Indian Journal of Pure & Applied Physics
Vol. 59, October 2021, pp. 671-677

Electrostatic Ion-cyclotron Wave Excitation by a Gyrating Ion Beam in a Magnetized Plasma Containing Heavy Positive Ions

Meena Yadav & Jyotsna Sharma*

Department of Physics, Amity School of Applied Sciences, Amity University Haryana, Manesar, Gurugram-122051, India.

Received 23 August 2021; accepted 24 September 2021

In the present paper, we have developed a theoretical model of cylindrical magnetized plasma column consisting of electrons, K^+ ions & heavy positive Ba^+ ion species. A gyrating ion beam of potassium having radius ~1.5 cm and energy 10eV is launched through one end of the cylinder parallel to the static magnetic field and the electrostatic ion cyclotron wave is assumed to propagate nearly perpendicular to the magnetic field. The presence of heavy positive Ba^+ ions in the collisionless magnetized plasma decreases the critical drift for the excitation of EIC waves and hence the injected beam drives the electrostatic ion cyclotron (EIC) waves to instability through Cerenkov interaction. Therefore, the plasma is observed to contains two excited EIC wave modes; a (light positive) K^+ ion mode and a (heavy positive) Ba^+ion mode. The frequencies and growth rates for both the positive unstable modes are worked out using fluid theory for the typical existing experimental parameters which are found to increase with the increase in ion beam energy and beam density. It is also observed that the unstable wave frequency and growth rate for both the modes increases with the increase in relative ion concentration of $K^+$and heavy Ba^+ positive ions respectively. Magnetic field is also one of the important factors which influence the plasma instabilities appreciably. As the magnetic field increases, the frequency of both the ion modes increases linearly.

Keywords: Growth rate, Unstable frequency, Cerenkov interaction, Beam energy

1 Introduction

Recently, there has been growing much interest in studying electrostatic ion-cyclotron (EIC) waves[1-40] in a wide variety of situations, ranging from laboratory experiments[1,2] to space plasma[22,23]. D' Angelo and Motley[1] firstly reported the EIC oscillations in the laboratory plasma. In this paper, the electrostatic ion-cyclotron oscillations were driven by applying a positive potential to a small electrode immersed in a single-ended Q-machine with a uniform magnetic field. These observed oscillations were identified as the current-driven EIC instabilities predicted theoretically later on by Drummond and Rosenbluth[3]. D' Angelo and Merlino[4] have also observed EIC wave modes in a plasma consisting of positive ions, negative ions and electrons. Sheehan[5] has studied EIC waves in a plasma containing negative iodine ions. Song *et al*.[6] have analyzed EIC waves in a plasma with negative ions in a Q- machine. Here, the authors have observed that the frequency of two EIC wave modes increases with the relative density of negative ions, while the critical electron drift velocities for the excitation of both the modes decreases with increasing ε. Suszcynsky *et al.*[8] have analysed the EIC waves in a two-ion component plasma in a single-ended Q-Machine containing different concentrations of $Cs^+$and K^+ions. In this case, the authors have observed that the result is of interest in the context of EIC wave excitation in the ionosphere. Rynn[10] discussed his and Lang's work in K^+/Ba^+plasma. He studied the ion heating by the electrostatic ion cyclotron waves. Sugai *et al*.[12] experimentally observed that the minority ion concentration of positive ions only a few percent give rise to a new cut off and resonance frequency near the minority-ion cyclotron frequency. Sarid *et al.*[14] analyzed cyclotron modes in an unmagnetized Mg ion plasma. Angelo[20] has observed low-frequency electrostatic ion-acoustic and ion-cyclotron waves in a magnetized dusty plasma. The author has investigated the influence of negatively charged dust grains on plasma and found that the mode frequencies increase with increasing relative density. Suszcynsky *et al*.[21] have reported current-driven EIC waves in the presence of a transverse dc electric field in a magnetized plasma. Michelson *et al*.[24] have studied ion-cyclotron waves in a quiescent plasma into which a low energy beam of sodium ion was injected. In this case, the authors have

*Corresponding author (E-mail: plasmajyoti@gmail.com)

observed that the instability appeared when the beam velocity becomes more than 12 times the thermal velocity. Ion beam driven ion-cyclotron instability in neutral beam injection schemes for fusion plasma heating is analysed by Stix[25] and in space plasma physics is analysed by Montogomery *et al.*[26]. Hendel *et al.*[27] have observed ion beam driven electrostatic ion-cyclotron instability. Wolf *et al.*[29] observed that the ion-cyclotron waves are considerably influenced by the presence of high-amplitude high-frequency waves. Merlino *et al.*[31] have explained the theoretical and experimental results on the study of low-frequency electrostatic waves in plasma containing negatively charged dust grains. Moreover, some interesting theoretical observations[37-40] are also shown by many authors for laser beam-plasma interactions in magnetized two-ion and multi-ion species plasma. However, in the present study, we report the excitation of electrostatic ion-cyclotron wave instability by a gyrating ion beam in a magnetized collisionless uniform plasma containing electrons, K^+ions and heavy Ba^+ions species. The numerical analysis of electrostatic ion-cyclotron wave instability is worked out using fluid theory. The response of beam and two positive unstable EIC modes are obtained and explained. We derived the expression for the frequencies and growth rates for the two EIC mode instabilities in the presence of light K^+positive and heavy Ba^+positive ions using first-order perturbation method. The discussion on results is provided in the last Section.

2 Dispersion relation of EIC wave instability

Consider a cylindrical magnetized uniform collisionless plasma column of radius$b_1$which contains electrons, light positive K^+ ions and heavy positiveBa^+ionspecies having densities as $n^{0e} = n^{0p} n^{0L+} = \alpha^L n^{0p}$and $n^{0H+} = \alpha^H n^{0p}$ respectively, immersed in a static magnetic field *i.e.*,$B_s \parallel \hat{Z}$. Here, let us consider,$\alpha^L$and

$\alpha^H (= 1 - \alpha^L)$ be the fractional concentration of K^+ions and Ba^+ions respectively and n^{0p} is the electron plasma density. The charge, mass and temperature of the three species (electrons, K^+ions and heavy Ba^+ions) are given as $(-e, m^e, T^e)$, (e, m^L, T^L) and (e, m^H, T^H) respectively, where $T^e \sim T^L \sim T^H$.A gyrating ion beam with velocity $\vec{v} = (v_\perp \hat{\theta} + v_{b0} \hat{Z})$, mass m^b, density n^{0b} and radius $r_{b0} = 1.5\, cm$ propagates through one end of the plasma cylinder of radius b_1 along the magnetic field $B_s \parallel \hat{Z}$.The beam plasma system prior to the perturbation is quasi-neutral, such that$-n^{0e} + n^{0L+} + n^{0H+} + n^{0b}$is approximately zero. There is no electric field present in the equilibrium and plasma is partially uniform.

The equilibrium is perturbed by an electrostatic perturbation to the potential

$$\phi_1 = \phi(r) e^{-i(\omega t - l\theta - k_z z)} \quad \dots (1)$$

The equations of motion and equations of continuity as given below

$$m^e \left[\frac{\partial \vec{v}_e}{\partial t} + (\vec{v}_e.\nabla)\vec{v}_e\right] = -e\vec{E} - \frac{e}{c}(\vec{v}_e \times B_s) - T^e \frac{\nabla n^e}{n^{0e}} \quad \dots (2)$$

$$m^L \left[\frac{\partial \vec{v}_L}{\partial t} + (\vec{v}_L.\nabla)\vec{v}_L\right] = e\vec{E} + \frac{e}{c}(\vec{v}_L \times B_s) - T^L \frac{\nabla n^L}{n^{0L+}} \quad \dots (3)$$

$$m^H \left[\frac{\partial \vec{v}_H}{\partial t} + (\vec{v}_H.\nabla)\vec{v}_H\right] = e\vec{E} + \frac{e}{c}(\vec{v}_H \times B_s) - T^H \frac{\nabla n^H}{n^{0H+}} \dots (4)$$

$$\left[\frac{\partial n^e}{\partial t} + \nabla.(n^e \vec{v}_e)\right] = 0 \quad \dots (5)$$

$$\left[\frac{\partial n^L}{\partial t} + \nabla.(n^L \vec{v}_L)\right] = 0 \quad \dots (6)$$

$$\left[\frac{\partial n^H}{\partial t} + \nabla.(n^H \vec{v}_H)\right] = 0 \quad \dots (7)$$

On linearization Eq. (2), we obtain the perturbed density of electrons given as

$$n^{1e} = \frac{n^{0e}\, e\phi_1}{T^e} = \frac{n^{0p}\, e\phi_1}{T^e} \quad \dots (8)$$

The linearization of Eq. (3) gives the perturbed velocities

$$v^{zL1+} = \frac{k_z e\phi_1}{m^L \omega} + \frac{T^L k_z n^{1L+}}{m^L \omega\, n^{0L+}} \quad \dots (9)$$

and

$$v^{\perp L1+} = \frac{e}{m^L}\left[\frac{i\omega\, \vec{E}_{1\perp} - \vec{E}_{1\perp} \times \vec{\omega}_{L+}}{(\omega^2 - \omega_{L+}^2)}\right] - \frac{T^L}{m^L \alpha_L n^{0L+}} \frac{(i\omega \nabla_\perp n^{1L+} + \nabla_\perp n^{1L+} \times \vec{\omega}_{L+})}{(\omega^2 - \omega_{L+}^2)} \quad \dots (10)$$

where superscript 1 represents the perturbed quantities and $\omega_{L+} (= eB_s/m^L c)$represents cyclotron frequencies for K^+ ions. $v^{zL1+}$and $v^{\perp L1+}$are the axial and perpendicular components of the perturbed velocities.

The response of K^+ ions is given by

$$\left(1 - \frac{c_L^2 k_z^2}{\omega^2}\right) + \frac{c_L^2 \nabla_\perp^2 n^{1L+}}{(\omega^2 - \omega_{L+}^2)} = -\frac{e n^{0L+}}{m^l}\left[\frac{\nabla_\perp^2 \phi_1}{(\omega^2 - \omega_{L+}^2)} - \frac{k_z^2}{\omega^2}\right] \quad \dots (11)$$

where $c_L(=T^e/m^L)^{1/2}$ is the thermal velocity of K^+ ions.

Using Eqs. (9) and (10) in the continuity equation [cf. Eq. (6)], we obtain the perturbed density for K^+ ions as

$$n^{1L+} = -\frac{n^{0p}\alpha^L e c_L^2}{T^L}\left[\frac{\nabla_\perp^2\phi_1}{\omega^2-\omega_{L+}^2} - \frac{k_z^2\phi_1}{\omega^2}\right] \quad \ldots (12)$$

Similarly, the perturbed density for heavy positive Ba^+ ions is given as

$$n^{1H+} = -\frac{n^{0p}\alpha^H e c_H^2}{T^H}\left[\frac{\nabla_\perp^2\phi_1}{\omega^2-\omega_{H+}^2} - \frac{k_z^2\phi_1}{\omega^2}\right] \quad \ldots (13)$$

where $c_H(=T^e/m^H)^{1/2}$ is the thermal velocity for heavy positive ions and $\omega_{H+}(=eB_s/m^H c)$ is the cyclotron frequency for heavy positive ions.

Following Sharma and Tripathi[9] the perturbed density of spiraling ion beam can be written as

$$n^{1b} = -\frac{N_0 e\,\delta(r-r_{b0})\left(\frac{l^2}{r^2}+k_z^2\right)\phi_1}{(\varpi-l\omega_{cb})^2\, m^b\, 2\pi\, r_{b0}} \quad \ldots (14)$$

Using Eqs. (8), (12), (13) and (14) in the Poisson's equation $\nabla^2\phi_1 = 4\pi e\,[n^{1e} - n^{1L+} - n^{1H+} - n^{1b}]$ … (15)

and using the values $\omega_{pe}^2 = \frac{4\pi n^{0p} e^2}{m^e}$, $\omega_{pL}^2 = \frac{4\pi\,\alpha^L\, n^{0p} e^2}{m^L}$, $\omega_{pH}^2 = \frac{4\pi\,\alpha^H\, n^{0p} e^2}{m^H}$ and $\varpi = \omega - k_z v_{b0}$,

we obtain a second-order differential equation in ϕ_1, which can be rewritten for the axially symmetric case as

$$\frac{\partial^2\phi_1}{\partial r^2} + \frac{1}{r}\frac{\partial\phi_1}{\partial r} + Q^2\phi_1 = -\frac{2N_0 e^2\,\delta(r-r_{b0})\left(\frac{l^2}{r^2}+k_z^2\right)\phi_1}{N\,(\varpi-l\omega_{cb})^2\, m^b\, r_{b0}} \quad \ldots (16)$$

where, $N = \left(1 - \frac{\alpha^L\omega_{pL}^2}{\omega^2-\omega_{L+}^2} - \frac{\alpha^H\omega_{pH}^2}{\omega^2-\omega_{H+}^2}\right)$ and $Q^2 = \left(\frac{-\frac{\omega_{pe}^2 m^e}{T^e} + \frac{\omega_{pL}^2\alpha^L k_z^2}{\omega^2} - k_z^2 + \frac{\omega_{pH}^2\alpha^H k_z^2}{\omega^2}}{\left(1 - \frac{\alpha^L\omega_{pL}^2}{\omega^2-\omega_{L+}^2} - \frac{\alpha^H\omega_{pH}^2}{\omega^2-\omega_{H+}^2}\right)}\right)$

Therefore, Eq. (16) can be rewritten as

$$\nabla_\perp^2\phi_1 + Q^2\phi_1 = -\frac{2N_0 e^2\,\delta(r-r_{b0})\left(\frac{l^2}{r^2}+k_z^2\right)}{N\,(\varpi-l\omega_{cb})^2\, m^b\, r_{b0}} \quad \ldots (17)$$

In the absence of a beam, Eq. (17) becomes

$$\frac{\partial^2\phi_1}{\partial r^2} + \frac{1}{r}\frac{\partial\phi_1}{\partial r} + \left(p_m^2 - \frac{l^2}{r^2}\right)\phi_1 = 0 \quad \ldots (18)$$

Here, Eq. (18) is a Bessel equation giving a solution $\phi_1 = AJ_l(p_m r)$. At $r = b_1$, $\phi_1 = 0 \Rightarrow p_m = \frac{X_n}{b_1}$ (n=1,2,3,4,5………)

i.e., X_n are the zeros of the Bessel function $J_0(x)$. In the presence of beam, the wave function ϕ_1 can be expressed in a series of orthogonal sets of the wave function

$$\phi_1 = \sum_m A_m J_0(p_m r) \quad \ldots (19)$$

Now, using Dirac Delta property and solving equations (17) and (19), we obtain

$$Q^2 - p_m^2 = -\frac{\omega_{pb}^2\left(\frac{l^2}{r_{b0}^2}+k_z^2\right)J_l^2(p_m r_{b0})}{N\,(\varpi-l\omega_{cb})^2 J_{l+1}^2(p_m b_1)} \quad \ldots (20)$$

After putting all the values, the above Eq. (20) can be rewritten as

$$-(k_z^2+p_m^2) - \frac{\omega_{pe}^2 m^e}{T^e} + \frac{\omega_{pL}^2\alpha^L k_z^2}{\omega^2} + \frac{\omega_{pH}^2\alpha^H k_z^2}{\omega^2} + \frac{\alpha^L\omega_{pL}^2\, p_m^2}{\omega_{L+}^2} + \frac{\alpha^H\omega_{pH}^2 p_m^2}{\omega_{H+}^2} = -\frac{\omega_{pb}^2\left(\frac{l^2}{r_{b0}^2}+k_z^2\right)J_l^2(p_m r_{b0})}{(\varpi-l\omega_{cb})^2 J_{l+1}^2(p_m b_1)} \quad \ldots (21)$$

After simplifying Eq. (21) and multiplying both sides by $\frac{(k_z^2+p_m^2)}{p_m^2}$, also assuming $\alpha^1 = \frac{(k_z^2+p_m^2)}{p_m^2} + \frac{\omega_{pe}^2}{v_{te}^2 p_m^2}$,

Eq. (21) can be written as

$$1 - \frac{\omega_{pL}^2\alpha^L k_z^2}{\alpha^1\omega^2 p_m^2} - \frac{\alpha^L\omega_{pL}^2}{\alpha^1(\omega^2-\omega_{L+}^2)} - \frac{\omega_{pH}^2\alpha^H k_z^2}{\alpha^1\omega^2 p_m^2} - \frac{\alpha^H\omega_{pH}^2}{\alpha^1(\omega^2-\omega_{H+}^2)} = -\frac{\omega_{pb}^2\left(\frac{l^2}{r_{b0}^2}+k_z^2\right)J_l^2(p_m r_{b0})}{\alpha^1\, p_m^2\,(\varpi-l\omega_{cb})^2 J_{l+1}^2(p_m b_1)} \quad \ldots (22)$$

In the presence of only K^+ ions, Eq. (22) can be rewritten as

$$1 - \frac{\omega_{pL}^2\alpha^L k_z^2}{\alpha^1\omega^2 p_m^2} - \frac{\alpha^L\omega_{pL}^2}{\alpha^1(\omega^2-\omega_{L+}^2)} = \frac{\omega_{pb}^2\left(\frac{l^2}{r_{b0}^2}+k_z^2\right)J_l^2(p_m r_{b0})}{\alpha^1\, p_m^2\,(\varpi-l\omega_{cb})^2 J_{l+1}^2(p_m b_1)} \quad \ldots (23)$$

Multiplying both sides by $\omega^2(\omega^2-\omega_{L+}^2)$ and after rearranging the terms, we obtain

$$\omega^4 - \omega^2\left(\omega_{L+}^2 + \frac{\omega_{pL}^2\alpha^L}{\alpha^1} + \frac{\alpha^L\omega_{pL}^2\, k_z^2}{\alpha^1\, p_m^2}\right) + \frac{\alpha^L\omega_{pL}^2\, k_z^2\,\omega_{L+}^2}{\alpha^1\, p_m^2} = \frac{\omega_{pb}^2\left(\frac{l^2}{r_{b0}^2}+k_z^2\right)\omega^2(\omega^2-\omega_{L+}^2)\; J_l^2(p_m r_{b0})}{\alpha^1\, p_m^2\,(\varpi-l\omega_{cb})^2 J_{l+1}^2(p_m b_1)} \quad \ldots (24)$$

where $\alpha^1 = \frac{(k_z^2+p_m^2)}{p_m^2} + \frac{\omega_{pe}^2}{v_{te}^2 p_m^2} = \frac{k^2}{p_m^2} + \frac{\omega_{pe}^2}{v_{te}^2 p_m^2} \approx \frac{\omega_{pe}^2}{v_{te}^2 p_m^2}$

Equation (24) can be rewritten as

$$(\omega^2 - c_1^2)(\omega^2 - c_2^2)(\varpi - l_{\omega_{cb}})^2 = \frac{\omega_{pb}^2\left(\frac{l^2}{r_{b0}^2}+k_z^2\right)\omega^2(\omega^2-\omega_{L+}^2)\ J_l^2(p_m r_{b0})}{\alpha^1\ p_m^2 J_{l+1}^2(p_m b_1)} \quad \ldots (25)$$

where, $c_1^2 = \ \omega_{L+}^2 + k^2\alpha^L R c_H^2$ ... (26)

and $c_2^2 = \frac{\omega_{L+}^2\alpha^L k_z^2\ R\ c_H^2}{\omega_{L+}^2+k^2\alpha^L R c_H^2}$ ... (27)

$R = \frac{M^H}{m^L}$ and $c_H^2 = \frac{T^H}{m^H}$

Here, $\omega \approx c_1$ corresponds to K^+ ion mode and $\omega \approx k_z v_{0b} + l_{\omega_{cb}}$ corresponding to the beam mode. But, we are looking for solutions, when $\omega \approx k_z v_{0b} + l_{\omega_{cb}}$. In this case the two factors on the left-hand side of Eq. (25) are simultaneously zero in the absence of beam. In the presence of beam, we can expand ω as $\omega \approx c_1 + \delta_1 \simeq k_z v_{0b} + l_{\omega_{cb}} + \delta_1$, where δ_1 is the small frequency mismatch due to finite right-hand side of Eq. (25). Then Eq. (25) gives the growth rate of unstable K^+ ion mode as

$$\Gamma = Im\ \delta_1 = \left[\frac{c_1}{2}\frac{(c_1^2-\omega_{L+}^2)\ \omega_{pb}^2\left(\frac{l^2}{r_{b0}^2}+k_z^2\right) J_l^2(p_m r_{b0}) c_L^2}{\omega_{pL}^2\ (\omega^2-c_2^2)\ J_{l+1}^2(p_m b_1)}\right]^{1/3} \frac{\sqrt{3}}{2} \quad \ldots (28)$$

The real frequency of unstable mode in term of beam energy is given by

$$\omega_r = k_z\left(\frac{2eV_b}{m^b}\right)^{1/2} - \frac{1}{2}\left[\frac{c_1}{2}\frac{(c_1^2-\omega_{L+}^2)\ \omega_{pb}^2\left(\frac{l^2}{r_{b0}^2}+k_z^2\right) J_l^2(p_m r_{b0}) c_L^2}{\omega_{pL}^2\ (\omega^2-c_2^2) J_{l+1}^2(p_m b_1)}\right] \ldots (29)$$

where eV_b is the beam energy. However, in the absence of K^+ ions and presence of heavy Ba^+ ions Eq. (22) can be rewrittenas

$$1 - \frac{\omega_{pH}^2\alpha^H k_z^2}{\alpha^1\ \omega^2 p_m^2} - \frac{\alpha^H\omega_{pH}^2}{\alpha^1\ (\omega^2-\omega_{H+}^2)} = \frac{\omega_{pb}^2\left(\frac{l^2}{r_{b0}^2}+k_z^2\right) J_l^2(p_m r_{b0})}{\alpha^1\ p_m^2\ (\varpi-l_{\omega_{cb}})^2 J_{l+1}^2(p_m b_1)} \quad \ldots (30)$$

Multiplying both sides by $\omega^2(\omega^2 - \omega_{H+}^2)$ and after rearranging the terms, we get

$$\omega^4 - \omega^2\left(\omega_{H+}^2 + \frac{\omega_{pH}^2\,\alpha^H}{\alpha^1} + \frac{\alpha^H\omega_{pH}^2\,k_z^2}{\alpha^1\ p_m^2}\right) + \frac{\alpha^H\omega_{pH}^2\,k_z^2\,\omega_{H+}^2}{\alpha^1\ p_m^2} = \frac{\omega_{pb}^2\left(\frac{l^2}{r_{b0}^2}+k_z^2\right)\omega^2(\omega^2-\omega_{H+}^2)\ J_l^2(p_m r_{b0})}{\alpha^1\ p_m^2\ (\varpi-l_{\omega_{cb}})^2 J_{l+1}^2(p_m b_1)} \quad \ldots (31)$$

Equation (31) can be rewritten as

$$(\omega^2 - d_1^2)(\omega^2 - d_2^2)(\varpi - l_{\omega_{cb}})^2 = \frac{\omega_{pb}^2\left(\frac{l^2}{r_{b0}^2}+k_z^2\right)\omega^2(\omega^2-\omega_{H+}^2)\ J_l^2(p_m r_{b0})}{\alpha^1\ p_m^2 J_{l+1}^2(p_m b_1)} \quad \ldots (32)$$

where, $d_1^2 = \ \omega_{H+}^2 + \frac{k^2\alpha^H c_L^2}{R}$ ... (33)

$$d_2^2 = \frac{\alpha^H\ k_z^2\omega_{H+}^2\ c_L^2}{R\ \omega_{H+}^2 + k^2\alpha^H c_L^2} \quad \ldots (34)$$

Here, $\omega \approx d_1$ corresponds to the Ba^+ ion mode and $\omega \approx k_z v_{0b} + l_{\omega_{cb}}$ corresponding to the beam mode. Eq. (32) gives the growth rate of the unstable mode as

$$\Gamma = Im\ \delta_1 = \left[\frac{d_1}{2}\frac{(d_1^2-\omega_{H+}^2)\ \omega_{pb}^2\left(\frac{l^2}{r_{b0}^2}+k_z^2\right) J_l^2(p_m r_{b0}) c_H^2}{\omega_{pH}^2\ (\omega^2-d_2^2) J_{l+1}^2(p_m b_1)}\right]^{1/3} \frac{\sqrt{3}}{2} \ldots (35)$$

The real frequency of unstable mode in terms of beam energy is given by

$$\omega_r = k_z\left(\frac{2eV_b}{m^b}\right)^{1/2} - \frac{1}{2}\left[\frac{d_1}{2}\frac{(d_1^2-\omega_{H+}^2)\ \omega_{pb}^2\left(\frac{l^2}{r_{b0}^2}+k_z^2\right) J_l^2(p_m r_{b0}) c_H^2}{\omega_{pH}^2\ (\omega^2-d_2^2) J_{l+1}^2(p_m b_1)}\right]$$

3 Results

In the numerical calculations, we have used the following typical plasma parameters of the existing experimental paper of Rynn[10] for EIC waves: electron plasma density $n^{0e} = n^{0p} = 10^{10}$ cm^{-3}, temperature of electron, K^+ and heavy positive Ba^+ ions as, $T^e \sim T^L \sim T^H$ =0.2eV, R $(m^H/m^L) = 3.5$, radius of plasma column b_l=2 cm, beam radius of potassium beam r_{b0} = 1.5 cm, beam energy E_b = 10eV, guiding magnetic field Bs ~ 3x 10^3 gauss and mode number n = 1, i.e., the first zero of the Bessel function. EIC waves are the longitudinal oscillations of electrons or ions which propagates nearly perpendicular to the magnetic field. We have studied the EIC waves driven to instability by gyrating ion beam. However, the frequency of any ion beam driven EIC instability is lower than any electron beams driven instability. But temperature of plasma influences its growth rates, as hotter is the plasma, more will be the growth rate. Here, due to an increase in the relative concentration of positive ions, the ion-cyclotron wave exhibits two ion modes, a light K^+ mode and a heavy Ba^+ mode. Using Eqs. (28) and (35), we have plotted Figs. 1 & 2 where it can be observed that the growth rate Γ (in sec^{-1}) of both the unstable electrostatic ion-cyclotron modes increases with their respective relative ion concentrations. In Fig.3, using Eqs. (29) & (36), we have plotted real unstable wave frequency ω_r versus beam energy (eV_b) for K^+ and heavy Ba^+ ions and observed that the increase in beam energy increases the unstable wave frequency and our theoretical plot of unstable wave frequency ω_r with beam energy in the presence of positive ions is qualitatively similar to Fig. 2 (b) of the experimental paper of Hendel *et al.*[15]

In Fig. 4, the normalized parallel phase velocity ω_r/k_zV_i [cf. Eq. (36)] as a function of beam velocity V_b/V_i is plotted and we found that the parallel phase velocity varies linearly with the beam velocity. In Fig. 5, the frequency of K^+ ion mode and Ba^+ ion mode as a function of magnetic field B_s for different value of relative ion concentrations of these ions is also plotted. Our theoretical plots are in line with the experimental observation of Suszcynsky *et al.*[8] [cf. Fig. 4]. In Fig. 6, the dispersion relation ω_r/ω_{H+} for heavy positive Ba^+ ion as a function of concentration $d = n^{Ba+}k_z/\omega_{H+}$ is plotted. Here, we have observed that the frequency increases with increasing concentration d, and for any fixed value of d, the frequency increases with the increase in concentration of Ba^+ ions. The theoretical observations of our study show that the growth rate of the unstable modes in the presence of K^+ ions and heavy positive Ba^+ ions increase with the beam density and scales as the one-third power of the beam density [cf. Equations (28) and (35)]. The real part of frequency varies as one-half power of beam energy.

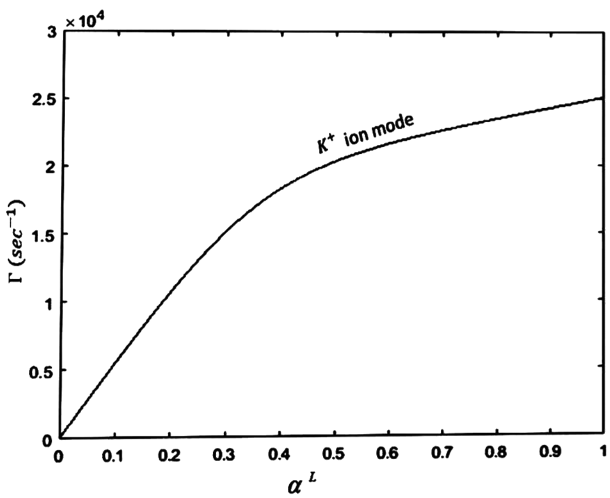

Fig. 1 — Growth rate Γ (in sec^{-1}) of the unstable K^+ion mode as a function of relative concentration α^L.

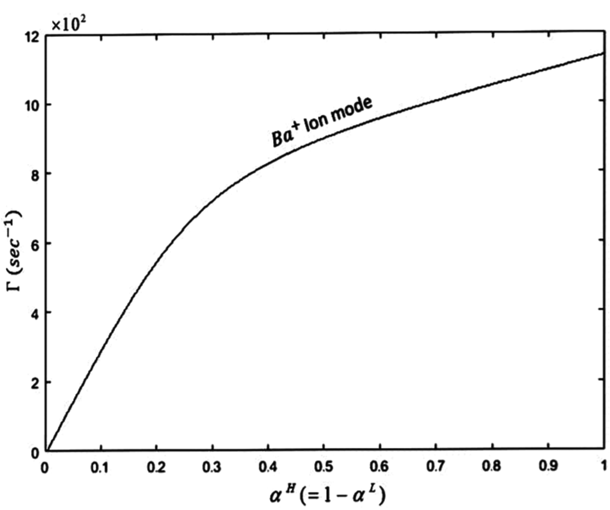

Fig. 2 — Growth rate Γ (in sec^{-1}) of the unstable ion mode as a function of relative concentration α^H.

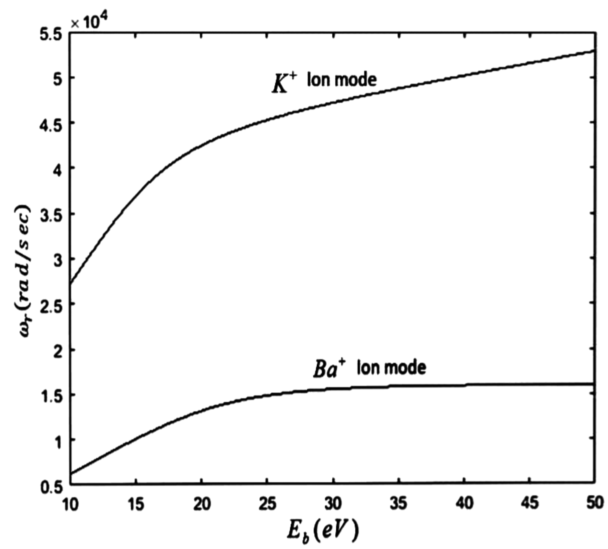

Fig. 3 — Unstable real wave frequency of K^+ and Ba^+ ion mode as a function of beam energy E_b (eV).

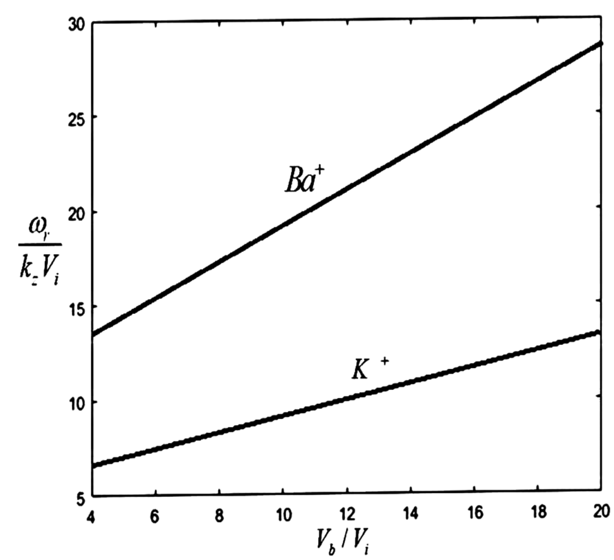

Fig. 4 — Normalized parallel phase velocity of $K^+$and Ba^+ions as a function of beam velocity V_b/V_i.

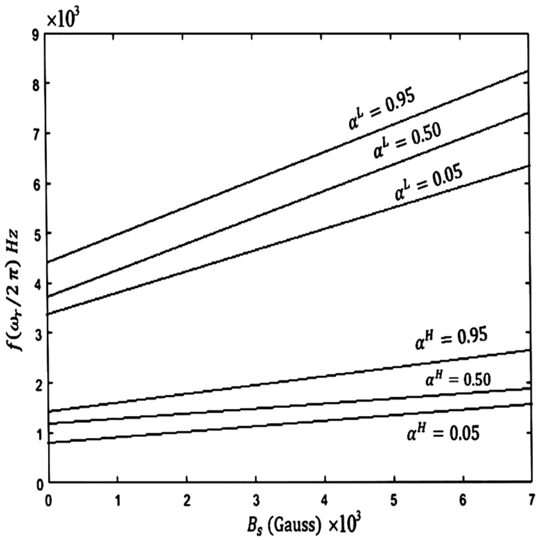

Fig. 5 — Unstable frequency f (Hz) plots of K^+ and Ba^+ion modes as a function of magnetic field for different values of their relative concentrations.

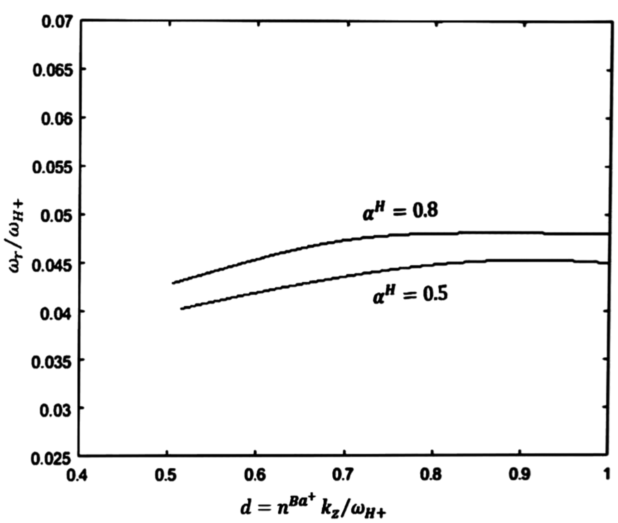

Fig. 6 — Dispersion relation ω_r/ω_{H+} of a EIC heavy Ba^+ion mode as a function of $d = n^{Ba+}k_z/\omega_{H+}$ for n^{Ba+}=2x10^7 cm^{-3} & $\alpha^H = 0.8$, $\alpha^H = 0.5$ keeping all the other parameters same as taken in Fig 1 or 2.

4 Discussion

Hence, a gyrating ion beam propagating through the collisionless magnetized plasma gives energy to the electrostatic ion cyclotron wave through Cerenkov interaction and excite it into two unstable ion modes; light positive $K^+$and heavy positive Ba^+. The growth rate of both the unstable modes in the presence of heavy positive ions increases with the beam density and scales as the one-third power of the beam density. Also, the parallel phase velocity varies linearly with the beam velocity. The unstable frequencies of both the modes are drastically influenced by the magnetic field also, the frequencies increase with the increase in magnetic field. However, this increase in the frequency is more rapid for the light positive ions as compared to the heavy positive ions.

The theoretical understanding of EIC waves in plasma/dusty plasma is very important in order to understand the theoretical aspects of the phenomenon occurring in the universe. On the other side, positive ion plasmas are found in the earth's ionosphere, magnetosphere and solar winds. Even the interstellar nebulae contain a mixture of several positive ion species, *e.g.*, H^+, O^+, He^+, O_2^+ & NO^+etc., while the concentration of these ions increases with height. Our theoretical model is predicting the excitation of electrostatic ion-cyclotron wave modes by the resonant ion beams and the dependence of growth rate of EIC wave instabilities on the relative concentrations of these positive ions. Our work may find various applications in understanding the space plasma, astronomy, astrophysical plasma as well as terrestrial plasma.

Acknowledgement

This work has been partially supported by SERB-DST, Government of India (EMR/2016/002699).

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