×10$^6$ | 227.26 |

$$f_v = \sqrt{\frac{3}{m_v}} \int \frac{d^3k}{(2\pi)^3}\phi(\vec{k})\sqrt{1+\frac{m_q}{E_k}}\sqrt{1+\frac{m_q}{E_k}} \times\left(1+\frac{k^2}{3(E_k+m_q)(E_k+m_{\bar{q}})}\right) \quad \ldots(9)$$

In the non relativistic limit, the above equation reduces to the well-known result of Van Royen and Weisskopf [26] for the meson decay constants:

$$f_v^2 = \frac{12|\psi_v(0)|^2}{m_v} \quad \ldots(10)$$

The Vector meson decay constant including a first order OCD correction factor is given by:

$$\bar{f}_v^2 = \frac{12|\psi_v(0)|^2}{m_v}C^2(\alpha_s) \quad \ldots(11)$$

where $C^2(\alpha_s)$ is given by[27]:

$$C(\alpha_s) = 1-\frac{\alpha_s}{\pi}\left(\Delta_v - \frac{m_q - m_{\bar{q}}}{m_q + m_{\bar{q}}}\ln\frac{m_q}{m_{\bar{q}}}\right) \quad \ldots(12)$$

where $\Delta_v = 8/3$.

The calculated values of $f_v$ and $\bar{f}_v$ for vector mesons are presented in Table 1.

### 3.3 E1 transitions

To calculate the transitions between quarkonium levels with emission of a photon, the standard multipole expansion is applied since quarkonium here is treated as non- relativistic system. In potential model approach, the spatial dependence of EM transition amplitudes reduces to functions of quark position and momentum between the initial and final state wave functions. Expanding the matrix elements in powers of photon momentum generates the electric and magnetic multipole moments and is also an expansion in powers of velocity. The leading order transition amplitudes are the electric dipole (E1) and magnetic dipole (M1) terms, which are given by the corresponding terms in the Hamiltonian, $H_{E1} = -e_q e(\vec{r}.\vec{E})$ and $H_{M1} = -\mu_q e(\vec{\Delta}.\vec{B})$, where $e_q$ is the charge of the quark, $\mu_q = e_b e/(2m_q)$ is magnetic moment, $\vec{E}$ and $\vec{B}$ are the electric and the magnetic fields and $\vec{\Delta} = \vec{\sigma_1} - \vec{\Delta_2}$ with $\vec{\sigma}_1$ and acting on the quark and the antiquark, respectively. The partial width for the transition from $^3S_1$ state to $^3P_J$ state is given by[28]:

$$T(^3S_1 \to \gamma^3P_J) = (2J+1)\frac{4}{27}e_q^2\alpha_{em}k_0^3|I_{PS}|^2 \quad \ldots(13)$$

where $k_0$ is the energy of the emitted photon, and $I_{PS}$ is the radial overlap integral which has the dimension of length.

$$I_{PS} = \int_0^\infty r^3R_P(r)R_S(r)dr \quad \ldots(14)$$

with $R_{S.P}(r)$ being the normalized radial wave function for the corresponding state. The transition from $^3P_J$ level to a $a^3S_1$ level is given by[28]:

$$T_{(^3P_J\to\gamma^3S_1)} = \frac{4}{9}e_q^2\alpha_{em}k_0^3|I_{SP}|^2 \quad \ldots(15)$$

The electric dipole term is responsible for the transition between the S and P states with the same spin S of the quark pair. E1 transitions do not change quark spin. These transitions have selection rules $\Delta l = \pm 1$ and $\Delta s$=0.

### 3.4 M1 transitions

The allowed M1 transitions are $^3S_1 \to \gamma^1S_0$ and $^1S_0 \to \gamma^3S_1$. From Ref. (29), the relevant transition rates can be calculated from the expressions:

$$T_{(n^3S_1\to\gamma m^1S_0)} = \frac{4}{3m_q^2}e_q^2\alpha_{em}k_0^3|I_{mn}|^2 \quad \ldots(16)$$

$$_{(n^1S_0\to\gamma m^3S_1)} = \frac{4}{m_q^2}e_q^2\alpha_{em}k_0^3|I_{mn}|^2 \quad \ldots(17)$$

where $I_{mn}$ is overlap integral for the unit operator between the coordinate wave functions of the initial and the final states[29].

$$I_{mn} = \int_0^\infty r^2R_{nS}(r)R_{mS}(r)dr \quad \ldots(18)$$

The ortho normality of states guarantees, in the limit of zero recoil, the spatial overlap is for states within the same multiplet and zero for transitions between multiplets which have different radial quantum numbers.

Magnetic transitions flip the quark spin and hence their amplitudes are proportional to the quark magnetic moment and therefore inversely proportional to the constituent quark mass. These transitions have $\Delta l$=0. The above expressions for decay widths are derived for the mesons having the same flavour wave functions $n\bar{n}$. For radiative decays involving mesons with mixed flavours such as

Table 2 — Radiative decay widths of light vector meson states in non relativistic phase space

| Transition | Expl. Value Γ(keV) | $k_0^{nrel}$ (MeV) Γ(keV) | Ref. [30] | Calculated value Γ(keV) |
|---|---|---|---|---|
| $\rho^+ \to \pi^+\gamma$ | 67.82±7.5 | 630 | 535.23 | 545.57 |
| $\rho^0 \to \pi^0\gamma$ | 102.48±25.69 | 634 | 547.02 | 556.03 |
| $\rho^0 \to \eta\gamma$ | 36.18±13.57 | 222 | 105.52 | 71.62 |
| $\omega \to \pi^0\gamma$ | 714.85±42.74 | 648 | 5238.71 | 5343.2 |
| $\omega \to \eta\gamma$ | 5.47±0.84 | 235 | 13.96 | 9.44 |
| $\phi(1020) \to \eta\gamma$ | 55.82±2.73 | 472 | 157.04 | 255.22 |
| $\phi(1020) \to \eta(958)\gamma$ | 0.53±0.31 | 62 | 0.35 | 0.29 |
| $K^*(892)^0 \to K^0\gamma$ | 116.15±10.19 | 398 | 361.34 | 372.63 |
| $K^*(892)^+ \to K^+\gamma$ | 50.29±4.66 | 398 | 318.28 | 252.23 |

Table 3 — Radiative decay widths of light vector meson states in relativistic phase space

| Transition | Expl. Value | $k_0^{nrel}$ (MeV) Γ(keV) | $E_B / m_A$ Γ(keV) | Ref. [30] | Calculated value Γ(keV) |
|---|---|---|---|---|---|
| $\rho^+ \to \pi^+\gamma$ | 67.82±7.55 | 373 | 0.52 | 56.98 | 58.41 |
| $\rho^0 \to \pi^0\gamma$ | 102.48±25.69 | 373 | 0.52 | 57.24 | 58.88 |
| $\rho^0 \to \eta\gamma$ | 36.18±13.57 | 190 | 0.75 | 49.79 | 33.67 |
| $\omega \to \pi^0\gamma$ | 714.85±42.74 | 380 | 0.51 | 543.44 | 549.53 |
| $\omega \to \eta\gamma$ | 5.47±0.84 | 200 | 0.74 | 6.37 | 9.44 |
| $\phi(1020) \to \eta\gamma$ | 55.82±2.73 | 363 | 0.64 | 45.90 | 255.22 |
| $\phi(1020) \to \eta(958)\gamma$ | 0.53±0.31 | 60 | 0.94 | 0.30 | 0.29 |
| $K^*(892)^0 \to K^0\gamma$ | 116.15±10.19 | 310 | 0.65 | 111.19 | 372.63 |
| $K^*(892)^+ \to K^+\gamma$ | 50.29±4.66 | 309 | 0.65 | 97.47 | 252.23 |

charged decays of $\rho((u\bar{u} - d\bar{d})/\sqrt{2})$ into $\pi\gamma$, the charge contents are calculated as in Ref. (30).

We have calculated the E1 transition widths for $^3P_j \to {}^2S_1$ decays for light vector meson states in the long wavelength approximation. These transitions are reported[31] in PDG. In our calculations, the experimental values of the meson masses have been used. Table 2 presents the calculated values of transition widths for various light vector meson states in non relativistic phase space. Here the energy of the photon $k_0$ is taken to be the difference between the resonances and the term $E_B(E_0)/m_a$ is equal to unity. Table 3 presents the calculated values of transition widths for various light vector meson states in relativistic phase space where the photon energy is taken to be $E_0 = m_a^2 - m_b^2 / 2m_a$ where $m_a$ and $m_b$ are the masses of initial and final resonances, respectively. A comparison with the Ref. (30) has been made. From the calculated values for E1 and M1 transition widths, we arrive at the conclusion that the present NRQM is not suited for light mesons. The calculations show that the relativistic phase space is better than the non-relativistic phase space and give good description of light mesons.

### 3.5 Theoretical uncertainties in the predictions of the model

An important drawback of the present model is not taking into account of the chiral symmetry. But the NRQM has several other basic features of QCD. But NRQM allows the direct calculations of the matrix elements of both spectra and decay widths which can be compared with the experiment. We have used harmonic oscillator basic which is known to increase wave function at the origin as there could be mixing of the higher order states. but, this could be settled only through lattice QCD calculations which will be most reliable as they are the calculations from the basic QCD. Also, in NRQM, there could be relativistic corrections which could be significant. But, in the relativistic model there is problem of separating the center of mass.

## 4 Conclusions

The main objective of the present work has been the study of some of the decay properties including radiative decays of light vector mesons within the framework of NRQM formalism using both non-relativistic and relativistic phase spaces. In present work, we have obtained the leptonic decay constants and radiative decay widths of light vector mesons using spectroscopic parameters from which we obtained the masses of vector and pseudo scalar mesons. The experimental data on radiative decay rates is understood theoretically in terms of a multipole expansion model. It is known that long wavelength approximation is fully justified in atomic physics but when applied to the meson sector, the transition energy is typically $E_\gamma = k_\gamma = 0.1$–$0.5$ GeV, and the size of the source is roughly $R$ = 0.5-1 *fm* = 2.5-5 GeV$^{-1}$ which does not justify the long wavelength condition $k_\gamma << 1$. This is the limitation of the model. The calculated results show that the non-relativistic phase space is not a correct prescription for the light mesons. Comparison between the photon energy and the mass of the emitting meson reveals that the relativistic phase space is more suited which is seen in our model calculations.

## Acknowledgement

The authors acknowledge the BRNS for funding the project.

## References

1 Buchmuller W & Tye S H H, *Phys Rev D*, 24 (1981) 132.
2 Quigg C & Rosner J L, *Phys Lett B*, 71 (1977) 153.
3 Martin A, Phys *Lett B*, 93 (1980) 338.
4 Danning Li, Mei Huang, *JHEP,* 11 (2013) 088;
5 Wang Z G, *Phys Rev*, 75 (2007) 034013.
6 Owen B *et al*, *PoS Lattice,* 2012 (2012) 254.
7 Oller J A, *Nucl Phys A,* 727 (2003) 353.
8 Palomar J E,Roca L, Oset E & Vicente Vacas M J, *Nucl Phys A,* 729 (2003) 743.
9 Mengesha E & Bhatnagar S, *Int J Mod Phys E,* 21 (2012) 1250084.
10 Kalashnikova Yu, Kudryavtsev A E, Nefediev A V, Haidenbauer J & Hanhart C, *Phys Rev C,* 73 (2006) 045203.
11 Black D, Harada M & Schechter J, *Phys Rev Le*tt, 88 (2002) 181603.
12 Nagahiro H, Roca L, Oset E & Zou B S, *Phys Rev D,* 78 (2008) 014012.
13 Nagahiro H, Roca L & Oset E, *Phys Rev D,* 77 (2008) 034017.
14 Roca L, Hosaka A & Oset E, *Phys Lett B,* 658 (2007) 17.
15 Dong Y B, Faessler A, Gutsche T & Lyubovitskij V E, *Phys Rev D,* 77 (2008) 094013.
16 Faessler A, Gutsche T, Lyubovitskij V E & Ma Y L, *Phys Rev D,* 76 (2007) 114008.
17 Tumaikin G M, *Proc. of the 10-th Int. Conf. on High Energy Particle Accelerators*. Serpukhov, (1977), vol.1, 443.
18 Bhavyashri, Vijaya Kumar K B, Hanumaiah B, Sarangi S & Zhou S G, *J Phys G: Nucl Part Phys,* 31( 2005) 981.
19 Bhavyashri, Sarangi S, Saldanha Godfrey & Vijaya Kumar K B, *Pramana J Phys,* 70 (2008) 75.
20 Vijaya Kumar K B, Bhavyashri, Yong-Liang Ma, Monteiro Antony Prakash, *Int J Mod Phy A,* 22 (2009) 4209.
21 Bali G S, *Phys Rep,* 343 (2001) 1.
22 De Rujula A, Georgi H & Glashow S L, *Phys Rev D,* 12 (1975) 147.
23 Vijaya Kumar K B, Hanumaiah B & Pepin S, *Eur Phys J A,* 19 (2004) 247.
24 Fayyazuddin & Riazuddin, *A Modern Introduction to Particle Physics, 2nd edn, World Scientific, Singapore,* (2000).
25 Lakhina O & Swanson E S, *Phys Rev D,* 74 (2006) 014012.
26 Royen R Van & Weisskopf V F, *Nuovo Cimento,* 50 (1967) 617.
27 Braaten E & Fleming S, *Phys Rev D,* 52 (1995)181.
28 Kwong W & Rosner J L *Phys Rev D,* 38 (1988) 279.
29 Eichten E, Gottfried K, Kinoshita T, Lane K D & Yan T M, *Phys Rev D,* 17(1978) 3090.
30 Bonnaz R, Silvestre-Brac B & Gignoux C, *Eur Phys J A,* 13 (2002) 363.
31 Nakamura K *et al*., (Particle Data Group) *J Phys G,* 37 (2010) 075021.