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# Screening mechanism and multi-vortices in dual QCD vacuum

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The screening effects in a chromodynamic vacuum which act as dual superconductor in the background of the magnetic condensation, have been studied. The colour charge and colour electric flux screening mechanism have been investigated and these screening effects are shown to be responsible for the colour confinement in dual QCD. It is also demonstrated that with the transition from the type-II to type-I in dual QCD vacuum at strong coupling constant $\alpha_s \approx 0.5$ , there exist n-vortex solutions with Bogomol'nyi-Prasad-Sommerfeld (BPS) conditions.

**Keywords:** Dual QCD, Charge/flux screening, Confinement, BPS conditions, Multi-vortex

## 1 Introduction

The Bardeen-Cooper-Schrieffer (BCS) theory of conventional superconductivity has played an influential role in bringing the spontaneous symmetry breaking to the elementary-particle physics community in diversified ways[1,2]. In particular, it has a direct analogue in QCD which explains various non-perturbative features[3]. In fact, the striking parallelism of QCD vacuum with the conventional super-conductivity, where the condensation of magnetically charged objects[3-5] (viz. monopoles and dyons), plays an important role in a way analogous to the Cooper pair condensation of electric charges is of prime importance in QCD to explain various issues especially related to the confinement and deconfinement (i.e. formation of quark-gluon plasma[6]) scenario[3,4]. As such, in naive QCD Lagrangian, the monopoles do not appear as dynamical variables, and thereby it is quite necessary to have these configurations as macroscopic variables in order to impart the chromomagnetic superconducting features to the QCD vacuum[3-19]. In fact, the superconducting nature of QCD vacuum is basically due to the coherent plasma of monopole or dyon pairs and leads to a covariant description of QCD vacuum as a magnetic superconductor[9,11,15]. In magnetic superconductors[3,9-16], the dual potentials coupled to a field operator similar to a complex scalar field corresponding to a monopole or dyon field[4,17,20,21] are the natural variables to describe the large-distance response of QCD vacuum. In such dual superconductor models[3], the topological interaction is spanned over at the different vacuum expectation values of the monopole or dyon field for various length scales in the spontaneously broken phase of symmetry. On the other hand, in view of the brilliant insights of the Abelian gauge fixing techniques[18] and lattice QCD calculations in maximally Abelian (MA) gauge[19], it is quite reasonable to pay due attention over the Abelian component which dominates in the non-Abelian gauge theories at large-distances and there has been a growing impetus[22] to speculate the Abelian dominance in QCD to study the colour confinement in QCD. Moreover, the physical vacuum with a non-Abelian gauge theory like QCD appears analogous to the ground state of an interacting many-body system and leads to the possibility of vacuum screening currents[23]. A non-vanishing vacuum expectation value (VEV) of the scalar field in the ground state of such vacuum then leads to the mass acquisition of the dual gauge field which pushes the QCD vacuum in superconducting phase. Further, such models are also renormalisable[24]. It is, therefore, imperative to further investigate the screening effects and vortex configurations[25,26], in dual QCD.

In the present paper, the screening effects led by the magnetic condensation of the QCD vacuum to investigate their enlightening impacts on the confinement scenario, have been studied. The novel element of this work is the inclusion of quarks together with the monopoles or dyons into the effective Lagrangian for the dual QCD phenomenologically and it is an extension of our previous work related to the screening current and dielectric parameters[10] in dual QCD.

## 2 Screening Effects and Confinement

The model with the effective Abelian field which describes the strong interaction in QCD (in presence of quarks) with the complex scalar monopole field can phenomenologically be considered by the Lagrangian[9, 28] in the following form:

$$L = -\frac{1}{4}\tilde{G}_{\mu\nu}\tilde{G}^{\mu\nu} + \left| (\partial_\mu + i\,\tilde{g}\,\tilde{C}_\mu)\,\phi \right|^2 + \bar{\psi}\,\gamma^\mu(\partial_\mu + i\,\tilde{g}\,\tilde{C}_\mu)\,\psi - m\psi\bar{\psi} - V(\phi\phi^*) \quad \ldots(1)$$

where $\tilde{G}_{\mu\nu} = \partial_\nu\tilde{C}_\mu - \partial_\nu\tilde{C}_\mu$ is the field strength tensor corresponding to the dual gauge field $\tilde{C}_\mu$, $\phi$ is the scalar field with the magnetic charge $\tilde{g} = g\,/\,(4\pi)$ and $\Psi$ is the quark field with $m$ as the free quark mass. Here $g$ is related to the strong coupling constant[9-11] defined as $\alpha_s = g^2/(4\pi)$. The effective potential with correct field-theoretic description of the monopoles from phenomenological view point[9,10] to incorporate the notions of symmetry breakdown given in Eq.(1) is then given by:

$$V(\phi\phi^*) = \Omega\,(\phi\phi^* - \eta^2)^2 \quad \ldots(2)$$

where $\eta^2 = \langle\varphi\varphi^*\rangle_0$ represents the VEV of complex scalar monopole field ($\phi$) and $\Omega = 3\lambda/\alpha_s^2$ which is a constant[9]. The potential given by Eq. (2) at $\phi = \eta$ gives the ground-state field configuration which captures the essential physics for the symmetry breakdown and thereby resulting the features corresponding to a particular theory. The field equations corresponding to the dual gauge, monopole and quark fields are then derived respectively in the following form:

$$\partial^\nu\tilde{G}_{\mu\nu} + i\,\tilde{g}\,(\varphi^*\overleftrightarrow{\partial}_\mu\varphi) - \tilde{g}\,\bar{\psi}\,\gamma_\mu\,\psi\,0 - \tilde{m}^2\,\tilde{C}_\mu = 0 \quad \ldots(3)$$

$$(\partial_\mu + i\,\tilde{g}\,\tilde{C}_\mu)^2\,\phi - 2\Omega\,(|\phi|^2 - \eta^2)\,\phi = 0 \quad \ldots(4)$$

$$(i\,\gamma_\mu\,\partial^\mu - \tilde{g}\gamma_\mu\tilde{C}^\mu - m)\,\psi = 0 \quad \ldots(5)$$

where $\tilde{m} = (8\pi/\alpha_s)^{1/2}\,\eta$ is the mass acquired by the dual gauge field as an immediate consequence of the symmetry breakdown. The total colour charge from Eq. (3) may then be calculated as follows:

$$\tilde{Q}_c = \int d^3x\,\tilde{C}_0 + \tilde{g}\int d^3x\,\psi\dagger\psi \quad \ldots(6)$$

Since all the observed hadrons are colour singlets, the total colour electric charge given by Eq. (6) on a quark system must vanish. In order to find the axial symmetric confined solution around *z*-axis for the field equations of the present model, let us first consider the following cylindrically symmetric ansatz[11,29]:

$$\tilde{\mathrm{C}} = -\mathrm{e}_\theta\,\tilde{C}(\rho) \quad \tilde{C}_0 = 0 \quad \phi(\rho) = \chi(\rho)\exp(in\theta) \quad \ldots(7)$$

where $n = \pm1, \pm2, \pm3, \cdot\cdot\cdot\cdot\cdot\cdot$, which measures the magnitude of the colour electric flux. In asymptotic regime for large ρ, the complex scalar field $|\varphi| = \eta$ with $\chi \rightarrow \eta$ plays the role of an order parameter like the Cooper wave function does in the Ginzburg-Landau (GL) theory of conventional superconductivity[29,30]. However, we consider the following form of ψ for the stationary solution to the bound quark[28]:

$$\psi = \exp(-i\varepsilon t)\begin{pmatrix} u\,(\rho,\theta,z) \\ v\,(\rho,\theta,z) \end{pmatrix} \quad \ldots(8)$$

where $u(\rho)$ and $v(\rho) \rightarrow 0$ as $\rho \rightarrow \infty$. Utilising ansatz in Eqs (7) and (8) for the total colour charge screening, the temporal gauge degrees of freedom of the dual gauge field must decay faster than $\rho^{-1}$ in view of the Gauss law, which results in the total colour charge as given by expression given in Eq. (6). The temporal gauge degrees of freedom for a single quark system (i.e. $\int d^3x \;\; \psi\dagger\psi = 1$) then leads to the following necessary condition for the total charge screening at large distances (cf.[28]),

$$\tilde{m}^2\tilde{C}_0 >> \tilde{g}\,\psi\dagger\psi \quad \ldots(9)$$

this, therefore, puts a constraint on a quark system. $\tilde{C}_0$ thus plays a role of screening potential[28]. Such QCD vacuum having non-zero equilibrium values cannot be achieved by using the perturbative techniques where the equilibrium values of all the fields are considered to be zero, which in turn will lead to $\tilde{m} = 0$. Further, it is only possible to generate a vacuum screening current in the non-perturbative sector of QCD having the non-vanishing VEV of the complex scalar field with the condition given by Eq. (9). The vacuum screening current with the massive dual gauge field then quickly leads to a desired onset for the colour confinement where a superconducting ground state of QCD vacuum is established as a coherent plasma of the magnetic charges[10,12].

In fact, in the present formulation of dual QCD vacuum, the dual field-strength tensor $\tilde{G}_{\mu\nu}$ has its field contents in terms of the colour electric ($\tilde{E}$) and magnetic ($\tilde{H}$) fields[10]. The field Eqs (3) and (4) in view of the ansatz given in Eq. (7) then acquire the form as given below:

$$\tilde{C}'' + \frac{\tilde{C}'}{\rho} - \frac{\tilde{C}}{\rho^2} - \frac{2n\tilde{g}}{\rho}\chi^2 - \tilde{m}^2\tilde{C} = 0 \qquad \ldots(10)$$

$$\tilde{C}_0'' + \frac{\tilde{C}_0'}{\rho} - \tilde{m}^2\tilde{C}_0 = 0 \qquad \ldots(11)$$

$$\chi'' + \frac{\chi'}{\rho} - \left(\frac{n}{\rho} + \tilde{g}\,\tilde{C}\right)\chi + \tilde{g}^2\tilde{C}_0^2\chi + 2\,\Omega\,(\chi^2 - \eta^2)^2\,\chi = 0 \qquad \ldots(12)$$

with the following boundary condition for the spatial part of the dual gauge field at large distances in addition to the Eq. (9) responsible for the colour charge screening:

$$\tilde{m}^2\rho\,\tilde{C} + 2n\tilde{g}\varsigma^2 >> \rho\,(\psi\dagger\gamma\psi) \qquad \ldots(13)$$

In Eqs (10)-(13), prime ($'$) denotes the differentiation with respect to $\rho$. The electric and magnetic fields in the Eqs (10) and (11) are defined respectively as follows:

$$\tilde{E} = -\frac{1}{\rho}(\rho\,\tilde{C})' \qquad \tilde{H} = \tilde{C}_0' \qquad \ldots(14)$$

Eqs (10)-(12) have the structural similarity with the equations those are derived for a GL-type superconductor and would,therefore, lead to similar consequences. With Eq. (14), the Eqs (10) and (11) can also be re-casted in terms of the colour electric and magnetic fields,respectively. The solution of the Eqs (10) and (11) is exponentially approachable in the asymptotic limit[11,31]. For instance, the colour electric field at a particular coupling evolves generally for $\rho \to \infty$ as follows:

$$\tilde{E} \to C\sqrt{\frac{n}{\rho}}\,\exp(-\tilde{m}\,\rho) + \text{non-leading terms} \qquad \ldots(15)$$

where $C$ is a constant. Eq. (15) depending on the gauge field mass clearly indicates that the colour electric flux screens out in the dual QCD vacuum up to a finite depth $\tilde{\lambda} = \tilde{m}^{-1}$ which determines the magnitude of the dual Meissner effect (DME) responsible for the confinement of the quarks[10-12]. Indeed, the colour electric flux leaks into the superconducting QCD vacuum a bit over $\tilde{\lambda}$ which may be well determined by the competition between the energy in the colour electric field and the mass gained by the dual gauge field. The penetration depth $\tilde{\lambda}$ is basically the thickness of the surface layer over which the colour electric flux falls to zero. The mass acquisition of dual gauge field as well as the screening potential (i.e. the temporal degrees of freedom of the dual gauge field) conclusively leads to the colour flux screening and hence, the confinement of quarks. This can also be understood in terms of the supercurrent whose magnitude is given as $\tilde{J} = \tilde{g}\,(n/\rho - \tilde{g}\,\tilde{C})\,\eta^2$, which enforces the colour electric field lines to form a flux tube in a small region and has a close relationship with the colour electric flux quantisation[16]. This may be seen through the kinetic energy term $|D_\mu\phi|^2$ where $D_\mu \equiv \partial_\mu + i\,\tilde{g}\,\tilde{C}_\mu$ in the Lagrangian given in Eq. (1) which, in turn, leads to the corresponding energy per unit length with ansatz given in Eq.(7) as follows:

$$K.E. = \int_0^\infty \rho\,d\rho \int_0^{2\pi} d\theta \left(\frac{1}{\rho}\frac{d\varphi}{d\theta} - \tilde{g}\,\tilde{C}\right)^2 \eta^2 \qquad \ldots(16)$$

In fact, the phase of the complex scalar field is associated with the local centre-of-mass momentum of the monopole pairs and the oscillations in this superconducting phase of QCD vacuum correspond physically to the plasma oscillations[16], which consequently defines the mode to which the longitudinal part of the dual vector potential is coupled. It is clear from the ansatz made for the complex scalar field that one of its degrees of freedom (the phase) is transferred to the dual gauge field to impart it the mass while the other degree of freedom (the modulus) is lost at particular coupling as it becomes a constant entity[2]. The minimisation of the kinetic energy given by Eq. (16) in superconducting phase of QCD vacuum i.e. $\eta \neq 0$ therefore, gives the quantisation of the colour electric flux in terms of the condition $2\pi n/\tilde{g} = \int \tilde{C}\rho\, d\theta = \oint \tilde{C}\cdot dl$. Now considering the line integral $\oint \tilde{C}\cdot dl$ around the circle $S^1$ at infinity, the Stokes theorem then leads to the total colour electric flux enclosed in the following form:

$$\Phi_{\tilde{E}} = \int (\nabla\times\tilde{C}).\, dS = \int \tilde{E}.\, dS = n\, Q_e \quad \ldots(17)$$

where $Q_e = g/2$ can be interpreted as the colour electric charge of a quark[12]. This quantisation condition is valid with the requirement that the complex scalar monopole field be continuous along any closed path in the dual superconducting QCD vacuum which encircles the colour electric flux. Such behaviour with a DME advocates a flux tube structure between a quark and anti-quark.

However, it still remains to see the n-vortex solutions and behaviour of multi-flux vortices[32] for such confined solutions in the present model of dual QCD vacuum.

## 3 Multi-flux Vortices and Energy Configuration

In view of such screening effects as discussed, the dynamics of confinement scenario given by the Lagrangian given in Eq.(1) is also derivable from the following Lagrangian in the absence of quarks in a equally capable manner[9-12]:

$$L = -\frac{1}{4}\tilde{G}_{\mu\nu}\tilde{G}^{\mu\nu} + \left| D_\mu\phi \right|^2 - V(\phi\phi^*) \quad \ldots(18)$$

Let us consider, the cylindrically symmetric monopole field in two dimensions in the broken phase of symmetry. In fact, the vortices are invariant under translations along any fixed axis and therefore, they can be viewed as finite energy solutions in two dimensions[33]. The free energy per unit length associated to them in two dimensions with the suppression of the temporal gauge degrees of freedom of the dual gauge field (i.e. $\tilde{C}_0 = 0$)[11] is then derived in the following form:

$$E = \int d^2x \left[ \frac{1}{4}\tilde{G}_{ij}^2 + \left| D_k\phi \right|^2 + \Omega(\phi\phi^* - \eta^2)^2 \right] \quad \ldots(19)$$

In view of the screening constraints on the Lagrangian given in Eq. (1) and upon symmetry breaking, the free energy from Eq. (1) or Eq. (19) then contains a term $\tilde{m}^2\tilde{C}_i^2$ which is precisely what we need to have the DME[13] as presented by Eq. (15) in the last section. The DME guarantees the confinement in dual QCD vacuum followed by a flux tube structure where the energy increases with the separation between quark and anti-quark[12]. However, the total energy contents can be re-written in terms of the following squared quantities by using Bogomol'nyi's trick[34],

$$E = \int \varepsilon\, d^2x = \int d^2x \begin{bmatrix} \frac{1}{2}\tilde{G}_{12}^2 + \left| D_1\phi \right|^2 + \left| D_2\phi \right|^2 \\ + \Omega(\phi\phi^* - \eta^2)^2 \end{bmatrix} \quad \ldots(20)$$

where $\tilde{G}_{12} = \partial_2\tilde{C}_1 - \partial_1\tilde{C}_2$. The integrand of the energy functional given by Eq. (20) can further be restructured in the form given below by using the Stokes theorem with the elimination of some terms along-with the necessary boundary condition for the monopole field at large distances for finite energy configurations:

$$\varepsilon = \begin{bmatrix} \left|(D_1 + iD_2)\,\phi\right|^2 + \frac{1}{2}\left|\tilde{G}_{12} + \tilde{g}\,(\phi\phi^* - \eta^2)^2\right|^2 \\ + (2\Omega - \tilde{g}^2)\,(\phi\phi^* - \eta^2)^2 + \tilde{g}\eta^2\Phi_{\tilde{E}} \end{bmatrix} \quad \ldots(21)$$

The boundary of the type-I and type-II superconducting dual QCD vacuum is of particular interest and if we set a condition $g^2 = 6\lambda$ then it automatically restricts the GL parameter $\kappa$ to its unit value as follows:

$$\kappa = \left[\frac{3\lambda}{2\pi\alpha_s}\right]^{\frac{1}{2}} = \left[\frac{6}{\lambda}\right]^{\frac{1}{2}} \quad \ldots(22)$$

In fact, the superconducting behaviour of QCD vacuum gradually changes with the running strong coupling constant along with the change in the characteristic mass/length scales from one type to other and the GL parameter is a physically important parameter to account the changes in chromomagnetic superconducting vacuum at different couplings. The GL parameter as given in the Eq. (22) is defined as the ratio of scalar mass mode to vector mass where the inverse of the scalar mass mode leads to the coherence length which sets the typical distance scale necessary for dual superconductivity to get established in dual QCD vacuum. The GL parameter may, therefore, be either $\kappa > 1$ or $\kappa < 1$ which correspond to the type-II and type-I superconducting regimes of QCD vacuum respectively[10,11]. However, the unit value of the GL parameter for the present model itself represents a transition from type-I to type-II QCD vacuum at a coupling $\alpha_s = 0.4778$ (λ = 1). With the condition given by Eq. (22), the energy contents now read as:

$$E = \int d^2x \left[ \begin{array}{l} \left|(D_1 + iD_2)\,\phi\right|^2 \\ +\dfrac{1}{2}\left|\tilde{G}_{12} + \tilde{g}\,(\phi\phi^* - \eta^2)^2\right|^2 + \tilde{g}\eta^2\Phi_{\tilde{E}} \end{array} \right] \quad \ldots(23)$$

With the vanishing of the squared entities in above expression (i.e. Bogomol'nyi case),

$$(D_1 + iD_2)\,\phi = 0 \quad \ldots(24)$$

$$\tilde{G}_{12} + \tilde{g}\,(\phi\phi^* - \eta^2)^2 = 0 \quad \ldots(25)$$

one can obtain the minimum energy of the system which corresponds to the last term in Eq. (23) as follows by using the quantisation condition given in Eq.(17):

$$E_n = 2\pi\eta^2 \quad \ldots(26)$$

Eq.(26) indicates the linear increase in the energy as the amount of flux increases and the vortices may, therefore, coalesce into macroscopic regions of colour electric field. Such condition only appears due to the typical balance of the propagation range of the dual gauge field and complex scalar monopole field in vacuum. However, it is believed that the type-I regime of a superconductor has analogies with bags while the type-II favours the flux tubes[35].

## 4 Results and Discussion

The total colour charge screening and colour electric flux exclusion in dual QCD vacuum necessarily demands the existence of the DME in the background of magnetic condensation. These screening effects as given in terms of the Eqs (9) and (15), in turn, guarantees the confined states of a quark system and the formation of vortices in QCD vacuum. In fact, the screening phenomenon of the total colour charge by the monopole scalar field paves the way for the colour electric flux screening responsible for the confined states of quarks. The energy for n-vortex configurations is obtained at the boundary of the type-I and type-II superconducting zones of the present dual QCD vacuum as given by Eq. (26). It indicates that at the transition point $\alpha_s \approx 0.5$, the quark system belongs to the multi-vortex solutions having a Bogomol'ny bound on the energy contents. Thus, the multi-vortices essentially exist at the border of two types of superconducting regimes of QCD vacuum, and they lost their individuality with the transition from one to other superconducting regime. However, it still remains to see the evolution of the interaction energy among vortices for different couplings/GL parameter (i.e. in the near and far Bogomol'nyi's regime) in both the superconducting zones of dual QCD to investigate the exact process of how the vortices attract/repel each other.


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