Indian Journal of Radio & Space Physics
Vol 46, June 2017, pp 55-65

Potential analysis of sunspot parameters and behaviour of random noise

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Received 29 April 2016; revised 11 September 2016; accepted 27 October 2016

Changes in solar magnetic field are responsible for initialisation and maintenance of different solar processes. Sunspots are clear manifestations of field variations and are good indicators of solar activity. Nature of activity can be well understood by analyzing the underlying sunspot dynamics. Techniques of potential analysis are used in this paper to investigate sunspot numbers and sunspot area, during the period 1875-2012, for finding out their stochastic behaviour. The presence of instabilities in the time series of sunspot numbers and sunspot area are examined in detail. The level of instability in sunspot numbers was observed to be maximum in the years 1953-1955, while that in sunspot area was maximum during 1887-1889. This study also concludes that random noise has a greater effect on dynamics of sunspot area than that on dynamics of sunspot numbers. Presence of high level of noise is noticed in both parameters during 1923-1925. Effect of random noise on the dynamics of sunspot number and area was shown to be very high during the years close to sunspot minima. Results reported can be helpful in predicting evolution of solar activity, which would be crucial in understanding solar-terrestrial phenomena.

Keywords: Sunspots, Solar activity, Solar magnetic field

1 Introduction

Sunspots are manifestations of solar magnetic activity. They are relatively cooler areas on the photosphere, formed by the suppression of convective heat transport by intense magnetic field generated in the solar convection zone[1]. And as such they are essentially magnetic phenomena by origin and thermal phenomena by appearance. The number of spots visible on the Sun and area they cover on the photosphere are the two most recognizable features of solar variability. Sunspot number (SN) shows good correlation with other indicators of solar activity like solar radio flux at 10.7 cm, solar total irradiance etc.[2] Origin of the magnetic field inside the Sun and its emergence to its surface are a long-standing mystery.

According to Babcock[3] and Parker[4], sunspots are formed from a magnetic loop emerging from the convective zone to the solar surface due to buoyancy. Magnetic loops were assumed to be caused by instabilities of large scale mean magnetic field generated by joint action of differential rotation and mean helicity of convection. The idea of mean field generation and its evolution has been supported by other studies[5-7] also. One model of sunspot formation suggests that a toroidal flux tube stored at the core/convective zone overshoot layer becomes unstable and erupts to the surface of the Sun when its field exceeds 10^5 G[8,9].

The importance of fluctuating fields in the generation of mean field was addressed in different studies[7,10]. Ruzmaikin[11] developed a model which demonstrates how sunspots could result from stochastic fluctuations superposed on a weaker mean field. This model shows that mean field plays a vital role in producing observed features of sunspot magnetic field. The fluctuating field is responsible for allowing the mean field to be observed and for producing cycle to cycle variations. Randomness in sunspots shows the importance of fluctuating fields and random processes (chaotic or stochastic) play an essential role in sunspot formation. While regular component of magnetic field dominates during normal times, the randomness or the fluctuating fields dominates during active times of Sun. Mutual relationship between regularity and randomness in sunspot series is not clear. However, Kakad *et al*.[12] have predicted descending time of forthcoming solar cycle by estimating Shannon entropy, a measure of randomness in solar cycle. There exist dynamo models developed to explain the randomness in SN which include stochastic processes[13-16].

SN is an indication of how frequently the solar dynamo produces solar activities in terms of sunspots.

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Although there are regularities in sunspot behaviour, there are many irregularities too. Sunspot may appear at random time and random place. The number of sunspots observed in a given cycle varies from cycle to cycle. Presence of random noise in SNs is clear from the fact that its short-term (days-months) variability exceeds observational uncertainties. Ruzmaikin[11,17] suggests that a sunspot is formed when a combination of the regular dynamo and random fields exceeds a buoyancy threshold. A model based on the above concept qualitatively reproduces some features of the sunspot cycle. Moss *et al*.[18] and Käpylä *et al*.[19] studied the role of random processes in the formation of sunspot cycle.

Sunspot area (SA) is a measure of how strongly the solar dynamo produces magnetic flux. It gives extra information about solar variability. Changes in SA cause irradiance variations and hence irradiance studies help in computing SA[20]. Chang[21] studied stochastic properties of north-south asymmetry in SA and found that it is characterized by random noise superimposed on a slowly varying sinusoidal background. A cepstrum analysis reveals that ~1.4 years, ~3.8 years and ~43 years are the periodicities in north-south asymmetry of SA due to stochastic random noise[22].

2 Potential Analysis

Potential analysis is a statistical method developed for noisy time series to investigate the critical behaviour of a system. Noise acts as a driving force in non-linear systems and drastically modifies its deterministic dynamics. In addition it is a function of a very large number of unknown variables. According to Thom[23], non-linear systems can be modelled by means of potential functions. These functions help to describe the system without knowing the internal parameters linked to its behaviour. When one of the parameters of non-linear system exceeds a critical value, potential function shows qualitative changes. These qualitative changes are named as bifurcations[24].

Potential analysis detects the number of states of a geophysical system from its recorded time series. A polynomial approximation further enables the detection of the number of states from the degree of polynomial. The number of states indicates the structural changes encountered by the underlying system potential. Changes in the number of states represent bifurcations of system. Alternatively, a bifurcation occurs when there is a loss of stability in the dynamical condition of a system.

Even though potential analysis cannot reveal the mechanisms causing structural changes, in spite of its ability to detect such changes, the method is a widely used in a variety of time series investigations. For example, application of potential analysis on ice-core proxy records of paleo-temperature identified the loss of warm interstadial state in Greenland climate[25]. Also the signs of appearance of alternative climate states in a particular region at particular times during the Holocene was detected using the method of potential analysis[26]. Hirota *et al*.[27] detected the presence of alternative stable states in tropical forest and savanna using this technique. Even though the technique of potential analysis was applied to several climatic studies, no significant application of the method to the solar activity parameters has been found in the literature.

The present study detects the number of states, and bifurcations caused by stochastic forcing in SN and SA. The following assumptions are made in the theory (i) multiple states can be approximated by a non-oscillatory potential, though it changes through time and (ii) transitions between states are triggered purely by stochastic noise.

Random noise acting on the normal sunspot dynamics is considered as stochastic forcing. According to the assumptions inherent in the potential analysis theory, random noise is therefore responsible for the non-linear behaviour of sunspot dynamics. Randomness in sunspots is obvious. But its nature and relationship with regular dynamics of sunspots are still unknown. By applying potential analysis on SN and SA data, an estimate of time variation of amplitude of random noise is obtained.

3 Data and Method of Analysis

Relative sunspot numbers and sunspot area required for the study were collected from http://www.ngdc.noaa.gov and http://www.solarscience.msfc.nasa.gov respectively.

The given time series is treated as a non-linear dynamical system which can possess multiple states, with shifts between different states induced by stochastic forcing. To describe a stochastic dynamical system, a noisy differential equation was first used by Brown[28]. In general, a noisy differential equation is a combination of slowly varying regular and rapidly varying random parts. In this study the time series is modelled using a stochastic differential equation given by Langevin[26].

$$dz = -U'(z)dt + \sigma dW \quad \text{... (1)}$$

$$dW = \xi(t)dt \quad \text{... (2)}$$

where $U(z)$ is the potential function, σ the noise level , W denotes the standard Weiner process and $\xi(t)$ represents random noise[29]. The state variable z represents the magnetic field produced by the solar dynamo and is here identified with solar activity proxies (SN and SA).

The shape of the potential can be approximated by a polynomial

$$U(z) = \sum_{i=1}^{L} a_i z^i \quad \text{... (3)}$$

where the order L is even and the leading coefficient a_L is positive for a stationary solution. The number of states associated with the system scales with the value of L.

Since Eq. (1) is a stochastic differential equation, it can be solved using statistical methods. The statistical analogue of Eq. (1) is the Fokker-Planck equation[26], a partial differential equation for the evolution of a probability density function. The Fokker-Planck equation corresponding to Eq. (1) is

$$\partial_t p(z,t) = \partial_z \left[U'(z)p(z,t)\right] + \frac{1}{2}\sigma^2 \partial_z^2 p[z,t] \quad \text{... (4)}$$

where $p(z,t)$ represents a probability density function. A stationary solution[30] of Eq. (4) is given by

$$p(z) \sim exp\left[-2U(z)/\sigma^2\right] \quad \text{... (5)}$$

The relation between the potential and the stationary probability density allows to reconstruct the potential as

$$U = -\frac{\sigma^2}{2} \log p_d \quad \text{... (6)}$$

where p_d is the empirical probability density of the data. A standard Gaussian kernel estimator is used to determine p_d and is given by

$$\hat{f}(z) = \frac{1}{nh} \sum_{i=1}^{n} K\left(\frac{z - z_i}{h}\right) \quad \text{... (7)}$$

where K denotes the Gaussian kernel, z_i are the data points, n is the length of the data set and h is the bandwidth controlling smoothness of the estimator. By following the methods used by Livina *et al.*[25], bandwidth is estimated as $h = 1.06s / n^{1/5}$, where s is the standard deviation of the data set. The best suitable polynomial representing the potential is obtained from least-square fits of $-\log p_d$. This preferred polynomial is characterised by the highest degree just before a sign change occurs in the leading co-efficient. Then the number of states S in the system is determined as

$$S = 1 + \frac{I}{2} \quad \text{... (8)}$$

where I is the number of inflection points of the fitted polynomial potential of degree L. Inflection points are the points at which the polynomial changes concavity. Limiting behavior of even-order polynomials with positive leading co-efficient implies that their inflection points can only occur in pairs. The polynomial determines the shape of the potential representing the given non-linear system and a polynomial showing zero inflection point is identified as a system having one state. An addition of each pair of inflection points adds another state to the system[25].

4 Results and Discussion

4.1 Sunspot numbers

A potential analysis has been done on daily SN time series (Fig. 1) for the years 1875-

2012. The whole set of SN values is divided into 46 equal intervals, with data for 3 years in each interval. This represents approximately 1000 daily SN values in one interval. According to Livina *et al.*[25], accuracy of detection of number of states in a time

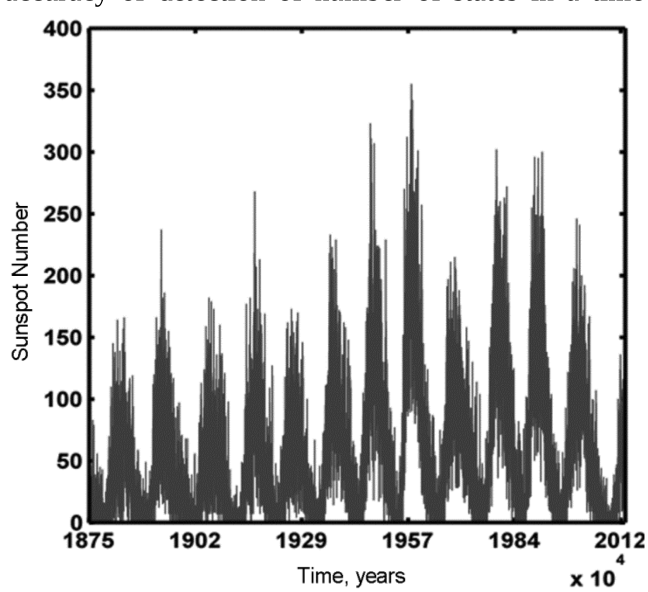

Fig. 1 — Daily SN data (1875-2012)

series using potential analysis depends on window size. If the window contains 400-500 data points, success rate is above 90%. As the window size increases, success rate also becomes high. 98% accuracy is obtained for a data length of 1000 points. Our analysis ensures 98% accuracy in the detection of number of states by taking a window size of 3 years (~1096 points).

A stochastic state is defined as the value of the dynamical variable for which the stationary probability distribution (SPD) is maximum. Number of states is a scalar parameter derived from probability density function in order to find out number of wells in the potential of an unknown dynamical system indirectly. Number of states can be considered as a measure of amplitude of random noise. Hence, potential analysis can find out time intervals of data having high amplitude random noise. Varying number of states with time gives an understanding of temporal variation of amplitude of random noise existed in sunspot dynamics during the past. Hence, the results obtained from this study may help in developing sunspot cycle model, with temporal random noise information. By incorporating a random field to a regular 11 year oscillatory dynamo Usoskin *et al.*[31] constructed a model that reproduces most of the fundamental properties of sunspot cycle during normal as well as active periods.

In the present study, SPD is used to track changes in the potential representing sunspot dynamics. Distributions of many natural phenomena are at least approximately normally distributed[32] and hence a Gaussian function was used for the detection of SPD. Potential analysis carried out on the complete SN data set reveals that there are 1-5 states for any 3 year interval. These states may be the same as in a noiseless case. But in a stochastic dynamical system, noise alters positions and even the number of stochastic states. Noise changes the shape of the potential function and noise-induced states are a nontrivial effect of noise. As assumed in the theory of potential analysis, the years with only one state are the intervals in which sunspot dynamics is not under the influence of noise. Higher numbers of states (2 to 5) are assumed to be induced by noise. In a particular interval, the number of wells in the potential is one less than the number of states. Changes in number of wells in the potential can be interpreted as bifurcation.

Numbers of states obtained in different 3 years intervals are shown in Table 1. Fig.2 is the histogram

Table 1 — Number of states in daily SN (1875-2012) during 46 intervals

Interval	Years	Number of states
1	(1875-1877)	2
2	(1878-1880)	3
3	(1881-1883)	1
4	(1884-1886)	3
5	(1887-1889)	4
6	(1890-1892)	2
7	(1893-1895)	3
8	(1896-1898)	2
9	(1899-1901)	1
10	(1902-1904)	3
11	(1905-1907)	2
12	(1908-1910)	2
13	(1911-1913)	1
14	(1914-1916)	2
15	(1917-1919)	2
16	(1920-1922)	3
17	(1923-1925)	4
18	(1926-1928)	1
19	(1929-1931)	2
20	(1932-1934)	3
21	(1935-1937)	2
22	(1938-1940)	2
23	(1941-1943)	2
24	(1944-1946)	2
25	(1947-1949)	2
26	(1950-1952)	2
27	(1953-1955)	5
28	(1956-1958)	2
29	(1959-1961)	1
30	(1962-1964)	3
31	(1965-1967)	2
32	(1968-1970)	2
33	(1971-1973)	2
34	(1974-1976)	2
35	(1977-1979)	1
36	(1980-1982)	1
37	(1983-1985)	2
38	(1986-1988)	3
39	(1989-1991)	1
40	(1992-1994)	2
41	(1995-1997)	3
42	(1998-2000)	1
43	(2001-2003)	2
44	(2004-2006)	2
45	(2007-2009)	1
46	(2010-2012)	1

showing number of occurrences of intervals with different number of states. Intervals with two states are predominant in number compared to intervals with one, three, four and five states. While there are 26, two-state intervals, there are only 10, one-state intervals, 7 three-state intervals, 2 four-state intervals and 1 five-state interval. Also, there are longer episodes of group of two state intervals like 21-26, 31-34, and 42-44 (Table 1).

The theory of potential analysis suggests that the presence of noise leads to an increase in the number of wells in potential curves. The number of states in any interval is one greater than the number of potential wells in that particular interval. In the present potential analysis on SN, the largest number of states (S=5) was obtained for the interval 27 (1953-1955). The corresponding potential profile with 4 wells is shown in Fig. 3. This interval is, therefore considered to be the most affected by noise. Interestingly, this interval coincides with year 1954, the sunspot minimum and one among the lowest minimum since 1913. Dominant periodicity of 9.8 years has been found in SN during the above years[33]. Similarly intervals 5 (1887-1889) and 17 (1923-1925) are having 4 states and correspondingly 3 potential wells each (Fig. 4). These intervals are also affected by considerable noise, and they coincide with various sunspot minima. It shall be noted that years 1887-1889 and 1923-1925 are close to the sunspot minimum of solar cycle 13 and 16 respectively. Therefore, the largest noise and highest number of states coincides with sunspot minimum periods. Also anomalously long oscillations in SN and sudden hike in solar and geomagnetic activities were obtained after the year 1923[34]. These are interpreted as non-linear nature of solar dynamo after 1923.

Bifurcations represent changes in the number of states between adjacent intervals. Bifurcations occurred in SN time series data are shown in Table 2. There are 6 transitions from 2 states to 1 state,

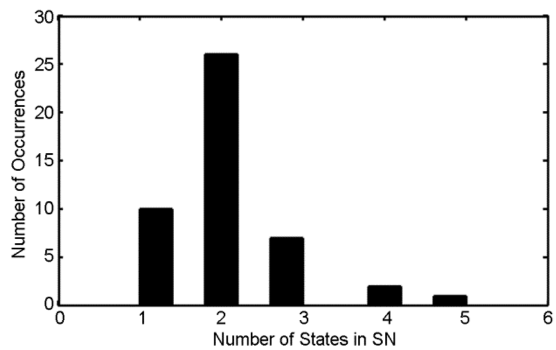

Fig. 2 — Histogram of number of states in SN (1875-2012)

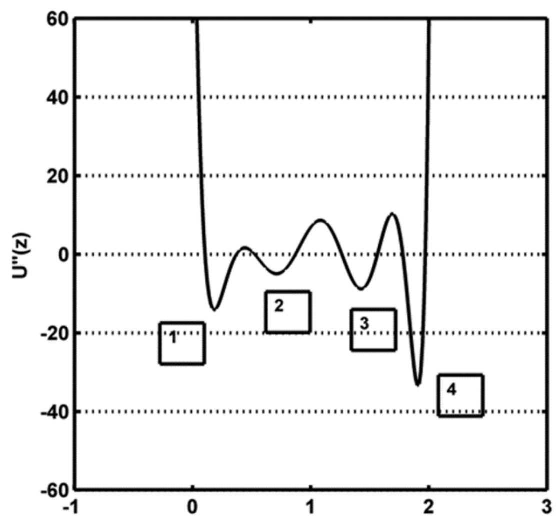

Fig. 3 — Potential variations in SN in the interval 27 (1953-1955) showing 4 wells

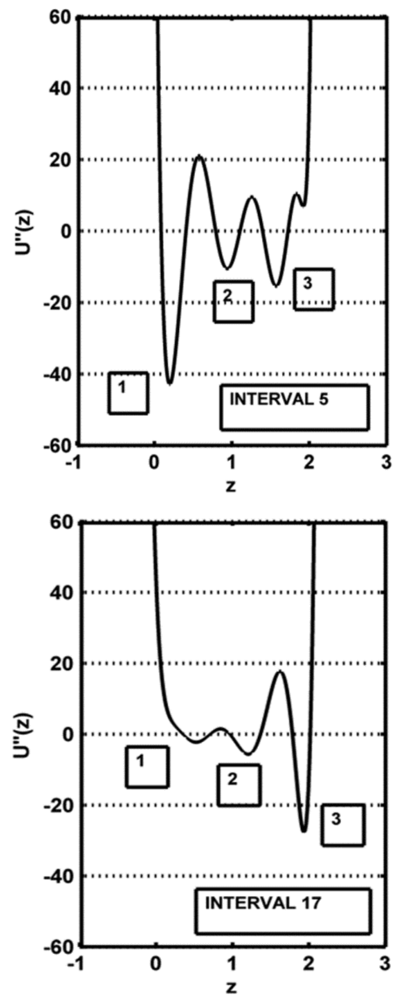

Fig. 4 — Potential variations in SN in the intervals 5 and 17 showing the three wells each

Table 2 — Bifurcations occurred in daily SN (1875-2012)

Between two intervals	Years concerned	Transition of states
1 and 2	(1875-1877) and (1878-1880)	2→3
2 and 3	(1878-1880) and (1881-1883)	2→1
3 and 4	(1881-1883) and (1884-1886)	1→2
4 and 5	(1884-1886) and (1887-1889)	3→4
5 and 6	(1887-1889) and (1890-1892)	4→2
6 and 7	(1890-1892) and (1893-1895)	2→3
7 and 8	(1893-1895) and (1896-1898)	3→2
8 and 9	(1896-1898) and (1899-1901)	2→1
9 and 10	(1899-1901) and (1902-1904)	1→3
10 and 11	(1902-1904) and (1905-1907)	3→2
12 and 13	(1908-1910) and (1911-1913)	2→1
13 and 14	(1911-1913) and (1914-1916)	1→2
15 and 16	(1917-1919) and (1920-1922)	2→3
16 and 17	(1920-1922) and (1923-1925)	3→4
17 and 18	(1923-1925) and (1926-1928)	4→1
18 and 19	(1926-1928) and (1929-1931)	1→2
19 and 20	(1929-1931) and (1932-1934)	2→3
20 and 21	(1932-1934) and (1935-1937)	3→2
26 and 27	(1950-1952) and (1953-1955)	2→5
27 and 28	(1953-1955) and (1956-1958)	5→2
28 and 29	(1956-1958) and (1959-1961)	2→1
29 and 30	(1959-1961) and (1962-1964)	1→3
30 and 31	(1962-1964) and (1965-1967)	3→2
34 and 35	(1974-1976) and (1977-1979)	2→1
36 and 37	(1980-1982) and (1983-1985)	1→2
37 and 38	(1983-1985) and (1986-1988)	2→3
38 and 39	(1986-1988) and (1989-1991)	3→1
39 and 40	(1989-1991) and (1992-1994)	1→2
40 and 41	(1992-1994) and (1995-1997)	2→3
41 and 42	(1995-1997) and (1998-2000)	3→1
44 and 45	(2004-2006) and (2007-2009)	2→1

5 transitions each from 1 state to 2 states, 2 states to 3 states and 3 to 2 states. Bifurcations with magnitude of differences of states greater than 2 (like 2→5, 4→1, etc.) are also present. Such higher magnitude bifurcations, even though less in number are more significant. A 2→1 transition corresponds to a change of potential from single well to zero well. In the daily SNs from 1875 to 2012, bifurcations showing maximum change in the number of states are those with a change of states equal to three. There are 3 such bifurcations observed in our analysis. These bifurcations cause major deformations in the potential. Change of three states causes addition or removal of two wells in the potentials corresponding to the intervals (Fig. 5). In 2→5 bifurcation, two wells are added to the potential. But in 4→1 and 5→2 bifurcations, two wells are removed from the potential. High deformation of potential taking place between adjacent intervals means a sudden addition or removal of large amplitude noise. Intervals involved in the consecutive bifurcations 2→5 and 5→2 are 26, 27 and 28. This means large amplitude noise is added to the system when it undergoes a transition from interval 26 to 27. But, during transition from interval 27 to 28 noise is removed from the system. A 4→1 bifurcation occurs between intervals 17 and 18 and this corresponds to removal of noise from the system. In the years (1923-1925) corresponding to interval 17, Duhau and Chen[34] have noticed vanishing of amplitude of cyclical oscillations of poloidal and toroidal field. A combined action of these fields determines sunspot evolution. The stochastic nature of dynamo α effect and fluctuations in the meridional flow are capable of producing variations in the amplitudes of solar cycles[14,35].

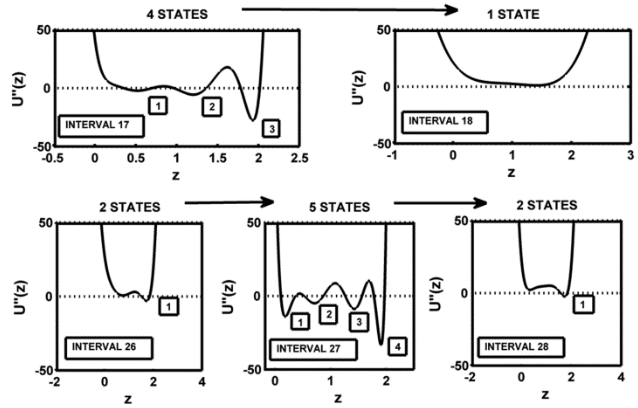

Fig. 5 — Changes in the potential representing SN during 4→1, 2→5 and 5→2 bifurcations

Intervals with a single state are treated as noiseless intervals and Table 3 lists bifurcations present between two noiseless intervals. There are seven episodes lying between noiseless intervals. In between two noiseless intervals, the noise is observed to increase and decrease, indicating that the noise affecting sunspot dynamics was not persistent throughout the period of study. It affects the dynamics occasionally (seven times in our analysis) and causes corresponding effects in sunspot formation. The level of noise reaches a maximum in a particular interval which is characterized by the highest number of states. In seven episodes considered, the noise becomes highest at intervals 5, 10, 17, 27, 30, 38 and 41 respectively. Interestingly these intervals fall nearer to solar minima of different solar cycles. Previous investigations concluded that poloidal field starts to accumulate at the poles by the Babcock-

Leighton process during solar minima. Choudhary[36] identified random noise in the process as one of the causes of irregularities in the solar cycle.

In SN dynamics, 2, 5, 7, 10, 16, 17, 20, 27, 30, 38 and 41 are intervals having higher number of states (three to five). All of them except interval 7 (1893-1895) coincide with years of sunspot minima such as 1878, 1889, 1901, 1923, 1933, 1954, 1964, 1986 and 1996. Number of states detected in SN and their temporal evolution are shown in Fig. 6. Years corresponding to higher number of states (3, 4 and 5) are marked in the figure. It is clear from this figure that the longest episode over which the system never returned to the noiseless states (number of states=1) is episode 5. In this episode, the system remained affected by the noise for 30 years from 1929 to 1958. This is also the interval with the largerest number of bifurcations present. The maximum number of states (5 in interval 27) also belongs to this episode. Therefore this episode can be considered as the one in which strong and very dynamic noise perturbations were prevailing.

Table 3 — Characteristics of bifurcations occurred in daily SN in between noiseless states

Episodes between noiseless intervals	Bifurcations in SN: Between intervals	Bifurcations in SN: Change of states	Highest number of states	Interval corresponding to highest number of states
	3 and 4	1→2		
1	4 and 5	2→4	4	5
	5 and 6	4→2		
	6 and 7	2→3		
	7 and 8	3→2		
	8 and 9	2→1		
	9 and 10	1→3		
2	10 and 11	3→2	3	10
	12 and 13	2→1		
	13 and 14	1→2		
3	15 and 16	2→3	4	17
	16 and 17	3→4		
	17 and 18	4→1		
	18 and 19	1→2		
4	19 and 20	2→3	5	27
	20 and 21	3→2		
	26 and 27	2→5		
	27 and 28	5→2		
	28 and 29	2→1		
	29 and 30	1→3		
5	30 and 31	3→2	3	30
	34 and 35	2→1		
	36 and 37	1→2		
6	37 and 38	2→3	3	38
	38 and 39	3→1		
	39 and 40	1→2		
7	40 and 41	2→3	3	41
	41 and 42	3→2		
	44 and 45	2→1		

4.2 Sunspot area

Daily SA time series during the years from 1875 to 2012 is shown in Fig. 7. Potential analysis is done on SA by dividing it into 46 equal intervals similar to SN. Various states obtained in SA for different three years intervals and their corresponding years of existence are shown in Table 4.

Like SN, SA also exhibits states from 1 to 5. Highest number of states in SA is 5 corresponding to the interval 5 (1887-1889). Comparatively higher number of states such as 4 states also occurs in intervals 9 (1899-1901), 17 (1923-1925) and 20

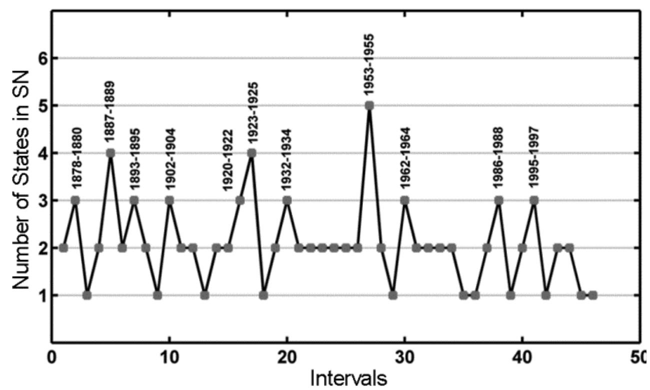

Fig. 6 — Number of states in SN and their evolution during 1875-2012

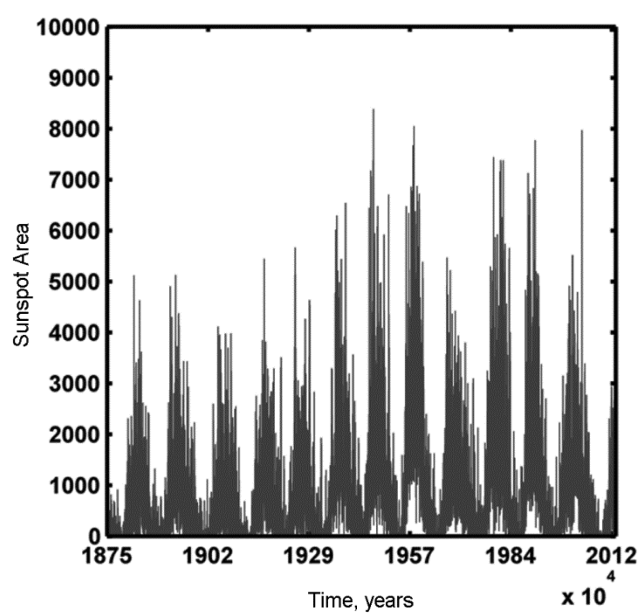

Fig. 7 — Daily SA data (1875-2012)

Table 4 — Number of states in daily SA during 46 intervals

Interval	Years	Number of states
1	(1875-1877)	3
2	(1878-1880)	1
3	(1881-1883)	2
4	(1884-1886)	1
5	(1887-1889)	5
6	(1890-1892)	2
7	(1893-1895)	2
8	(1896-1898)	2
9	(1899-1901)	4
10	(1902-1904)	3
11	(1905-1907)	2
12	(1908-1910)	2
13	(1911-1913)	1
14	(1914-1916)	2
15	(1917-1919)	1
16	(1920-1922)	2
17	(1923-1925)	4
18	(1926-1928)	1
19	(1929-1931)	2
20	(1932-1934)	4
21	(1935-1937)	1
22	(1938-1940)	1
23	(1941-1943)	2
24	(1944-1946)	1
25	(1947-1949)	1
26	(1950-1952)	1
27	(1953-1955)	3
28	(1956-1958)	1
29	(1959-1961)	1
30	(1962-1964)	3
31	(1965-1967)	1
32	(1968-1970)	2
33	(1971-1973)	2
34	(1974-1976)	1
35	(1977-1979)	1
36	(1980-1982)	1
37	(1983-1985)	2
38	(1986-1988)	2
39	(1989-1991)	1
40	(1992-1994)	2
41	(1995-1997)	3
42	(1998-2000)	2
43	(2001-2003)	1
44	(2004-2006)	2
45	(2007-2009)	1
46	(2010-2012)	3

(1932-1934). From Fig. 8, intervals with 1 and 2 states are more regular compared to other intervals having states 3, 4 and 5.

Table 5 shows bifurcations detected in SA. Bifurcations involving a change of states greater than two are from 1 state to 5 states, 4 states to 1 state and 5 states to 2 states.

Episodes between consecutive noiseless levels and corresponding bifurcations in SA are shown in Table 6. Of the 12 episodes, the longer one between two noiseless periods is episode 2, which lasts for 23 years (1887-1910). The maximum number of states (5) corresponding to the maximum noise perturbation, in the whole period of analysis also falls in this episode. The highest number of states in the 12 episodes is occurring in intervals 3, 5, 11, 14, 17, 20, 23, 27, 30, 32, 37, 41 and 44 with intervals 5, 17, 20, 23, 27, 30, 37 and 41 close to solar minima. The correlation between solar minima and the intense noise (represented by the number of states) in SA is quite interesting. A similar relationship was obtained for SN noise as well.

Figure 9 shows the number of states detected in SA and their temporal evolution. In SA, intervals 1, 5, 9, 10, 17, 20, 27, 30, 41 and 46 shows relatively higher number of states (3-5 states). Among these, eight out of ten intervals are close to the years of various sunspot minima such as 1889, 1901, 1923, 1933, 1954, 1964 and 1996.

Results obtained reveal that intervals with 2 states are most often occurring in the potential analysis of SN and SA indicating that intervals with a single well potential are more frequently occurring. This suggests that the noise in sunspot dynamics is best represented by a single well potential. It is also observed that the SN was affected by maximum noise in the interval 27

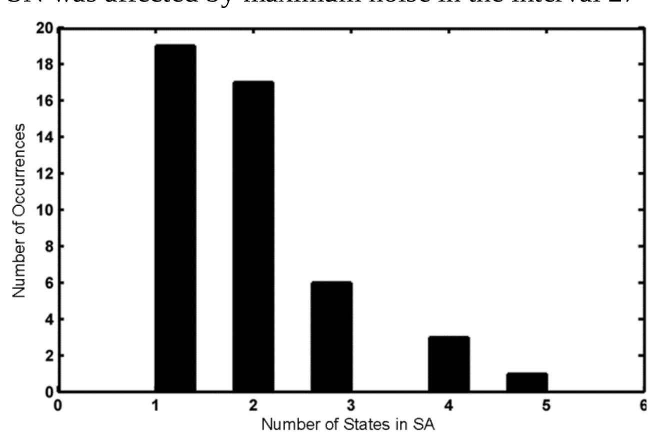

Fig. 8 — Histogram of number of states in SA (1875-2012)

Table 5 — Bifurcations occurred in daily SA (1875-2012)

Intervals between two intervals	Years concerned	Transition of states
1 and 2	(18751877) and (1878-1880)	3→1
2 and 3	(1878-1880) and (1881-1883)	1→2
3 and 4	(1881-1883) and (1884-1886)	2→1
4 and 5	(1884-1886) and (1887-1889)	1→5
5 and 6	(1887-1889) and (1890-1892)	5→2
8 and 9	(1896-1898) and (1899-1901)	2→4
9 and 10	(1899-1901) and (1902-1904)	4→3
10 and 11	(1902-1904) and (1905-1907)	3→2
12 and 13	(1908-1910) and (1911-1913)	2→1
13 and 14	(1911-1913) and (1914-1916)	1→2
14 and 15	(1914-1916) and (1917-1919)	2→1
15 and 16	(1917-1919) and (1920-1922)	1→2
16 and 17	(1920-1922) and (1923-1925)	2→4
17 and 18	(1923-1925) and (1926-1928)	4→1
18 and 19	(1926-1928) and (1929-1931)	1→2
19 and 20	(1929-1931) and (1932-1934)	2→4
20 and 21	(1932-1934) and (1935-1937)	4→1
22 and 23	(1938-1940) and (1941-1943)	1→2
23 and 24	(1941-1943) and (1944-1946)	2→1
26 and 27	(1950-1952) and (1953-1955)	1→3
27 and 28	(1953-1955) and (1956-1958)	3→1
29 and 30	(1959-1961) and (1962-1964)	1→3
30 and 31	(1962-1964) and (1965-1967)	3→1
31 and 32	(1965-1967) and (1968-1970)	1→2
33 and 34	(1971-1973) and 1974-1976)	2→1
36 and 37	(1980-1982) and (1983-1985)	1→2
38 and 39	(1986-1988) and (1989-1991)	2→1
39 and 40	(1989-1991) and (1992-1994)	1→2
40 and 41	(1992-1994) and (1995-1997)	2→3
41 and 42	(1995-1997) and (1998-2000)	3→2
42 and 43	(1998-2000) and (2001-2003)	2→1
43 and 44	(2001-2003) and (2004-2006)	1→2
44 and 45	(2004-2006) and (2007-2009)	2→1
45 and 46	(2007-2009) and (2010-2012)	1→3

(1953-1955), where as the SA was affected by maximum noise in the interval 5 (1887-1889). In both SN and SA, this peak noise occurred at episodes of longest noise persistence. Also, in this analysis most of the years close to sunspot minima since 1875 are associated with higher number of states and hence affected by high amplitude random noise. Years 1913 and 2008 are sunspot minima having relatively high number of spotless days. True that during these years of extreme minima, random noise is found absent in both SN and SA. The reason for this exceptional behaviour in the two periods is still to be understood. Work in that direction requires further analysis and is in progress. During the present interval (1875-2012) of study, total number of bifurcations occurred in SA is greater than that in SN.

Table 6 — Characteristics of bifurcations occurred in daily SA in between noiseless states

Episodes between noiseless intervals	Bifurcations SA		Highest number of states	Intervals corresponding to highest number of states
	Between intervals	Change of states		
1	2 and 3	1→2	2	3
	3 and 4	2→1		
2	4 and 5	1→5		
	5 and 6	5→2	5	5
	8 and 9	2→4		
	9 and 10	4→3		
	10 and 11	3→2		
	12 and 13	2→1		
3	13 and 14	1→2	2	14
	14 and 15	2→1		
4	15 and 16	1→2		
	16 and 17	2→4	4	17
	17 and 18	4→1		
5	18 and 19	1→2		
	19 and 20	2→4	4	20
	20 and 21	4→1		
6	22 and 23	1→2	2	23
	23 and 24	2→1		
7	26 and 27	1→3	3	27
	27 and 28	3→1		
8	29 and 30	1→3	3	30
	30 and 31	3→1		
9	31 and 32	1→2	2	32
	33 and 34	2→1		
10	36 and 37	1→2	2	37
	38 and 39	2→1		
11	39 and 40	1→2		41
	40 and 41	2→3	3	
	41 and 42	3→2		
	42 and 43	2→1		
12	43 and 44	1→2	2	44
	44 and 45	2→1		

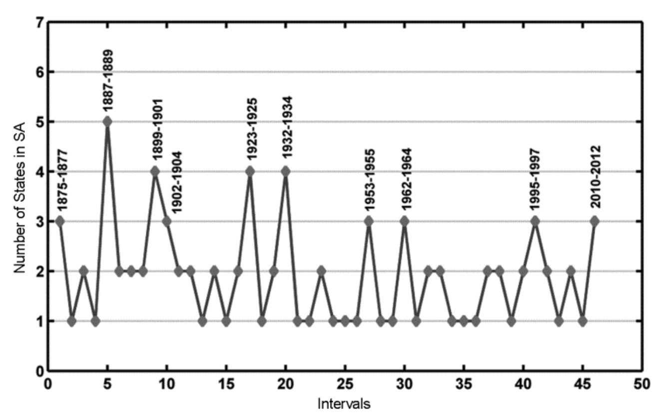

Fig. 9 — Number of states in SA and their evolution during 1875-2012

Total number of bifurcations is 34 and 31 for SA and SN respectively (shown in Tables 2 and 5). More number of bifurcations in SA compared to SN indicates that dynamics of the former is more susceptible to random noise than latter. Kakad[37] developed a model for predicting peak SN and ascent time of the upcoming solar cycle by considering random fluctuations in SN during different phases of solar cycle. Significance of considering randomness during sunspot prediction is noticeable. Chang[38] find out random noise superimposed cyclical evolution of SA asymmetry.

5 Conclusions

Potential analysis of SN and SA for the years 1875 to 2012 is an effort to understand the underlying dynamics of sunspots due to stochastic forcing. The basic nature of noise is shown to be the one creating potentials with single wells. The maximum noise in SN and SA coinciding with various sunspot minimum periods is an interesting result which requires further studies and analysis to explain.

The study establishes the presence of random noise during their formation. Sources of random noise producing amplitude fluctuations in the dynamo models of solar cycle are fluctuations of large scale flows in the convective envelope such as differential rotation and meridional circulation[39]. A threshold magnetic field of 10^5G, required for the appearance of sunspots is provided by fluctuating magnetic fields superimposed on the mean field. These fluctuating magnetic fields are treated as noise and are also generated by dynamo action.

Our results support the view that the formation of sunspots is strongly affected by random noise. From previous studies, it is clear that fluctuations in the differential rotation and meridional circulation in the convective zone act as random noise in the evolution of sunspots. Production of toroidal flux at the solar convection zone leading to sunspot formation is highly sensitive to values of magnetic diffusivity[40]. Also fluctuations in magnetic diffusivity had caused asymmetries in sunspot distribution between two hemispheres during Maunder minimum [41]. Hence variations in magnetic diffusivity are also considered as a crucial factor in the context of random fluctuations in SNs. We used two proxies - SN and SA- to study random noise. Random noise affecting SN and SA dynamics shows one particular characteristic in common: In both the parameters, the highest peak of random noise appears during the longest interval of noise persistence. In addition the amplitude grows and declines very quickly. Two- state noise or single potential well noise was shown to be occurring most frequently in both of these proxies. This shows that random noise is an integral part of sunspot formation. Results obtained from potential analysis of sunspot parameters provides a new result that potential representing sunspot area undergoes more number of deformations due to the presence of random noise than that observed in the potential of sunspot number evolution during the period of years from 1875-2010. Thus, SA dynamics is most susceptible to random noise than SN. SN and SA represent two aspects of solar dynamo. Action of variables influencing the above two processes may be different. Hence, they may have different time evolution. But in both the parameters, random noise plays a crucial factor in their time behaviour. Hence on the basis of assumptions made in theory of potential analysis, it can be concluded that stochastic forcing reflected as non-linear behaviour in the observational features of sunspots is discontinuous with varying amplitudes and intervals of persistence. Therefore, dynamo models with a random noise component can effectively reproduce irregularities shown by the sunspot cycle. Insufficient knowledge about the nature of irregularities will however make the accurate prediction of sunspots almost impossible. In this regard, potential analysis exploring the nature of noise and irregularities in solar parameters is an effective tool of investigation.

Acknowledgements

The authors would like to express their gratitude to NGDC and NASA's Marshall Space Flight Center for making relative SN data and SA-data available online. One of the authors, G L Jayalekshmi, acknowledges the junior research fellowship granted to her by University of Kerala, Thiruvananthapuram.

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