

Dust acoustic waves in warm dusty plasmas

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The nonlinear propagation of dust acoustic waves in warm dusty plasma system containing Boltzmann distribution of electrons and ions, arbitrary charged dust grains has been investigated by employing the reductive perturbation technique (RPT). The nonlinear waves have been observed in the case of negative charged dust grains from the stationary solution of the Korteweg-de Vries (*K-dV*) equation, Burgers equation and Korteweg-de Vries-Burgers (*KdV-Burgers*) equation. The analytical solution of *K-dV-Burgers* and Burgers equation are numerically analyzed and DA shock waves propagation is reported. The critical values of the nonlinear coefficient *A* and the difference of shock wave profile of Burgers equation and *K dV-Burgers* equation for same parameters, have been observed.

Keywords: Dusty plasma, Solitary wave, Shock wave, Viscosity

1 Introduction

Now-a-days, the study of dusty plasmas represents one of the most rapidly growing branch of plasma physics. This is because in recent years dusty plasma plays a vital role in understanding different types of collective process in space environment, namely lower and upper mesosphere, radiofrequency plasma discharge, planetary ring, plasma crystals, commentary trail, asteroid zones, planetary magnetosphere, interplanetary spaces, interstellar medium, earth's environment etc¹⁻⁶. The dust grains are negatively charged because of a number of charging processes, such as field emission, ultra-violet radiation, plasma current, etc⁷⁻⁹. Similarly, it has been found that there are some plasma system, particularly in space plasma environments, namely cometary tails^{10,11} upper mesosphere¹² Jupiter's magnetosphere¹³ etc. where the positively charged dust particle plays some significant roles. Dusty plasma was first established as theoretical and experimental study, such as dust-acoustic mode^{14,15} in low phase velocity dust-acoustic wave¹⁶, dust-ion-acoustic mode¹⁷, dust-cyclotron mode¹⁸, dust-drift mode^{19,20}, etc. The dusty plasma condition has been found more or less, every model of plasma in space and laboratory plasma and which is why the field has been growing fast to explain many features in astrophysics problems and highlights the salient features of nonlinear plasma

acoustic waves viz. soliton dynamics²¹⁻²⁴, shock wave^{25,26}, double layers²⁷⁻²⁹, sheath formation in laboratory as well as in space which could explain special features on the formation of nebulous³⁰ (crystallization of dust clouds over the surface of moons and asteroids) observation of the solid body (e.g. moon) in astro-plasma. To make it more realistic, many researchers recently study in the direction of relativistic effect^{31,32}, quantum plasma³³⁻³⁵ and magnetized dusty plasma^{36,37} on different plasma model. Since the concept of studying the solitary waves by the well-known reductive perturbation technique, Sagdeev potential developed by many heuristic observation in various plasma configuration³⁸ (theoretically as an ideal model) in astrophysics¹ and supported by many experiments³⁹. All the observations depend on the plasma modally existing in different region of space plasma⁴⁰. Rao *et al*¹⁰. have studied the dust-acoustic solitary waves in an unmagnetized dusty plasma by using RPT which is only valid for a small but finite amplitude limit and Mamun *et al*¹². have investigated the nonlinear dust-acoustic waves in a two-component unmagnetized dusty plasma consisting of a negatively charged cold dust fluid and Maxwellian ions. In the present paper, unmagnetized dusty plasma; Boltzmann distributed electron and ions with arbitrary positive and negative dust charge have been studied.

2 Basic Equation

To study the nonlinear plasma acoustic wave, it is assumed that the finite size dust grains contaminate the plasma. The Boltzmann distributed electron and ion as a neutral plasma background have been considered. Following the basic equations, governing the dust charge grains that are in fluid description, the equations of continuity and momentum which can be written in the following form:

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_d) = 0 \quad \dots(1)$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} + \frac{\sigma_d}{n_d} \frac{\partial p_d}{\partial x} = z_d \frac{\partial \phi}{\partial x} + \eta \frac{\partial^2 u_d}{\partial x^2} \quad \dots(2)$$

$$\frac{\partial p_d}{\partial t} + u_d \frac{\partial p_d}{\partial x} + 3p_d \frac{\partial u_d}{\partial x} = 0 \quad \dots(3)$$

supplemented by the Poisson's equation as

$$\frac{\partial^2 \phi}{\partial x^2} = z_d \mu n_d + (1 - \mu) n_e - n_i \quad \dots(4)$$

The electron and ion-density may be described by a Boltzmann distribution i.e.

$$n_e = n_{e_0} \exp(\phi) \quad \dots(5)$$

$$n_i = n_{i_0} \exp(-\gamma\phi) \quad \dots(6)$$

When $n_\alpha (\alpha = d, e, i)$, u_d , p_d , ϕ , x , t are the dust particle number density, electron number density, ion number density, dust fluid velocity, dust fluid pressure, electrostatics potential, space variable and time, respectively and they have been normalized by n_{d_0} (unperturbed dust particle number density), n_{e_0} (unperturbed electron particle number density) and n_{i_0} (unperturbed ion particle number density);

$$\mu = \frac{n_{d_0}}{n_{i_0}}, \quad \gamma = \frac{T_e}{T_i}, \quad \sigma_d = \frac{T_d}{T_e} \quad \text{where } T_\alpha (\alpha = d, e, i)$$

is the temperature for dust, electron and ion. μ_d is the fluid velocity normalized to the dust acoustic speed

$$C_d = \left(\frac{z_d n_{d_0} e \mu + 3 \sigma_d K_B T_e q}{m_d q} \right)^{\frac{1}{2}} \quad \text{with}$$

$q = (1 - \mu) n_{e_0} + \gamma n_{i_0}$ and K_B , m_d and z_d being the Boltzmann constant, dust acoustic mass and charged number of dust particles. p_d is the pressure normalized to $n_{d_0} K_B T_d$; here $3 = (2 + N) / N$ with N being the number of degrees-of-freedom; ϕ is the electrostatic wave potential normalized by $\left(\frac{K_B T_i}{e} \right)$,

with e being the electron charged; the space variable normalized to the dust Debye length

$$\lambda_d = \left(\frac{3 \sigma_d K_B T_e m_d}{4 \pi e n_{d_0} (z_d^2 + qe)} \right)^{1/2} \quad \text{and the time variable is}$$

normalized to the dust plasma period

$$\omega_{pd}^{-1} = \left(\frac{m_d}{4 \pi n_{d_0} z_d^2 e^2} \right)^{1/2}. \quad \text{The coefficient of viscosity}$$

η is a normalized quantity given by $\omega_{pm} \lambda_m^2 m_d n_{d_0}$. All other symbols have their usual meanings.

The overall charge neutrality condition has been maintained throughout the plasma by the following relation:

$$z_d \mu n_{d_0} + (1 - \mu) n_{e_0} = n_{i_0} \quad \dots(7)$$

3 Derivation of Korteweg-de Vries (K-dV) Equation

In order to derive the K -dV equation, the following stretched coordinates are used:

$$\xi = \epsilon^{1/2} (x - \lambda t), \quad \tau = \epsilon^{3/2} t \quad \dots(8)$$

where λ is the phase velocity of the wave along the x direction and normalized by acoustic velocity and ϵ is a smallness dimensionless expansion parameter which measuring strength of the dispersion. The physical variables of plasma parameters namely n_d , u_d , p_d , ϕ are expanded in power series written in general form as:

$$S = S^{(0)} + \epsilon S^{(1)} + \epsilon^2 S^{(2)} + \epsilon^3 S^{(3)} + \dots \quad \dots(9)$$

we get $S^{(0)} = 0, 1$ for ϕ , u_d and n_d , p_d , respectively.

We are substituting Eqs. (8, 9) into the basic Eqs (1-6), and thereafter, equating the coefficient of ϵ , the lowest order of ϵ derives:

$$\begin{aligned}
n_d^{(1)} &= -\left\{ \frac{(1-\mu)n_{e_0} + \gamma n_{i_0}}{z_d \mu} \right\} \phi^{(1)} \\
u_d^{(1)} &= -\lambda \left\{ \frac{(1-\mu)n_{e_0} + \gamma n_{i_0}}{z_d \mu} \right\} \phi^{(1)} \\
p_d^{(1)} &= -3 \left\{ \frac{(1-\mu)n_{e_0} + \gamma n_{i_0}}{z_d \mu} \right\} \phi^{(1)} \\
\lambda^2 &= 3\sigma_d + \frac{z_d}{m_d} \left\{ \frac{z_d \mu}{(1-\mu)n_{e_0} + \gamma n_{i_0}} \right\} \quad \dots(10)
\end{aligned}$$

For the next higher order of \mathcal{E} , we get

$$\frac{\partial n_d^{(1)}}{\partial \tau} - \lambda \frac{\partial n_d^{(2)}}{\partial \xi} + \frac{\partial u_d^{(2)}}{\partial \xi} + \frac{\partial (n_d^{(1)} u_d^{(1)})}{\partial \xi} = 0 \quad \dots(11)$$

$$\frac{\partial u_d^{(1)}}{\partial \tau} - \lambda \frac{\partial u_d^{(2)}}{\partial \xi} + \sigma_d \frac{\partial p_d^{(2)}}{\partial \xi} = z_d \frac{\partial \phi^{(2)}}{\partial \xi} + z_d n_d^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} \quad \dots(12)$$

$$\begin{aligned}
\frac{\partial p_d^{(1)}}{\partial \tau} - \lambda \frac{\partial p_d^{(2)}}{\partial \xi} + u_d^{(1)} \frac{\partial p_d^{(1)}}{\partial \xi} + 3p_d^{(1)} \frac{\partial u_d^{(1)}}{\partial \xi} \\
+ 3 \frac{\partial u_d^{(2)}}{\partial \xi} = 0 \quad \dots(13)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} &= n_{e_0} (1-\mu) \phi^{(2)} + n_{e_0} (1-\mu) \frac{1}{2} (\phi^{(1)})^2 \\
&+ z_d \mu n_d^{(2)} + \gamma n_{i_0} \phi^{(2)} - \frac{\gamma}{2} n_{i_0} (\phi^{(1)})^2 \quad \dots(14)
\end{aligned}$$

eliminating $n_d^{(2)}$, $u_d^{(2)}$, $\phi^{(2)}$, $p_d^{(2)}$ form Eqs (11 - 14) and using Eq. (10), the K -dV equation has been derived as:

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + B \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0 \quad \dots(15)$$

where

$$\begin{aligned}
A &= \frac{z_d \mu \{ \gamma^2 n_{i_0} - (1-\mu)n_{e_0} \} (\lambda^2 - 3\sigma_d)}{2\lambda z_d \mu \{ (1-\mu)n_{e_0} + \gamma n_{i_0} \}^2} \quad \dots(16)
\end{aligned}$$

and

$$B = \frac{\lambda^2 - 3\sigma_d}{2\lambda \{ (1-\mu)n_{e_0} + \gamma n_{i_0} \}} \quad \dots(17)$$

By using the tanh-method⁴¹ the stationary wave solution of the K -dV Eq. (15) is derived as:

$$\phi^{(1)} = \phi_m \operatorname{sech}^2 \left(\frac{\chi}{\omega} \right) \quad \dots(18)$$

where quantities $\phi_m = \frac{3M}{A}$ and $\omega = 2 \left(\frac{B}{M} \right)^{1/2}$ are the amplitude and width of the solitary waves, respectively and the independent variables ξ and τ to $\chi = \omega(\xi - M\tau)$ where M is the Mach number and imposing the appropriate boundary condition, viz.

$$\phi^{(1)} \rightarrow 0, \phi^{(1)'} \rightarrow 0, \phi^{(1)''} \rightarrow 0 \text{ at } |\chi| \rightarrow \infty \quad \dots(19)$$

The soliton profile depends on A , B , which are functions of plasma parameters and Mach number.

Now our aim is to investigate how the nature of the solitary wave solution of Eq. (15) depends on the value of charge density ratio μ . We shall see that the solution of Eq. (18) describes a compressive or rarefactive solitary wave whether $A > 0$ or $A < 0$. From Eq. (16), the nonlinear coefficient $A \rightarrow \infty$ for $n_d \rightarrow 0$. Therefore, the nonlinear coefficient A appearing in the amplitude ϕ_m , ϕ_m will be zero, i.e. if the equilibrium dust density n_{d_0} is zero then Eq. (18) has no soliton and the nonlinear coefficient A can be zero. In this case, A has a critical value. For the critical values of nonlinear coefficient A i.e. $A \rightarrow 0$ appearing in the amplitude ϕ_m , ϕ_m will be infinity i.e. $\phi_m \rightarrow \infty$. In this particular case, the Eq. (18) has no solitary waves. For various conditions on the plasma parameter, A will be positive, negative and zero as shown in the following Fig. 1.

4 Derivation of the Burgers Equation

In order to derive the Burgers equation, the following stretched coordinates are used:

$$\xi = \mathcal{E}(x - \lambda t), \quad \tau = \mathcal{E}^2 t \quad \dots(20)$$

Following similar methodology, using the new set of stretching coordinate along with the perturbation

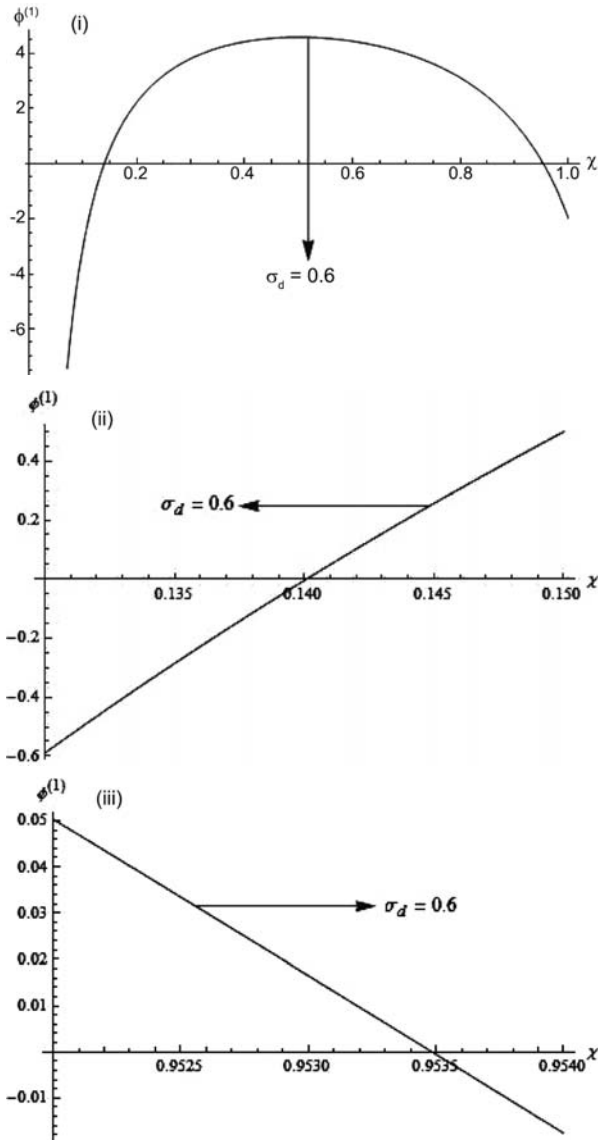


Fig. 1 — (i) Variation of nonlinearity parameter A is plotted against μ for $\lambda=0.1, z_d=10, \gamma=0.2$ and different values of $\sigma_d = 0.6$, (ii) μ varies from 0.12 to 0.15 and (iii) μ varies from 0.952 to 0.953

scheme Eq. (9) in to the set of basic Eqs (1-6), we get a similar set of relation as Eq. (10) for the first order terms in ϵ . However to the next higher order terms in ϵ , one obtains the Eqs (11) and (13) same as before and:

$$\frac{\partial u_d^{(1)}}{\partial \tau} - \lambda \frac{\partial u_d^{(2)}}{\partial \xi} + \sigma_d \frac{\partial p_d^{(2)}}{\partial \xi} = \frac{z_d}{m_d} \frac{\partial \phi^{(2)}}{\partial \xi} + \frac{z_d n_d^{(1)}}{m_d} \frac{\partial \phi^{(1)}}{\partial \xi} + \eta \frac{\partial^2 u_d^{(1)}}{\partial \xi^2} \quad \dots(21)$$

and

$$z_d (1 - \mu) \phi^{(2)} + z_d (1 - \mu) \frac{1}{2} (\phi^{(1)})^2 + \mu n_d^{(2)} + \gamma \phi^{(2)} - \frac{\gamma}{2} (\phi^{(1)})^2 = 0 \quad \dots(22)$$

repeating the same procedure to eliminate $n_d^{(2)}, u_d^{(2)}, \phi^{(2)}, p_d^{(2)}$ form Eqs. (11), (13), (21), (22) and using Eq. (10), we can obtain the Burgers equation as:

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} = C \frac{\partial^2 \phi^{(1)}}{\partial \xi^2} \quad \dots(23)$$

where the value of A is the same as before and C is given by:

$$C = \frac{\eta}{2\lambda} \quad \dots(24)$$

By using the tanh-method⁴² the shock wave solution of Burgers Eq. (23) is derived as:

$$\phi^{(1)} = \phi_m \left\{ 1 - \tanh \left(\frac{\chi}{\omega'} \right) \right\} \quad \dots(25)$$

where

$$\chi = (\xi - Mt), \phi_m = \frac{M}{A} \text{ and } \omega' = 2 \left(\frac{C}{M} \right) \quad \dots(26)$$

The quantities ϕ_m and ω' are the amplitude and width of the shock waves, respectively, and M is the Mach number. The shock wave profile depends on the variables A and C, which are functions of plasma parameters. It is clear from Eq. (25) that the shock potential profile is positive (negative) when A is positive (negative).

5 Derivation of Korteweg-de Vries-Burgers Equation

To derive the K-dV-Burgers equation for further study of shock wave solution in dusty plasma, we introduce a new scaling with Eq. (8) as $\eta = \epsilon^{1/2} \eta_0$, then Eq. (12) becomes:

$$\frac{\partial u_d^{(1)}}{\partial \tau} - \lambda \frac{\partial u_d^{(2)}}{\partial \xi} + \sigma_d \frac{\partial p_d^{(2)}}{\partial \xi} = \frac{z_d}{m_d} \frac{\partial \phi^{(2)}}{\partial \xi} + \frac{z_d n_d^{(1)}}{m_d} \frac{\partial \phi^{(1)}}{\partial \xi} + \eta_0 \frac{\partial^2 u_d^{(1)}}{\partial \xi^2} \quad \dots(27)$$

and Eqs (11), (13) and (14) remain same as before. Now replacing Eq. (12) by Eq. (27) and performing all mathematical steps as we did in order to derive the *K-dV* Eq. (15), we obtain *K-dV-Burgers* equation as:

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + B \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = C \frac{\partial^2 \phi^{(1)}}{\partial \xi^2} \quad \dots(28)$$

where the coefficients *A*, *B* are the same as above and

$$C = \frac{\eta_0}{2\lambda} \quad \dots(29)$$

Now to solve the *K-dV-Burgers* equation we used the transformation $\chi = \xi - U\tau$, and considering $\phi^{(1)}(\xi, \tau) = \psi(\chi)$ which gives:

$$B \frac{d^2 \psi}{d\chi^2} - C \frac{d\psi}{d\chi} + A \frac{\psi^2}{2} - U\psi = 0 \quad \dots(30)$$

To derive the required solution of *K-dV-Burgers* Eq. (28), we used well-known *tanh*-method and for that the transformation $z = \tanh(\chi)$, $\psi(\chi) = w(z)$ is introduced and Eq. (30) becomes:

$$(1-z^2)^2 \frac{d^2 w}{dz^2} - \left(2z + \frac{C}{B}\right) (1-z^2) \frac{dw}{dz} + \frac{A}{2B} w^2 - \frac{U}{B} w = 0 \quad \dots(31)$$

For finding the series solution of Eq. (31) substituting $w(z) = \sum_{r=0}^{\infty} a_r z^{\rho+r}$ and for leading order analysis of finite terms gives $r=2$ and $\rho=0$ and then the $w(z)$ becomes $w(z) = a_0 + a_1 z + a_2 z^2$

Now substituting the value of $w(z)$ in Eq. (31), gives the values of a_0, a_1 and a_2 as:

$$a_0 = \frac{1}{A}(U + 12B), \quad a_1 = -\frac{12C}{5A}, \quad a_2 = -\frac{12B}{A} \quad \dots(32)$$

This gives the solution of *K-dV-Burgers* equation as:

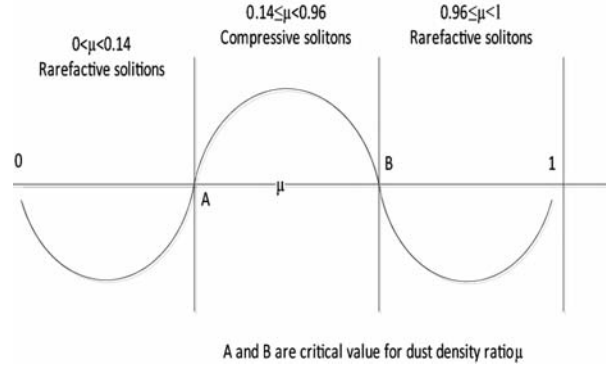


Fig. 2 — Region for rarefactive and compressive solitons of *K-dV* equation

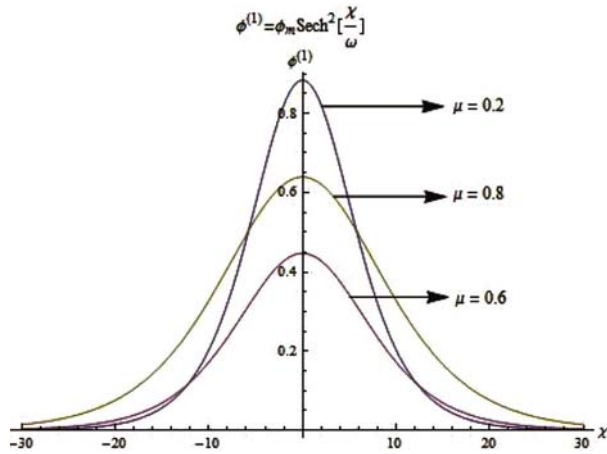


Fig. 3 — Amplitude of *K-dV* ($\phi^{(1)}$) soliton plotted against χ , showing the formation of compressive soliton

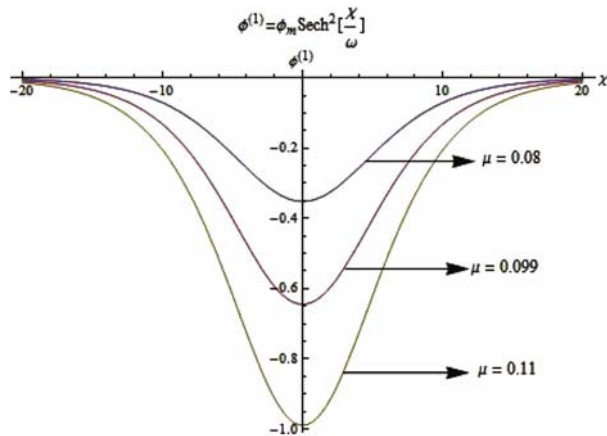


Fig. 4 — Amplitude of the *K-dV* ($\phi^{(1)}$) soliton plotted against χ , showing the formation of rarefactive soliton

$$\phi^{(1)} = \frac{U}{A} - \frac{12C}{5A} \tanh(\chi) + \frac{12B}{A} \operatorname{sech}^2(\chi) \quad \dots(33)$$

where $C^2 = 100B^2$ and $U = 24B$.

6 Results and Discussion

In Fig. 1, the nonlinear coefficient A is plotted against dust density ratio μ for different values of dust temperature σ_d and $\lambda=0.1, z_d=10, \gamma=0.2, M=1.001$. Figure 1 shows that A will be zero where $0.135 < \mu < 0.14$ and $0.95 < \mu < 0.96$. There are two critical dust densities for this system roughly

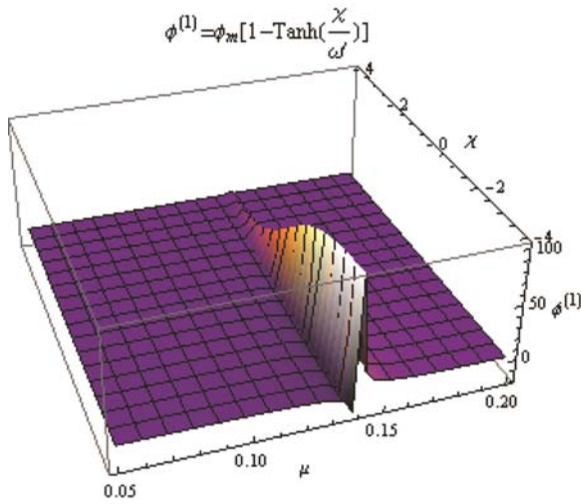


Fig. 5 — Neighborhood of $A=0$, the increases in the shock wave profile for dust density n_{d0}

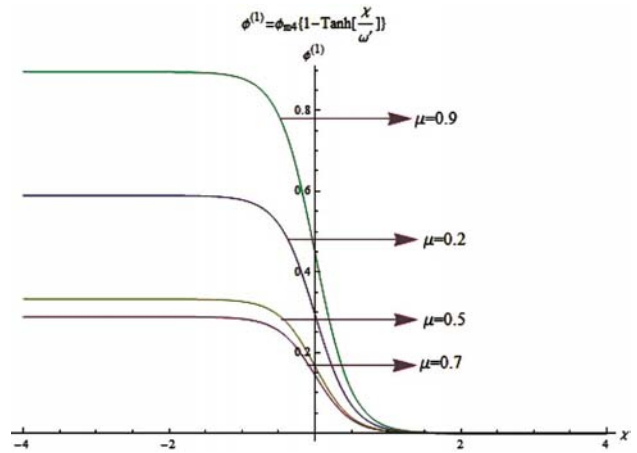


Fig. 6 — Variation of the DA shock wave potential $\phi^{(1)}$ plotted against χ for different plasma parameter

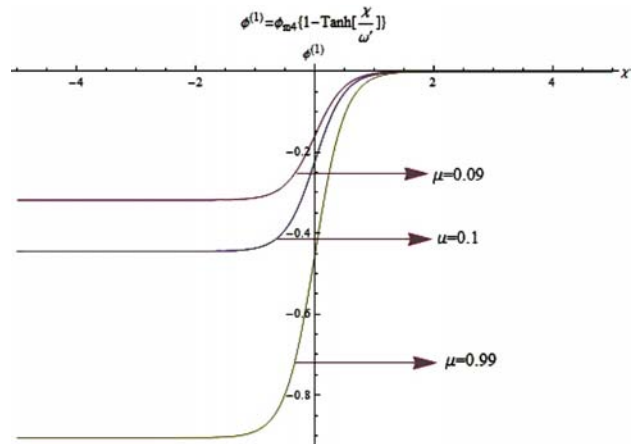


Fig. 7 — Variation of the DA shock wave potential $\phi^{(1)}$ plotted against χ for different plasma parameter

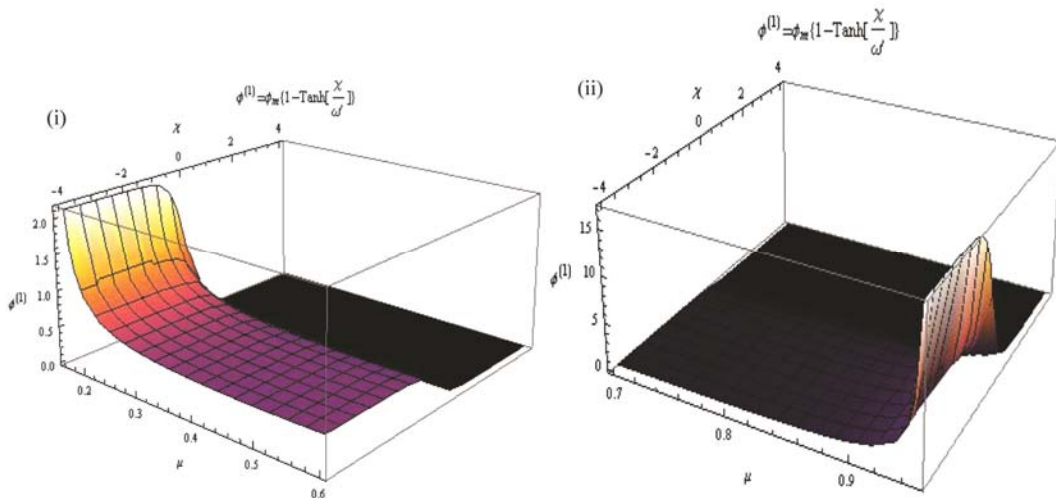


Fig. 8 — Variation of the positive DA shock wave potential $\phi^{(1)}$ plotted against spatial coordinate χ for different plasma parameter, where (i) μ varies from 0.2 to 0.6 and (ii) μ varies from 0.7 to 0.9

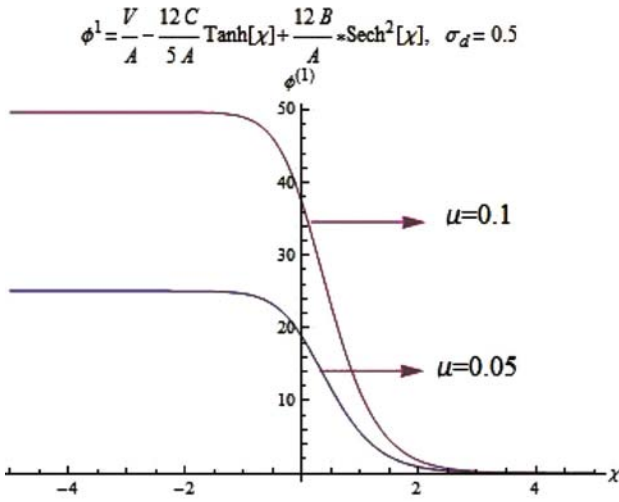


Fig. 9 — Variation of the DA shock wave potential $\phi^{(1)}$ plotted against χ for K-dV-Burgers equation

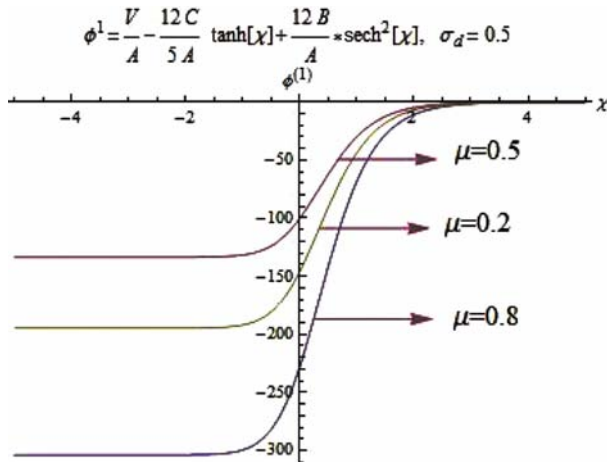
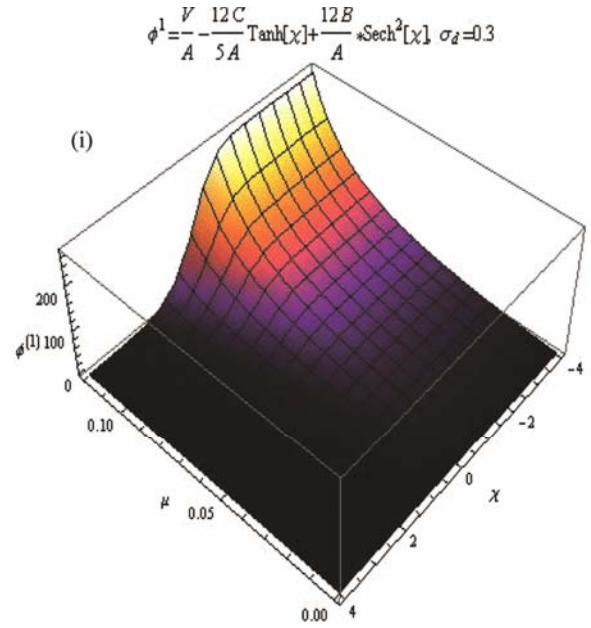


Fig. 10 — Variation of the DA shock wave potential $\phi^{(1)}$ plotted against χ for K-dV-Burgers equation

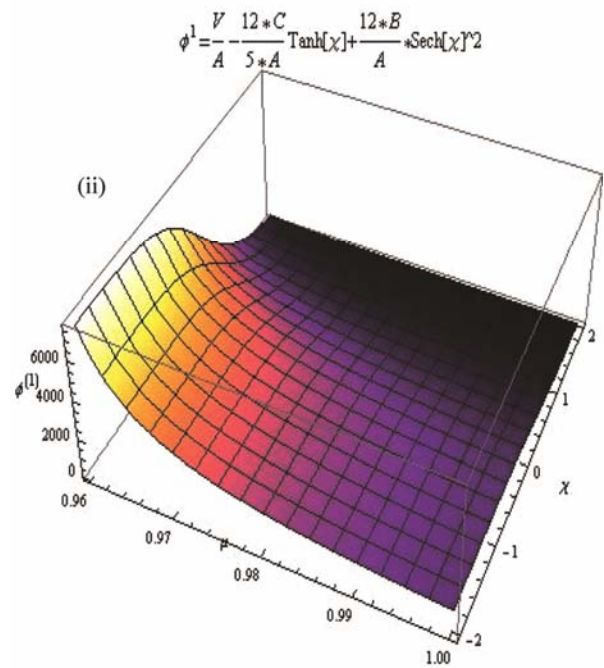


Fig. 11 — Variation of the positive DA shock wave potential $\phi^{(1)}$ plotted against spatial coordinate χ for KdV-Burgers equation, where (i) μ varies from 0.0 to 0.1 and (ii) μ varies from 0.96 to 1

$n_{dc_1} = 1411 \times 10^{-4}$ and $n_{dc_2} = 95349 \times 10^{-5}$ from Fig. 1(ii) and 1(iii). Therefore, Eq. (18) has no solitary wave for these two critical values. Again Fig. 1 shows that A will be negative where $0 < \mu < 0.1411$ and positive where $0.954 < \mu < 1$ and positive where $0.1412 \leq \mu \leq 0.9535$. The regions of compressive and rarefactive solitons are schematically shown in Fig. 2. In Figs 3 and 4, amplitude of the compressive and rarefactive solitary wave solutions Eq. (18) of the K-dV equation are plotted against χ for different values of μ .

We note that in Fig. 5, shock wave profiles are positively and negatively increasing for the neighbourhood of the critical value. The electrostatic shock profiles, caused by the balance between

nonlinearity coefficient and dissipation coefficient, are shown in Figs (6-8). The shock wave profile of Eq. (25) is positive for $A > 0$ and negative for $A < 0$ with same plasma parameter. In Figs (9-11) amplitude of the shock waves solutions Eq. (33) of the K-dV-Burgers equation is plotted against χ for different values of μ . It is observed basically the shock waves

profiles are positive for $A < 0$ and negative for $A > 0$. It is clearly seen that, shock wave profile of K -dV-Burgers equation is opposite of Burgers equation's of shock profile.

7 Conclusions

The formation of nonlinear structures in dusty plasma with a Boltzmann distributed electrons, ions considering constant dust charged and entering influence component of pressure with the help of K -dV equation, Burgers equation and K -dV-Burgers equation, have been studied. It is found that the nature of solitary waves is compressive and rarefactive as per positive and negative values of A . For weak dispersive coefficient in K -dV - Burgers equation will get shock wave solutions. Dust acoustic shock wave solutions of Burgers equation and K -dV-Burgers equations are directly dependent on dust density. It is observed mainly for Burgers equation with the increasing dust density, a transition of shock front occurs from the negative to positive and again from positive to negative potential but for K -dV-Burgers equation, with increasing dust density, a transition of shock front occurs from the positive to negative and again from negative to positive potential. From our graphical presentation, it is concluded that, both Burgers and K -dV-Burgers equation give us shock waves but opposite characteristic.

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