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MHD mixed convection boundary layer flow on a vertical permeable stretching sheet embedded in a porous medium with slip effects

A K Jhankal[a], R N Jat[b] & Deepak Kumar[b]

[a]Department of Mathematics, Army Cadet College, Indian Military Academy, Dehradun 248 007, India

[b]Department of Mathematics, University of Rajasthan, Jaipur 302 004, India

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In this paper, we investigate the problem of two-dimensional MHD mixed convection flow over a vertical permeable sheet embedded in a porous medium, with partial slip condition at the boundary. The nonlinear coupled boundary-layer equations have been transformed using an appropriate similarity transformation and resulting ordinary differential equations have been solved by Runge-Kutta fourth order method along with shooting technique. The influence of magnetic parameter M, permeability parameter K, buoyancy or mixed convection parameter λ, suction parameter S, slip parameter δ and Prandtl number Pr has been studied. It is found that these parameters have essential effects on the features of flow and heat transfer. Further, the present solutions are also validated by comparing with the existing solutions.

Keywords: MHD, Boundary layer mixed convection, Vertical stretching sheet, Porous medium, Partial slip

1 Introduction

The subject of MHD is largely perceived to have been initiated by Swedish electrical engineer Hannes Alfveén[1] in 1942. Under the influence of magnetic field, electrically conducting fluid induces currents. This also creates force on the fluid. MHD can be used for the control of fluid flow. MHD has been employed for many engineering applications in aerodynamic heating, electrostatic precipitation, petroleum industry, purification of oil and fluid droplets and sprays, etc. The confinement of hot plasma is of great importance in nuclear fusion devices where vast amount of energy is released. The MHD may be used for magnetically pinching the hot plasma[2]. In addition, the MHD flow of electrically conducting fluid through porous media has been motivated by its immense importance and continuing interest in many engineering and technological field, for example, soil mechanics, petroleum engineering, transpiration cooling, food preservation, cosmetic industry blood flow and artificial dialysis, etc.

The flow due to stretching sheet in a porous medium is a very important problem in fluid dynamics due to its significant applications in polymer processing industries, several biological processes and many others. In his pioneering work, Sakiadis[3] developed the flow field due to a flat surface, which is moving with a constant velocity in a quiescent fluid. Crane[4] extended the work of Sakiadis[3] for the two-dimensional problem where the surface velocity is proportional to the distance from the flat surface. As many natural phenomena and engineering problems are worth being subjected to MHD analysis, the effect of transverse magnetic field on the laminar flow over a stretching surface was studied by Pavlov[5]. Andersson[6] then demonstrated that the similarity solution derived by Pavlov[5] is not only a solution to the boundary layer equations, but also represents an exact solution to the complete Navier-Stokes equations. Liu[7] extended Andersson's results by finding the temperature distribution for non-isothermal stretching sheet, both in the prescribed surface temperature and prescribed surface heat flux conditions, in which the surface thermal conditions are linearly proportional to the distance from the origin. Wang[8] first studied the natural convection on a two-dimensional vertical stretching sheet. The effect of suction/blowing at the surface on the flow over vertical stretching surface was investigated by Vajravelu[9] and Gorla and Sidawi[10]. Chen[11] demonstrated the mixed convection laminar flow adjacent to continuously stretching vertical sheet. Elbashbeshy and Bazid[12] studied the heat transfer over a stretching surface in a porous medium, with

*Corresponding author (E-mail: anujjhankal@yahoo.com)

internal heat generation and suction or injection. Cortell[13] discussed the flow and heat transfer of a fluid in a porous medium over a stretching surface with internal heat generation/absorption and suction/blowing. Ishak *et al.*[14] analyzed the hydromagnetic effects to mixed convection flow near vertical stretching sheet, and Mukhopadhyay[15] investigated the effect of thermal radiation on unsteady mixed convection flow and heat transfer over a porous stretching surface in porous medium. The effect of transverse magnetic field on the laminar flow over a stretching surface was also studied by researchers Chakrabarthi and Gupta[16], Chiam[17], Ghaly[18], Raptis[19], Muhaimin *et al.*[20], Noor *et al.*[21], Jat and Chaudhary[22], Jhankal and Kumar[23], etc.

The non-adherence of the fluid to a solid boundary, known as velocity slip, is a phenomenon that has been observed under certain circumstances. Fluid in micro electro mechanical systems encounters the slip at the boundary. In the previous investigations, it is assumed that the flow field obeys the no-slip condition at the boundary. But, this no-slip boundary condition needs to be replaced by partial slip boundary condition in some practical problems. Beavers and Joseph[24] considered the fluid flow over a permeable wall using the slip boundary condition. The effects of slip at the boundary on the flow of Newtonian fluid over a stretching sheet were studied by Anderson[25], Wang[26] and Ariel *et al.*[27] analyzed the flow of a viscoelastic fluid over a stretching sheet with partial slip. Ariel[28] also studied the slip effects on the two dimensional stagnation point flow of an elastoviscous fluid. Bhattacharyya *et al.*[29] showed the slip effects on boundary layer mixed convective flow adjacent to a vertical permeable stretching sheet in porous medium. Mukhopadhyay *et al.*[30] analyzed the effects of temperature dependent viscosity on MHD boundary layer flow and heat transfer over stretching sheet.

In the present paper, an attempt is made to analyze the two-dimensional laminar MHD mixed convection flow over a vertical permeable sheet embedded in a porous medium, with partial slip condition at the boundary. The governing boundary layer equations have been transformed to a two-point boundary value problem in similarity variables and the resultant problem is solved numerically using the fourth order Runge-Kutta method with shooting technique. The effects of various governing parameters on the fluid velocity and temperature are shown in figures and analyzed in detail.

2 Formulation of the Problem

Let us consider the steady two-dimensional laminar MHD mixed convection flow over a vertical permeable sheet embedded in porous media. The fluid is an electrically conducting incompressible viscous fluid. The flow model and physical coordinate system is illustrated in Fig. 1 where x and y are the coordinates along and normal to the flat plate. The free stream temperature T_∞ is constant.

Using the Boussinesq and boundary layer approximation and neglecting the viscous dissipation term, the governing equations are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \ldots (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\nu}{k_1}u-\frac{\sigma_e B_0^2}{\rho}u\pm g\beta(T-T_\infty) \qquad \ldots (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \qquad \ldots (3)$$

Where u and v are the components of velocity in the x and y directions, respectively, ρ is the density; ν is the kinematic viscosity; k_1 is the permeability of the porous medium; σ_e is the electrical conductivity; B_0 is the externally imposed magnetic field in y-direction. The induced magnetic field effect is neglected for small magnetic Reynolds number flow. It is also assumed that the external electric field is zero, the

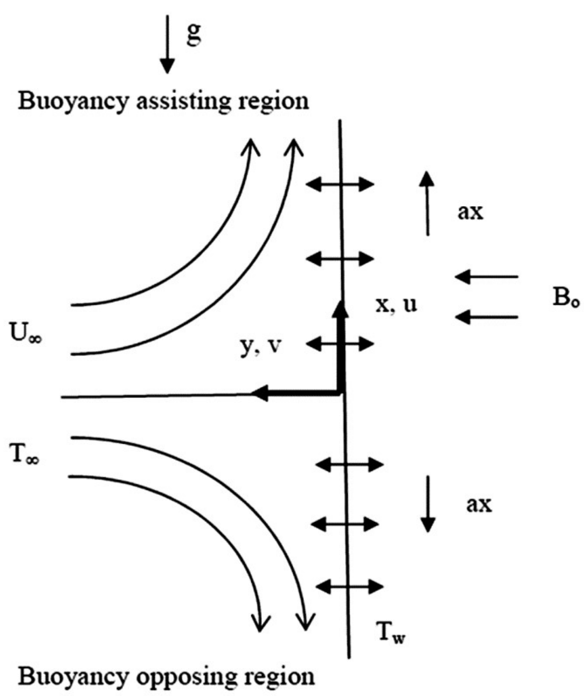

Fig. 1 — Flow model and physical coordinate system.

electric field owing the polarization of charges and the Hall effect is neglected. g is acceleration due to gravity; β is the coefficient of thermal expansion; T is the temperature and α is the equivalent thermal diffusivity.

In Eq. (2), the density variation is taken into account by the Boussinesq approximation. Also, the last term on the right-hand side of Eq. (2) which stands for the buoyancy effect on the flow field has ± signs. The plus sign denotes buoyancy assisting flow (the buoyancy force has a component in the direction of the free-stream velocity) whereas the negative sign indicates buoyancy opposing flow (the buoyancy force component is opposite to the direction of the free-stream velocity). The boundary conditions are defined as follows:

$$y = 0: u = ax + L\frac{\partial u}{\partial y}, v = v_w, T = T_w(x) = T_\infty + T_0 x$$

$$y \to \infty: u = 0, T = T_\infty \qquad \ldots (4)$$

where v_w is the uniform surface mass flux; a is a positive stretching constant; L denotes the slip length; $T_w(x) = T_\infty + T_0 x$ is variable temperature of the sheet with T_0 is positive.

The continuity Eq. (2) is satisfied by introducing a stream function Ψ such that $u = \frac{\partial \Psi}{\partial y}$ and $v = -\frac{\partial \Psi}{\partial x}$. The momentum and energy equations can be transformed into the corresponding ordinary nonlinear differential equation by the following transformation:

$$\eta = y\sqrt{a/\nu}, \Psi = \sqrt{a\nu}\, x f(\eta), T = T_\infty + (T_w - T_\infty)\theta(\eta) \qquad \ldots (5)$$

Where η is the independent similarity variable. The transformed non-linear ordinary differential equations are:

$$f''' + ff'' - f'^2 - Kf' + Mf' + \lambda\theta = 0 \qquad \ldots (6)$$

$$\theta'' + Pr(f\theta' - f'\theta) = 0 \qquad \ldots (7)$$

The boundary conditions are rewritten as follows:

$$f(0) = S, f'(0) = 1 + \delta f'', \theta(0) = 1, f'(\infty) \to 1, \theta(\infty) \to 0 \qquad \ldots (8)$$

In the foregoing equations, the primes denote the differentiation with respect to η. $K = \frac{\nu}{k_1 a}$ is the permeability parameter of the porous medium, $M = -\frac{\sigma B_0^2}{\rho a}$ is the magnetic parameter, $\lambda = \pm\frac{Gr_x}{Re_x^2}$ is the buoyancy or mixed convection parameter with $\lambda>0$ and $\lambda<0$ corresponding to assisting flow and opposing flow, respectively, $Gr_x = g\beta(T_w - T_\infty)x^3/\nu^2$ is the local Grashof number, $Re_x = ax^2/\nu$ is the local Reynolds number, $Pr = \frac{\mu}{\alpha}$ is the Prandtl number, $S = -v_w/\sqrt{a\nu}$; $S>0$ is corresponding to suction and $S<0$ is corresponding to blowing parameter, and $\delta = L\sqrt{a/\nu}$ is the slip parameter.

3 Results and Discussion

The system of governing Eqs (6) and (7) together with the boundary condition (8) is non-linear ordinary differential equations depending on the various values of the permeability parameter K, the magnetic parameter M, the buoyancy or mixed convection parameter λ, the suction parameter S, the slip parameter δ, and the Prandtl number Pr. The system of Eqs (6) and (7) is solved by Runge-Kutta fourth order scheme with a systematic guessing of by the shooting technique until the boundary conditions at infinity are satisfied. The step size $\Delta\eta = 0.01$ is used while obtaining the numerical solution and accuracy up to the seventh decimal place, i.e., 1×10^{-4}, which is very sufficient for convergence. The computations were done by a programme which uses a symbolic and computer language Matlab. In order to verify the accuracy of our present method, we have compared our results with those of Andersson[25] and Bhattacharyya *et al.*[29] Table 1 compares the values of the skin friction coefficient $f''(0)$ for various values of the slip parameter δ with $M = 0$, $K = 0$, $\lambda = 0$, $S = 0$ and Pr = 1. The comparisons are found to be in good agreement. The impacts of the magnetic parameter M on the velocity and temperature profiles are very significant in practical point of view. Figure 2 exhibits

Table 1 — Numerical values of skin friction coefficient $f''(0)$ for various values of the slip parameter δ with $M = 0$, $K = 0$, $\lambda = 0$, $S = 0$ and Pr = 1.

δ	$f''(0)$		
	Andersson[25]	Bhattacharyya *et al.*[29]	Present study
0.0	-1.0000	-1.00000	-1.000000
0.1	-0.8721	-0.872083	-0.872810
0.2	-0.7764	-0.776377	-0.777100
0.5	-0.5912	-0.591195	-0.591900
1.0	-0.4302	-0.430160	-0.430900
2.0	-0.2840	-0.283980	-0.284650
5.0	-0.1448	-0.144840	-0.145300
10.0	-0.0812	-0.081242	-0.081570
20.0	-0.0438	-0.043789	-0.043980
50.0	-0.0186	-0.0118597	-0.018676
100.0	-0.0095	-0.009551	-0.009581

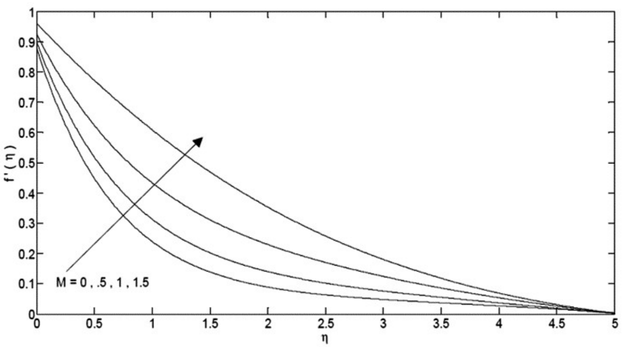

Fig. 2 — Velocity profiles for various values of M when K= 0.5, λ = 0.07, S = 0.5, δ = 0.1, Pr = 1.

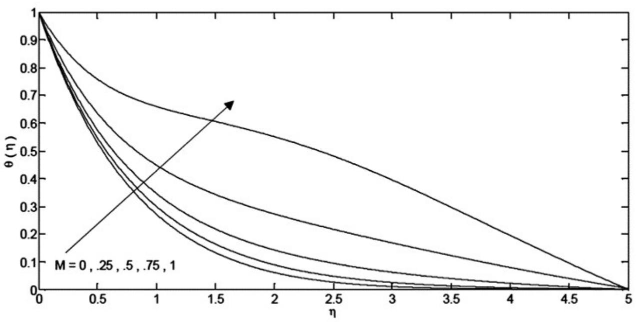

Fig. 3 — Temperature profiles for various values of M when K= 0.5, λ = 0.07, S = 0.5, δ = 0.1, Pr =1.

the effect of magnetic parameter *M* on the velocity profiles. The velocity profiles increase with increasing values of *M*. Accordingly, the thickness of momentum boundary layer decreases. This happens due to the Lorentz force arising from the interaction of magnetic and electric fields during the motion of the electrically conducting fluid. To reduce momentum boundary layer thickness the generated Lorentz force enhances the fluid motion in the boundary layer region. On the other hand, Fig. 3 depicts the effect of magnetic parameter *M* on the temperature profiles. The temperature profiles increases with increasing values of magnetic parameter *M*. Figures 4 and 5 show the effect of permeability parameter *K* on the velocity and temperature profiles, respectively. It is obvious that the presence of porous medium causes higher restriction to the fluid flow, which in turn slows its motion. Therefore, with increasing permeability parameter, the resistance to fluid motion increases and hence velocity decreases. For temperature profiles, it is obvious that the presence of porous medium reduced the temperature distribution. Thus, the thermal boundary layer thickness increases as permeability parameter increases.

Figures 6 and 7 show the effect of the buoyancy or mixed convection parameter λ on velocity and temperature profiles. It is noted that for increasing value of λ, the velocity increases. This phenomenon corresponds with the assumption of pure Darcy flow. Whereas increasing the λ decreases the thermal boundary layer thickness. This is because the increasing the λ increases the velocity near the surface. The high velocity near the surface carries more heat out of the surface, thus decreases the thermal boundary layer thickness. These buoyancy effects on the velocity profiles and the temperature profiles are very important for the physical and practical points of view.

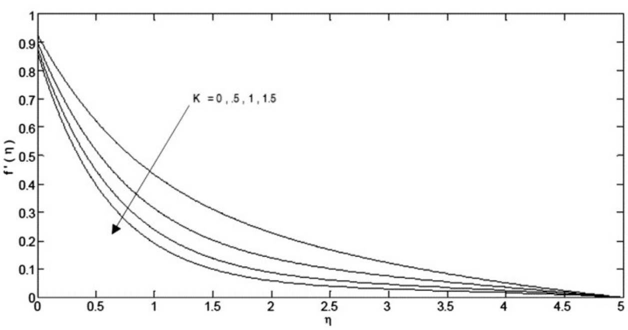

Fig. 4 — Velocity profiles for various values of K when M=0.5, λ = 0.07, S = 0.5, δ = 0.1, Pr = 1.

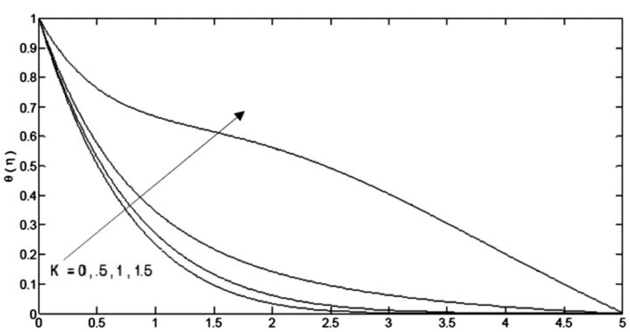

Fig. 5 — Temperature profiles for various values of *K* when *M*=0.5, λ = 0.07, *S* = 0.5, δ = 0.1, Pr =1.

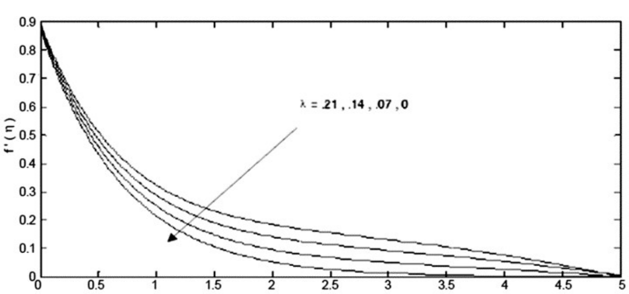

Fig. 6 — Velocity profiles for various values of λ when *M*=0.5, *K*=0.5, *S*=0.5, δ=0.1, Pr =1.

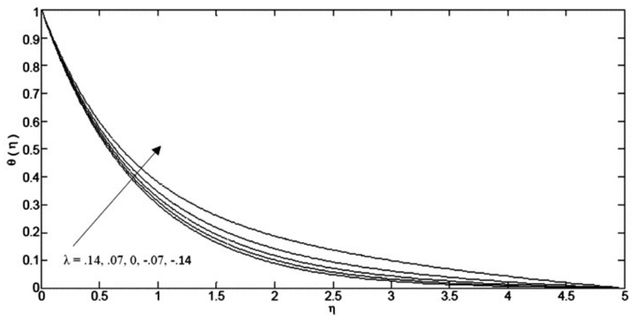

Fig. 7 — Temperature profiles for various values λ of when M = 0.5, K = 0.5, S = 0.5, δ = 0.1, Pr =1.

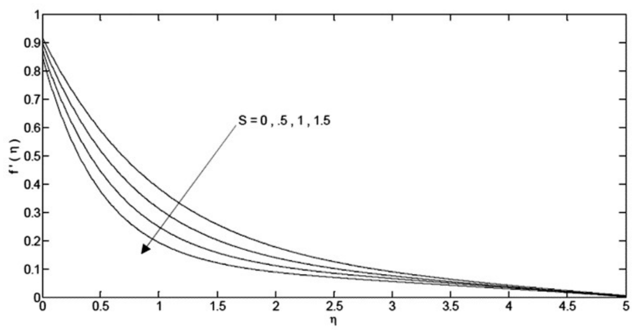

Fig. 8 — Velocity profiles for various values of S when M = 0.5, K = 0.5, λ = 0.07, δ = 0.1, Pr = 1.

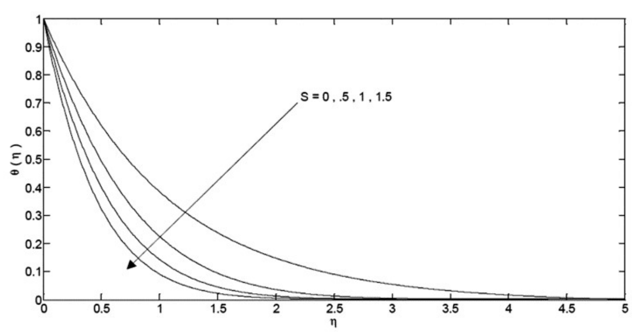

Fig. 9 — Temperature profiles for various values S when M = 0.5, K = 0.5, λ = 0.07, δ = 0.1, Pr = 1.

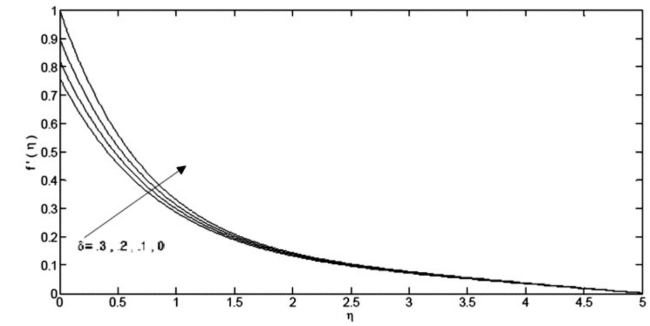

Fig. 10 — Velocity profiles for various values of δ when M = 0.5, K = 0.5, λ = 0.07, S = 0.5, Pr = 1.

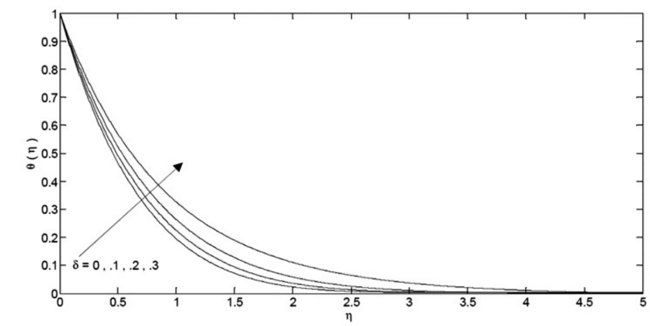

Fig. 11 — Temperature profiles for various values of δ when M = 0.5, K = 0.5, λ = 0.07, S = 0.5, Pr = 1.

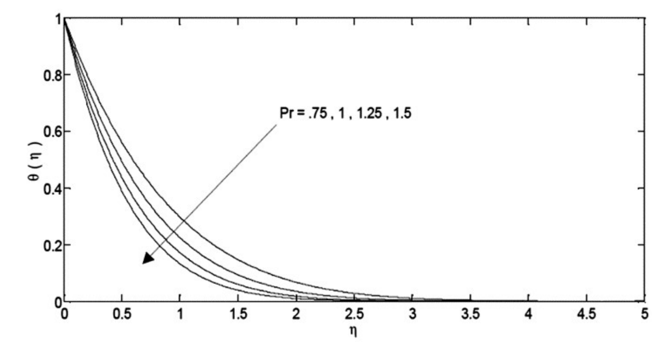

Fig. 12 —Temperature profiles for various values of Pr when M = 0.5, K = 0.5, λ = 0.07, S = 0.5, δ = 0.1.

Figures 8 and 9 show the effect of suction parameter S on velocity and temperature profiles respectively. We observe that the effect of increasing values of S the velocity and temperature decreases. This agrees with the natural phenomena.

Figures 10 and 11 show the effect of the slip parameter δ on velocity and temperature profiles, respectively. From the Fig. 9 it is noticed that the dimensionless velocity decreases for increasing strength of δ, but reversed phenomenon is observed for temperature profiles (Fig. 10).

The effect of Prandtl number (Pr) heat temperature may be analyzed from the Fig. 12. It is observed that the increase of Pr results in the decrease of temperature distribution. The reason is that smaller values of Pr are equivalent to increasing thermal conductivity and therefore heat is able to diffuse away from the heated surface more rapidly than for higher values of Pr.

4 Conclusions

The steady two-dimensional laminar MHD mixed convection flow over a vertical permeable sheet embedded in a porous medium has been investigated, with partial slip condition at the boundary. The coupled similar equations were obtained by using a suitable variable transformation and solved by Runge-Kutta fourth order method along with shooting technique. Numerical results were given for the dimensionless velocity profiles and dimensionless temperature profiles for various values of the magnetic parameter *M*, the permeability parameter *K*, the buoyancy or mixed convection parameter λ, the suction parameter *S*, the slip parameter δ, and the Prandtl number Pr. The present solutions are validated by comparing with the existing solutions. Our results show a good agreement with the existing work in the literature. The main physical results of the paper may be summarized as follows:

(i) The effect of magnetic parameter *M* increases the fluid velocity and temperature.
(ii) The effect of permeability parameter *K* decreases the fluid velocity and increases the temperature of the fluid.
(iii) The effect of the buoyancy or mixed convection parameter λ is to increase the fluid velocity and to decrease the temperature of the fluid.
(iv) The effect of suction parameter *S* decreases the fluid velocity and temperature.
(v) The effect of slip parameter δ is to decrease the fluid velocity and to increase the temperature of the fluid.
(vi) The boundary layers are highly influenced by Prandtl number Pr. The effect of Pr is to decreases the thermal boundary layer thickness.

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Nomenclature

- a Stretching constant
- B_0 Constant applied magnetic field
- f Dimensionless stream function
- g Gravity acceleration
- Gr_x Grashof number
- k_1 Permeability of porous medium
- K Permeability parameter
- L Slip length
- M Magnetic parameter
- Pr Prandtl number
- Re_x Reynolds number
- S Suction parameter
- T Temperature of the fluid
- u, v Velocity component of the fluid along the x and y directions, respectively
- x, y Cartesian coordinates along the surface and normal to it, respectively

Greek symbols

- η Similarity variable
- ρ Density of the fluid
- μ Viscosity of the fluid
- α Thermal diffusivity
- β Thermal expansion coefficient
- σ_e Electrical conductivity
- Ψ Stream function
- ν Kinematic viscosity
- λ Buoyancy parameter
- δ Slip parameter
- θ Dimensionless temperature

Superscript

- ʻ Derivative with respect to η

Subscripts

- w Properties at the plate
- ∞ Free stream condition

References

1 Alfven H, *Nature*, 150 (1942) 405.
2 Chandrasekhar S, *Hydrodynamic and hydromagnetic stability*, (Oxford University Press: England), 1961.
3 Sakiadis B C, *AIChE J*, 7 (1961) 221.
4 Crane L, *ZAMP*, 21 (1970) 645.
5 Pavlov K B, *Magn Gidrondin*, 4 (1974) 146.
6 Andersson H I, *Acta Machanica*, 113 (1995) 241.
7 Liu C, *Int Commun Heat Mass Transfer*, 32 (2005) 1075.
8 Wang C Y & *Z Angew Math Mech*, 69 (1989) 418.
9 Vajravelu K, *J Math Anal Appl*, 188 (1994) 1002.
10 Gorla R S R & Sidawi I, *Appl Sci Res*, 52 (1994) 247.
11 Chen C H, *Heat Mass Transfer*, 33 (1998) 471.
12 Elbashbeshy E M A & Bazid M A A, *Appl Math Comput*, 158 (2004) 799.
13 Cortell R, *Fluid Dyn Res*, 37 (2005) 231.
14 Ishak A, Nazar R & Pop I, *Heat Mass Transfer*, 44 (2008) 921.
15 Mukhopadhyay S, *Int J Heat Mass Transfer*, 52 (2009) 3261.
16 Chakrabarthi A & Gupta A S, *Q Appl Math*, 37 (1979) 73.
17 Chiam T, *Comput Math Appl*, 26 (1993) 1.
18 Ghaly A Y, *Chaos Soliton Fract*, 13 (2002) 1843.
19 Raptis A, *Appl Math Comput*, 153 (2004) 645.
20 Muhaimin, Kandasamy R, Hashim I & Khamis A B, *Theor Appl Mech*, 36 (2009) 101.
21 Noor N F M, Abdulaziz O & Hashim I, *J Num Methin Fluid*, 63 (2010) 357.
22 Jat R N & Chaudhary S, *App Math Sci*, 4 (2010) 65.
23 Jhankal A K & Kumar M, *Int J Eng Sci*, 2(3) (2013) 9.
24 Beavers G S & Joseph D D, *J Fluid Mech*, 30 (1967) 197.
25 Anderson H I, *Acta Mech*, 158 (2002) 121.
26 Wang C Y, *Chem Eng Sci*, 57 (2002) 3745.
27 Ariel P D, Hayat T & Asghar S, *Acta Mech*, 187 (2006) 29.
28 Ariel P D & *Z Angew, Math Mech*, 88 (2008) 320.
29 Bhattacharyya K, Uddin M S & Layek G C, *Acta Technica*, 58 (2013) 27.
30 Mukhopadhyay S, Layek G C & Samad S A, *Int J Heat Mass Transfer*, 48 (2005) 4460