Indian Journal of Pure & Applied Physics
Vol. 53, July 2015, pp. 488-491

Instability of relativistic electron beam-dielectric system for backward microwave amplification[#]

Wang Hong-Yu[1,3], Kong Ling-Bao[2], Sun Peng[1]* & Hou Zhi-Ling[4]
[1]School of Physics, Anshan Normal University, Anshan 114005, People's Republic of China
[2]School of Electronics Engineering and Computer Science, Peking University, Beijing 100871, P.R. China
[3]Shanghai Bright-Tech Information Technology Co. Ltd., Shanghai 200136, People's Republic of China
[4]School of Science, Beijing University of Chemical Technology, Beijing 100029, People's Republic of China
*E-mail: sunpeng169@126.com

Received 7 February 2014; revised 28 August 2014; accepted 30 January 2015

The linear wave dispersion relation of relativistic electron beam propagating along the negative direction of a constant guiding magnetic field in a slow wave structure has been presented by cold fluid limit. The instability occurs when the velocity of beam electrons exceeds the phase velocity of the wave. The numerical simulations of microwave amplification due to the instability indicate that the maximum interaction efficiency achieves about 16%. The maximum transverse drifting distance of the beam electrons calculated from single particle theory is about 0.02 cm and this result is also verified by Particle-In-Cell simulations.

Keywords: Relativistic electron beam, Slow wave system, Dispersion relation, Microwave amplification

1 Introduction

The linear dispersion relation reveals the electromagnetic wave propagation, damping and amplification in the beam-wave interaction process and the dispersion relation has fundamental importance in beam or plasma physics for the purpose of charged particle beam acceleration or high power microwave or millimeter wave generation or amplification.

The nonlinear dispersion relation of relativistic plasma has attracted much interest in the theoretical or laboratory studies[1-3]. The dispersion equation for arbitrary wave amplitude, as well as moderate plasma temperature for ultra-intense electron plasma wave propagation has been presented in infinite and uniform plasma[1]. The nonlinear theory of multidimensional plasma waves with phase velocities near the speed of light has been studied[2] for the potential applications of particle acceleration. The linearized theory of relativistic plasma in a homogeneous cosmological background has been presented in Ref. (3).

For the purpose of high-power microwave generation in gyrotrons, the dispersion relation of the relativistic electron beam and the fast transverse electromagnetic wave interaction have been intensively studied[4-6]. The instability occurs when gyrated electron beam interacting with a fast transverse electromagnetic wave in a guiding magnetic field. The electron beam bunches in the transverse direction and transfers energy to the wave so that coherent microwave amplification is produced. Gyrotron is one of the most successful coherent radiation sources of high power and bandwidth which has many applications in plasma heating, advanced materials processing, radar systems, particle accelerations, and electromagnetic wave weapons[7-10].

However, the gyrating beam and strong guiding magnetic field in gyrotrons result in much more sophistication in practical manipulation. So the study of slow wave electron cyclotron maser may be an appropriate supplement to the microwave generation or amplification devices. And the nonlinear interaction processes of slow wave electron cyclotron have been extensively studied by many researchers[11-18]. In the present paper, the dispersion relation of an initially rectilinear relativistic electron beam and slow wave system has been studied. The instability occurs when

[#]Project supported by the National Natural Science Foundation of China (Grant No. 11275007, 51102007 and 11175023). The first author is also supported by Program for Liaoning Excellent Talents in University (LJQ2012098).

the beam electron velocity exceeds the phase velocity of the slowed electromagnetic wave in the dielectric media. The instability reveals the possibility of microwave amplification using slow wave electron cyclotron mechanism with a rectilinear beam.

2 Dispersion Relation of the Relativistic Electron Beam-Dielectric System

An electron beam with initial velocity v_0 propagating along the negative z-axis in a medium with a dielectric constant ε, and the medium is immersed in a constant magnetic field $\vec{B}_0$ along the z-axis has been considered. A bunch of microwave in parallel with the electron beam is injected into the system. On the basis of Lorentz force, the equation for the beam electron motion in the medium can be written as:

$$\frac{d\vec{p}}{dt} = -e\left[\vec{E} + \frac{1}{c}\vec{v}\times\left(\vec{B}+\vec{B}_0\right)\right], \qquad (1)$$

where $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields of the wave, $\vec{p} = m_0\gamma\vec{v}$ the electron momentum, m_0 the rest mass of the electron, $\gamma = 1\big/\sqrt{1-v^2/c^2}$ the relativistic factor and c is the velocity of light in vacuum. The linearized equation of Eq. (1) can be presented as:

$$\left(\frac{\partial}{\partial t} + \vec{v}_0\cdot\nabla\right)\left[m_0\gamma_0\vec{v}' + \frac{m_0\gamma_0^3\left(\vec{v}_0\cdot\vec{v}'\right)}{c^2}\vec{v}_0\right] = -e\left[\vec{E}' + \frac{\left(\vec{v}'\times\vec{B}_0 + \vec{v}_0\times\vec{B}'\right)}{c}\right] \qquad \ldots(2)$$

where $\gamma_0 = \left(1 - v_0^2/c^2\right)^{-1/2}$ is the initial relativistic factor of the beam electron, and the prime symbols in Eq. (2) denote the perturbed quantity. Furthermore, it is assumed that all of the perturbed quantity varies with a factor $\exp\left(ikz + i\omega t\right)$, and electromagnetic wave propagates along the negative z direction. Here, k is the wave vector and ω is the angular frequency.

According to Faraday's law, we have the relation of the perturbed electric and magnetic fields as:

$$\vec{B}' = -\frac{kc}{\omega}\vec{e}_z \times \vec{E}' \qquad \ldots(3)$$

The wave equation for perturbed fields on the basis of the continuation equation is given by:

$$\nabla\cdot\vec{J}' + \frac{\partial}{\partial t}\rho' = 0 \qquad \ldots(4)$$

$$\frac{\varepsilon}{c^2}\frac{\partial}{\partial t^2}\vec{E}' + \nabla\times\nabla\times\vec{E}' = -\frac{4\pi}{c^2}\frac{\partial}{\partial t}\vec{J}' \qquad \ldots(5)$$

Combining Eq. (2) with Eqs (3)-(5), we find the self-consistent equation for the electric fields $\mathbf{D}\cdot\vec{E}' = 0$, where $\mathbf{D}$ is the dispersion tensor and has the following components:

$$D_{11} = D_{22} = k^2c^2 - \varepsilon\omega^2 + \frac{\omega_p^2}{\gamma_0}\cdot\frac{\left(\omega + kv_0\right)^2}{\left(\omega+kv_0\right)^2 - \Omega_0^2} \qquad \ldots(6)$$

$$D_{12} = D_{21}^* = \frac{i\omega_p^2\Omega_0\left(\omega+kv_0\right)}{\gamma_0\left(\omega+kv_0\right)^2} \qquad \ldots(7)$$

$$D_{33} = \frac{c^2\omega_p^2\omega^2}{\gamma_0^3\left(\omega+kv_0\right)^2} - \varepsilon\omega^2 \qquad \ldots(8)$$

$$D_{13} = D_{23} = D_{31} = D_{32} = 0 \qquad \ldots(9)$$

where $\omega_p = \left(4\pi n_0 e^2/m_0\right)^{1/2}$ is the beam plasma frequency, n_0 the initial beam density, $\Omega_0 = eB_0/m_0\gamma_0 c$ is the initial relativistic electron cyclotron frequency and the asterisk in Eq. (7) denotes complex conjugate. Then the dispersion relation is given by $\det\left|\mathbf{D}\right| = 0$ and yields:

$$\varepsilon\omega^2 - k^2c^2 - \frac{\omega_p^2\left(\omega+kv_0\right)}{\gamma_0\left(\omega+kv_0 \pm \Omega_0\right)} = 0. \qquad \ldots(10)$$

In Eq. (10), the positive and negative sign corresponds to right and left circularly polarized waves, respectively. It is obvious that the right circularly polarized wave exhibits the instability while the other one is stable, and the instability occurs at $\omega + kv_0 + \Omega_0 \approx 0$, *i.e.*, $\omega_0 = -kv_0 - \Omega_0 = kc/\sqrt{\varepsilon}$. Assuming $\omega = \omega_0 + \delta\omega$ in Eq. (10), furthermore $\delta\omega << \omega_0$ and $\delta\omega << \Omega_0$, then for the right polarized wave, we have:

$$\varepsilon\left(\omega_0+\delta\omega\right)^2-k^2c^2-\frac{\omega_p^2\left(\Omega_0-\delta\omega\right)}{\gamma_0\delta\omega}=0 \qquad \ldots(11)$$

Which results in $\delta\omega=\pm i\,\Omega_0\omega_p^2/2\varepsilon\gamma_0\omega_0$ and the linear wave growth rate ω_I can be finally presented as:

$$\omega_I=\operatorname{Im}\delta\omega=\omega_p\left(\frac{\Omega_0}{2\varepsilon\omega_0\gamma_0}\right)^{1/2} \qquad \ldots(12)$$

3 Numerical Simulations of Backward Microwave Amplification

Some numerical simulations of backward microwave amplification under resonance condition $\omega+kv_0+\Omega_0\approx 0$, have been presented. The components of Eq. (1) can be written as:

$$\frac{dp_x}{dt}+\Omega p_y=-\frac{eE}{\omega}\left[\omega+k\frac{dz}{dt}\right]\cos\left(\omega t+kz\right) \qquad \ldots(13)$$

$$\frac{dp_y}{dt}-\Omega p_x=\frac{eE}{\omega}\left[\omega+k\frac{dz}{dt}\right]\sin\left(\omega t+kz\right) \qquad \ldots(14)$$

$$\frac{dp_z}{dt}=\Omega\frac{nE}{B_0}\left[p_x\cos\left(\omega t+kz\right)-p_y\sin\left(\omega t+kz\right)\right] \ldots(15)$$

where $\Omega=eB_0/m_0\gamma c$ is the relativistic electron cyclotron frequency.

When the instability is occurring, the longitudinal movement will convert to the transverse cyclotron motion. Then, the particle will emit the cyclotron radiation and the particle kinetic energy will be transferred to the microwave. By the energy conservation, the interaction efficiency $\eta=-(\gamma-\gamma_0)/(\gamma_0-1)$ can be calculated straightway according to Eqs (13-15) and is shown in Fig. 1. For both simulations, the refractive index is 2 and $E=200$ SV/cm. For 30 GHz, the interaction parameters are B=0.2T and $\gamma_0=1.351$, and the maximum interaction efficiency is about 16%. However, for 90 GHz, *B*=0.5 T and γ_0=1.211 the maximum interaction efficiency is about 8%. We can see that the maximum interaction efficiency of the former is larger than the latter. Because the electron cyclotron frequency in the former is smaller than the latter, so the electron interacts with the wave in a longer time before the angle between the transverse velocity of the electron and the electric field of the wave approach $\pi/2$, then the beam electron is trapped in the decelerating phase in a longer time and results in a higher interaction efficiency.

The three dimensional trajectory of the beam electron is shown in Fig. 2 with simulating parameters: *f*=90 GHz, *B*=0.5 T, γ=1.211 and *E*=200 Stat V/cm. The maximum transverse drifting distance of the beam electron calculated from single particle theory is about 0.02 cm which is very optimum in the design of a practical microwave amplification device.

Three dimensional trajectory of the beam electron is also verified by 2D3V Particle-In-Cell (PIC) simulations[19]. The simulation[20] is executed by

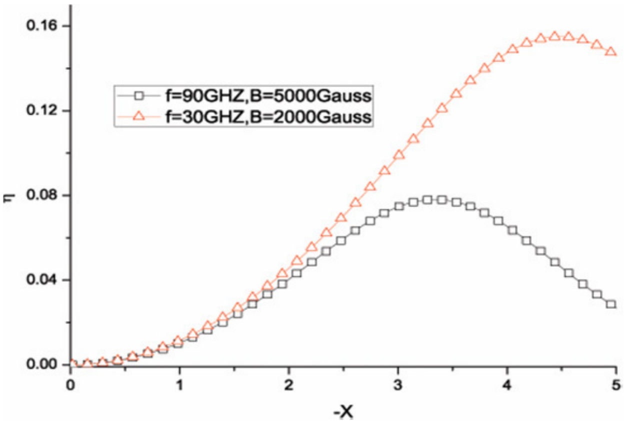

Fig. 1 — Interaction efficiency v.s. interaction length

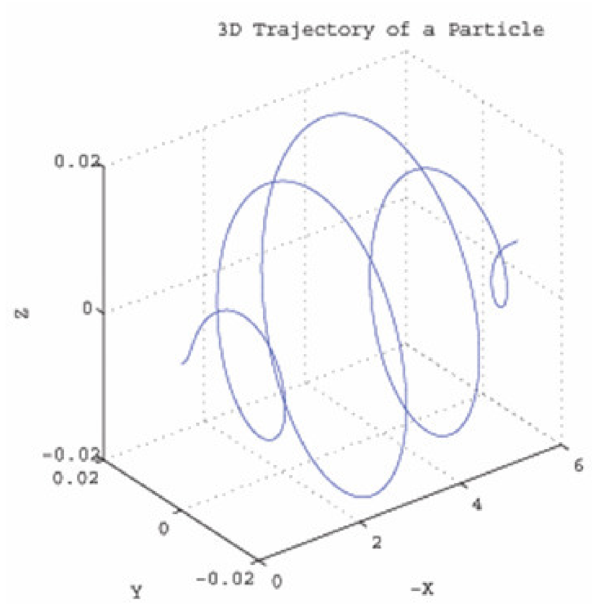

Fig. 2 — 3D trajectory of the beam electron with f=90 GHZ and B=0.5T$_0$

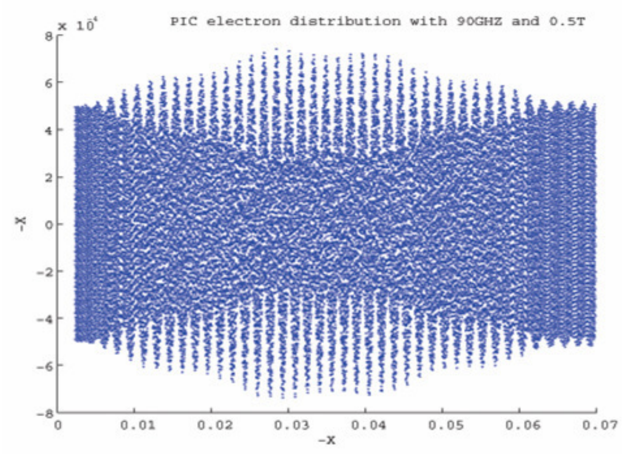

Fig. 3 — Electron X-Y distributions in PIC simulation

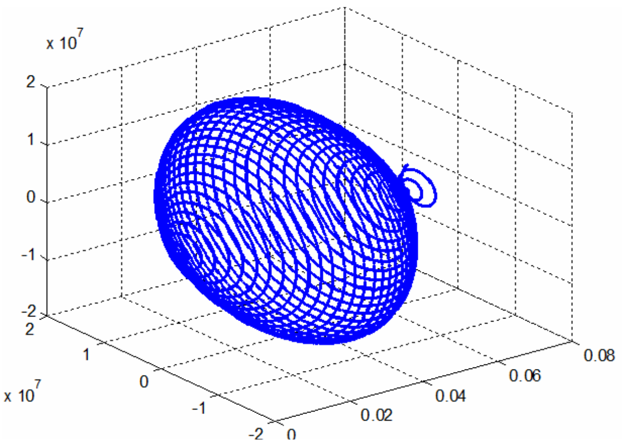

Fig. 4 — Electrons phase space X-Py-Pz from PIC simulation.

VORPAL. The simulating space is 7 cm*4 cm and is divided to 700*200 cells. In X direction, the absorbing boundary conditions (Perfect matched layer) are applied at the two sides. In Y direction, the periodic boundary conditions are applied. The electrons are a beam with 1mm width and γ=1.211. The injected electromagnetic wave is a plane wave with circle polarized like the theory. Its frequency is 90 GHZ and its amplitude is about 200 Stat V/cm, same as the single particle simulation. The guiding magnetic field is 0.5 T. In the PIC simulation, we find the result is very like the single particle theory. The particle distribution from the PIC simulation is shown in Fig. 3. The particle phase space distribution (X-Py-Pz) is shown in Fig. 4. For the interaction in quasi-stable, the space distribution is very similar to the time varying of the single particle. In the Figs 3 and 4, one can find the resonance of electrons and the wave, while the conversion and the evolution of cyclotron motion is shown clearly. The particle energy and momentum distribution from PIC can be viewed and show good agreement with the single particle theory.

4 Conclusions

The linear wave dispersion relation of relativistic electron beam and dielectric system in a uniform guiding magnetic field has been obtained. The instability occurs when the beam electron velocity exceeds the wave phase velocity and coherent microwave radiation can be produced under resonance condition. The numerical simulations show that the maximum interaction efficiency approaches 6%-16%, and the maximum drifting distance of the electron is about 0.2 cm. Because the interaction efficiency is the ratio of the energy which is injected to the microwave from the beam, these results indicate a potential design for a coherent microwave amplification device.

References

1 Jovanović M S & Dimitrijević D R, *Phys Plasmas*, 10 (2003) 1514.
2 Lu W, Huang C, Zhou M, Mori W B & Katsouleas T, *Phys Rev Lett*, 96 (2006) 165002.
3 Banerjee A, Chatterjee S, Sil A & Banerjee N, *Phys Rev D*, 50 (1994) 1161.
4 Sprangle P & Drobot P A, *IEEE Trans on Microwave Theory & Tech*, 25 (1977) 528.
5 Pritchett P L, *Phys Fluids*, 29 (1986) 2919.
6 Chu K R, *Rev Mod Phys*, 76 (2004) 489.
7 Davidson R C & Qin H, Physics of Intense Charged Particle Beams in High Energy Accelerators, London Imperial College Press, (2001) 3.
8 Kartikeyan M V, Borie E & Thumm M K A, Gyrotrons High-Power Microwave and Millimeter Wave Technology, Berlin Springer, (2004) 183.
9 Nusinovich G S, Introduction to the Physics of Gyrotrons, Baltimore The Johns Hopkins University Press, (2004) 235.
10 Barker R J, Luhmann N C, Booske J H & Nusinovich G S, Modern Microwave and Millimeter-Wave Power Electronics, New York Wiley Interscience, (2005)765.
11 Ginzburg N S, *Radiophys & Quant Electro*, 22 (1979) 323.
12 Vomvoridis J L, *IEEE Trans on Nucl Sci*, 30 (1983) 3124.
13 Kho T H & A T Lin, *Phys Rev A*, 38 (1988) 2883.
14 Vomvoridis J L & Hambakis M A, *Int J Electron*, 71 (1991) 167.
15 Vomvoridis J L & Iatrou C T, *Int J Electron*, 71 (1991) 145.
16 Nusinovich G S, Korol M & Jerby E, *Phys Rev E*, 59 (1999) 2311.
17 Kong L B, Liu P K & Xiao L, *Phys Plasmas*, 14 (2007) 053108.
18 Kong L B, Hou Z L & Xie C R, *Appl Phys Lett*, 98 (2011) 261502.
19 Birdsall C K & Langdon A B, Plasma Physics via Computer Simulation, New York Mc Graw-Hill (1985).
20 Nieter C & Cary J R, *J Comput Phys*, 196 (2004) 448.