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Renyi entropy based Bi-histogram equalization for contrast enhancement of MRI brain images

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The quality of the MRI brain images is dependent on the sensor. It is essential to have a pre-processing technique to meet the finest quality at the sensor's cost. A pre-processing algorithm has been proposed in this paper to enhance the low contrast MRI brain images. The input image's histogram has been divided into two sub histograms using its median value to uphold the input image's mean brightness. After calculating the Renyi entropy from the sub histogram, histogram clipping has been done to regulate the enhancement rate. The clipping limit has been selected automatically from the minimum value of the mean, median of the distribution function, and itself. Additionally, the proposed algorithm has incorporated the Discrete Cosine Transform (DCT) to improve the enhancement. Experimental results have shown that the proposed algorithm enhances the input image and maintains the mean brightness.

Keywords: Adaptive clipping limit, Contrast improvement index, Discrete cosine transform, Gradient magnitude similarity deviation, Spatial distribution

1 Introduction

The image enhancement techniques are used extensively as part of medical imaging, especially in Magnetic Resonance Imaging (MRI). The test that uses attractive field and radio pulses to capture body images, specifically the brain or cerebrum, is called MRI[1]. The method of MRI is chosen for low tissue density because it provides adequate molecular level knowledge. But extracting the information needed from these images is an inappropriate activity. Due to ambient noise and low light conditions, understanding such images is very difficult. Such images are of inferior quality, low contrast, and contain a great deal of ambiguity. It makes the process of diagnosis more complicated. Therefore, the radiologist and surgeons use medical image enhancement techniques to diagnose illness, identify fractures, detect tumors, and identify any organ in our body's excess development.

Because of its ease of execution and better performance, the most widely used spatial domain algorithm is Histogram Equalization (HE). This method's main benefit is that it is a simple technique and an invertible operator. However, it does cause the low contrast images to over-enhance, resulting in a noisy appearance in the enhanced images. One of the most commonly used techniques is Local Histogram Equalization (LHE) for image enhancement. The entire image in LHE is encompassed in a window in which the actual pixel is locally equalized within the given window by the histogram. Many algorithms have been developed to improve the efficiency of HE because of the difficulty and choice of window size[2,3]. Kim introduced Brightness, Preserving Bi-Histogram Equalization (BBHE)[4] in 1997 to improve the mean brightness shifting problems of HE. BBHE divides the input low contrast image's histogram into two by the average pixel intensity and equalizing the sub-histograms separately. It maintains the mean brightness while reducing the influence of distortion and eliminating irregular enhancement and unnecessary artifacts.

Following BBHE, Dualistic Sub-Image Histogram Equalization (DSIHE)[5] has been proposed, which uses the median value instead of the mean value to distinguish the histogram of the input image. The generalization scheme for BBHE and DSIHE, Recursive Mean-Separate Histogram Equalization (RMSHE), and Recursive Sub-Image Equalization (RSIHE)[6] has also been developed by recursive division of the input histogram other than once based on the mean and median values, respectively, RSIHE and RMSHE generate 2^r sub-histograms. Scalable brightness conservation is permitted by the recursive property of RMSHE and RSIHE techniques. RMSHE

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and RSIHE's common difficulty is the difficulty of defining the best value for r. The output image will be near-identical to the input image when r becomes very high, and there will be no enhancement.

A minimum of the mean and median values obtained from the histogram is subject to the clipping limit imposed on each sub-image histogram. The adjustment of over-enhancements resulted from these variations, but the conservation of mean brightness in the output image was not decisively reflected. In addition to the mean and median values for histogram clipping, Adaptive Image Enhancement Bi-Histogram Equalization (AIEBHE)[7] uses the initial histogram's minimum values. Although these algorithms were used to convey adequate results, rationality in choosing the minimum, mean, or median values to differentiate the input image was not acceptable. Since each image has its feature content, the values cannot be generalized against different images.

Edge Enhancing Bi-Histogram Equalization (EEBHEF)[8] has been used adaptive clipping limit and guided image filter based transformation. The average intensity value of the image has been used to break the histogram. With the aid of the segmented histogram's entropy values, plateau limits have been determined to prevent over-enhancement. The limits of the plateau adjust the histogram. The modified histogram, cumulative density function, and coefficients of the guided image filters have been determined for each pixel in the input image. The enhanced image has been generated by using the mapping function and linear filter coefficients. This algorithm's enhanced picture has the maximum information details. A new enhancement technique has been proposed by P. Shanmugavadivu *et al*.[9] to increase the contrast and retain the information. This technique has utilised Otsu's method for thresholding and HE to boost the image quality. Using the principle of gamma correction and the PSO algorithm, M. Kanmani has implemented an algorithm for improving the content of information and image contrast[10].

There are more background regions in the MRI images. So, in the enhanced images, it has created an offset intensity effect. Most of the current enhancement techniques has resulted in more data loss due to complexity in such images and cannot increase the contrast. Some methods produced noisy images over enhanced ones. The main objective of this proposed technique is to enhance the contrast enhancement thereby reducing artifacts, preserving edges and preventing over enhancement. The primary contributions of our proposed work are enumerated below:

Renyi entropy-based bi-histogram equalization is proposed to improve the visual perception of MRI brain images. Renyi entropy is employed to obtain a discrete function to prevent information loss and edge details.

Incorporation of the Discrete Cosine Transform (DCT) into Bi-histogram equalization for improved contrast enhancement.

The proposed technique is validated with comparing the results of state of the art bi-histogram equalization techniques with six different metrics for a publically available MRI database.

2 Materials and Methods

A low contrast input image I with size $M \times N$ was considered. The dynamic range of the input image was represented as $[I_{min}, I_{max}]$. I_{min} represented the minimum gray intesity and I_{max} represented the maximum gray intensity of the input image. The histogram of the input image was calculated by

$$H(k) = n_k,\ I_L \le k \le I_U \quad \ldots (1)$$

where, n_k represented the occurrence of the intensity k. To preserve the mean brightness of the input image, the histogram of the input image was divided into two parts by using a threshold value. The threshold value was the median of unique intensities of the input image and it is represented as I_{med}. It was defined as,

$$I_{med} = median([I_L, I_U]) \quad \ldots (2)$$

The threshold value splitted the histograms into two sub-histograms namely H_L and H_U.

$$H_L(k) = [h_k(x,y);\ 1 \le x \le X,\ 1 \le y \le \mathrm{Y}\} \quad \text{where, } I_{\min} \le \mathrm{k} < I_{med} \quad \ldots (3)$$

$$H_U(k) = \{h_k(\mathrm{x}, \mathrm{y});\ 1 \le \mathrm{x} \le X,\ 1 \le y \le Y\} \quad \text{where, } I_{med} \le \mathrm{k} \le I_{max} \quad \ldots (4)$$

The histograms of the intensities were calculated concerning the spatial locations. The input image was divided into non-overlapping spatial grids. The number of spatial grids in the image was determined from the aspect ratio and the number of unique intensities present in the image.

$$X = \lfloor (K \times ar)^{0.5} \rfloor,\ Y = \left\lfloor \left(\frac{K}{ar}\right)^{0.5} \right\rfloor \quad \ldots (5)$$

where, K was represented by the number of unique intensities present in the input image and ar denotes the aspect ratio of the image. From the sub-histograms, the Renyi entropy was calculated by using the following computation.

$$R_L(k) = \frac{1}{1-\alpha}\log_2\left(\sum_{x=1}^{X}\sum_{y=1}^{Y}\left(h_k(x,y)\right)^{\alpha}\right) \quad \dots (6)$$

$$R_U(k) = \frac{1}{1-\alpha}\log_2\left(\sum_{x=1}^{X}\sum_{y=1}^{Y}\left(h_k(x,y)\right)^{\alpha}\right) \quad \dots (7)$$

From the Renyi entropy the discrete function was obtained by the following equation.

$$f_L(k) = \frac{R_L(k)}{\sum_{i=I_{\min}}^{I_{med}-1} R_L(i) for\ \mathrm{i}\neq k} \quad \dots (8)$$

$$f_U(k) = \frac{R_U(k)}{\sum_{i=I_{med}}^{I_{\max}} R_U(i) for\ \mathrm{i}\neq k} \quad \dots (9)$$

The obtained density functions were normalized by using the following equations.

$$f_L(k) = \frac{f_L(k)}{\sum_{i=I_{\min}}^{I_{med}-1} f_L(i)} \quad \dots (10)$$

$$f_U(k) = \frac{f_U(k)}{\sum_{i=I_{med}}^{I_{\max}} f_U(i)} \quad \dots (11)$$

From the normalized discrete function adaptive clipping limit was calculated and based on the clipping limit, the modified normalized discrete function was obtained.

$$f_{Lnew}(k) = \min[f_L(k), mean(f_L), median(f_L)] \quad \dots (12)$$

where, $k = I_{\min}, I_{\min}+1, \dots, I_{med}-1$.

$$F_U(k) = \sum_{i=I_{med}}^{k} \mathrm{f}_{Unew}(i) \quad \dots (13)$$

where, $k = I_{med}, I_{med}+1, \dots, I_{\max}$. From the normalized density functions the cumulative distributions were obtained as follows

where, $k = I_{\min}, I_{\min} + 1, \dots, I_{med}-1$. … (14)

$$F_U(k) = \sum_{i=I_{med}}^{k} \mathrm{f}_{Unew}(i) \quad \dots (15)$$

where, $k = I_{med}, I_{med} + 1, \dots, I_{\max}$ after calculating the clipping limit, the mapping function was obtained,

$$J(k) = \begin{cases} I_0 + (I_{med} - 1 - I_0) \times F_{Lk} \\ where,\ k = I_{\min}, I_{\min}+1, \dots, I_{med}-1 \\ I_{med} + (255 - I_{med}) \times F_{Uk} \\ where,\ k = I_{med}, I_{med}+1, \dots, I_{\max} \end{cases} \quad \dots (16)$$

To enhance the image further, Discrete Cosine Transformation (DCT) was applied to the image J. where, J is the enhanced image obtained from the mapping function.

$$C = DCT(J) \quad \dots (17)$$

The DCT coefficients of the image J was altered by multiplying the weighting function $w(m, n)$

$$C_{\mathrm{mod}}(m,n) = C(m,n)w(m,n) \quad \dots (18)$$

where, m and n vary from 1 to M and 1 to N respectively. $w(m, n)$was calculated by

$$w(m,n) = \left(1 + \frac{\beta-1}{M-1} \times m\right)\left(1 + \frac{\beta-1}{N-1} \times n\right) \quad \dots (19)$$

where, $\beta \geq 1$. To get higher enhancement, β value of should be high. For the automatic selection of β, it was estimated from the normalized density function.

$$\beta = \left(\sum_{k=I_{\min}}^{I_{\max}} f_k \log_2(f_k)\right)^{\gamma} \quad \dots (20)$$

where, $0 \leq \gamma \leq 1$. Inverse DCT (IDCT) of the modified image resulted in a overall enhanced image J_{en}.

$$J_{en} = IDCT(C_{\mathrm{mod}}) \quad \dots (21)$$

3 Results and Discussion

In the medical imaging system, to diagnose the abnormality in any part of the human body, improved visual quality of images were required. Proper contrast enhancement techniques can achieve it. When comparing the image's perceived quality, the image's properties were considered, such as contrast improvement, artifacts, and over enhancement.

The proposed method for contrast enhancement deliberated in Section 2 was experimented on an MRI brain image database and evaluated by qualitative and quantitative analysis. Qualitative analysis was based on visual examination and offers details about annoying artifacts, unusual enhancement, and over enhancement. Quantitative analysis was done through the performance measures such as Absolute Mean Brightness Error

(AMBE), Contrast Improvement Index (CII), Structural Similarity Index (SSIM), Discrete Entropy (DE), Power Signal to Noise Ratio (PSNR), and Gradient Magnitude Similarity Deviation (GMSD). In this sectoin, qualitcve and quantitative analysis, along with the results, were discussed.

3.1 Qualitative analysis

I1, I2, I3, I4, and I5 were chosen to demonstrate the effect of contrast enhancement produced by the algorithms for the various MRI brain images[11]. The first column of Fig. 1 showed the input images; the second column to the fifth column represented the contrast-enhanced images of HE, BBHE, AIEBHE, EEBHE, and the proposed method, respectively. From the second column of Fig. 1, it was noticed that the HE produced an over enhanced image with visual artifacts at the edges. To resolve the preceding limitations, Bi-histogram equalization methods like BBHE and AIEBHE were proposed. From the third column, it was observed that BBHE results in an over-enhanced image, but it diminishes the artifacts near edges. The over-enhancement due to the histogram spikes presented input low contrast image. This algorithm had no regulation over the rate of enhancement that may contribute to an abnormally bright image. From the second column of Fig. 1, it was noticed that the HE produced an over enhanced image with visual artifacts at the edges. Bi-histogram equalization methods like BBHE and AIEBHE have been proposed to resolve the preceding limitations. From the third column, it wasobserved that BBHE results in an over-enhanced image, but it diminished the artifacts near edges. The over-enhancement due to the histogram spikes presented input low contrast image. This algorithm had no regulation over the rate of enhancement that may contribute to an abnormally bright image.

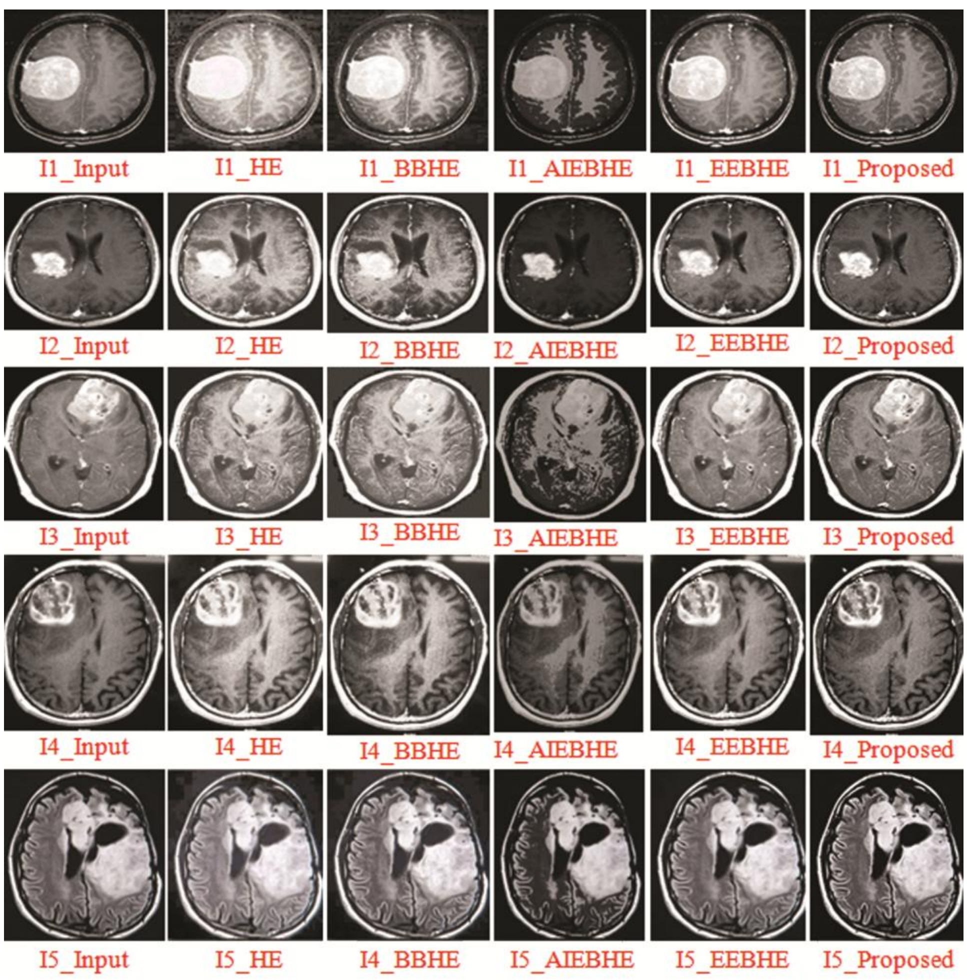

Fig. 1— Contrast enhanced images from MRI brain image database[11]. (a) low contrast images (I1_Input-I5_Input), (b) enhanced images (I1_HE-I5_HE), (c) Over-enhanced image (I1_BBHE-I5_BBHE)[4], (d) (I1_AIEBHE-I5_AIEBHE)[7], (e) (I1_EEBHE-I5_EEBHE)[8], and (f) proposed method (I1_Proposed-I5_Proposed).

Table 1 — Various performance measures[12-17]

Measures	Mathematical representation	Highlights	Range
AMBE[12]	$AMBE = \lvert M_I - M_J \rvert$	1. Preservation of mean brightness. 2. Amount of mean shifting.	0 to any positive value within the dynamic range.
CII[13]	$CII = \frac{E(C_{loc}(J))}{E(C_{loc}(I))}$	Local contrast improvement concerning neighbours.	More than input image.
SSIM[14]	$SSIM(I,J) = \frac{(2\mu_I\mu_J+c_1)(2\sigma_{I,J}+c_2)}{(\mu_I^2+\mu_J^2+c_1)(\sigma_I^2+\sigma_J^2+c_2)}$;$c_1$=6.5;$c_2$=58.5	Degradation due to processing	[0,1]
DE[15]	$E(I) = -\sum_{l=1}^{P} p(i_l)\log_2 p(i_l)$	Amount of information loss.	Maximum value: 8 bits/symbol.
PSNR[16]	$PSNR = 10\log_{10}\left[\frac{(L-1)^2}{\frac{1}{M\times N}\sum_x\sum_y \lvert I(x,y)-J(x,y)\rvert^2}\right]$	1. Reconstruction quality. 2. Amount of deviation between original and enhanced image.	30 - 50 dB for an 8-bit image
GMSD[17]	$GMSD = \sqrt{\frac{1}{M\times N}\sum_{i=1}^{M\times N}(G(i)-GM)^2}$	Structural deviation between the input and the processed image.	[0,1]

From the third column of Fig. 1, it was observed that the over-enhancement problem can be overcome by AIEBHE method, which employed an adaptive clipping limit to control the over enhancement. Still, the clipping limit lead to information loss, and it might produce histogram pits so that it resulted in an under enhanced image. In order to enhance the edge details, EEBHE method was proposed. From the fourth column it was perceived that EEBHE resulted in an the enhanced images with improved edge strength, due to the guided filtering employed in the EEBHE method. But the resultant image did occupy the entire dynamic range of the grayscale that may lead to less contrast improvement in the local region of the enhanced image. The fifth column showed the enhanced images which were produced by the proposed method. From Fig. 1, it was observed that the proposed method resulted in an enhanced image with improvements in contrast and without loss of information details due to the employment of Renyi entropy the edge information were preserved in the enhanced image. It was visible from the artifact free edges of the enhanced images. From the qualitative analysis, the proposed resulted, in contrast, enhanced and artifact-free images when compared to the methods available in the literature.

3.2 Quantitative analysis

The qualitative analysis was validated numerically with the help of quantitative analysis. The quantitative metrics used in this paper are discussed in the Table 1.

The proposed Renyi entropy-based method aimed to increase the contrast of the low contrast images while preserving the image's information details and preserving the edge information. The CII metric was used to compute the local contrast improvement that meets the objectives. DE calculated the preservation of information details, and SSIM metrics evaluated the preservation of edge information. Mean brightness preservation was one of the objectives of HE based algorithms, and AMBE measures it. The quantitative metrics' values for the sample images were tabulated in Table 2 to Table 7, and the most acceptable values were highlighted for each measure.

The average value of the sample images of the various metrics was assessed in Fig. 2. The completed

Table 2 — AMBE[12] values of various contrast enhancement techniques

Methods/Image	*HE*[2]	*BBHE*[4]	*AIEBHE*[7]	*EEBHE*[8]	*Proposed*
I1	51.7	28.26	26.59	13.96	2.64
I2	46.76	31.05	33.72	9.18	3.09
I3	21.17	30.05	36.76	7.82	3.81
I4	35.34	17.09	20.16	8.35	3.71
I5	31.43	38.04	18.85	6.82	5.78
Avg	37.26	25.04	27.22	9.2	3.8

Table 3 — CII[13] values of various contrast enhancement techniques

Methods/ Image	*HE*[2]	*BBHE*[4]	*AIEBHE*[7]	*EEBHE*[8]	*Proposed*
I1	0.77	0.82	1.12	0.88	1.34
I2	0.85	0.96	1.06	1	1.38
I3	0.97	0.92	1.6	1.01	1.45
I4	0.93	0.99	1.16	1.09	1.9
I5	1.12	1.14	1.12	1.05	1.29
Avg	0.93	0.97	1.23	1.01	1.6

database had been used in all the methods under consideration, including the proposed approach to get a more accurate analysis. Table 8 brought out the average of the performance metrics for all the databases.

The AMBE values for the sample images were listed in Table 2. It was observed that the proposed method produces a minimum shift compared to the other methods. It showed that the separation of histogram using its median value results in higher preservation of mean brightness.

Contrast improves index was used to measure the local contrast improvement in the enhanced image. The local improvement, in contrast, helped to differentiate the objects from one another. For medical images to diagnose the abnormalities, the objects should be distinguished by employing their contrast. The CII values were listed in Table 3. It was noticed from the table that the proposed method results in a high CII value compared to the other techniques. It indicated that the contrast improvement helps to identify the abnormalities.

SSIM values for the sample images were tabulated in Table 4. From the Table 4, it was inferred that the process under consideration provides a high SSIM value. It indicated that the proposed method preserved

Table 4 — SSIM[14] values of various contrast enhancement techniques

Methods/ Image	*HE[2]*	*BBHE[4]*	*AIEBHE[7]*	*EEBHE[8]*	*Proposed*
I1	0.6	0.65	0.64	0.87	0.9
I2	0.59	0.59	0.72	0.85	0.89
I3	0.68	0.68	0.61	0.83	0.83
I4	0.82	0.86	0.85	0.8	0.84
I5	0.79	0.85	0.8	0.73	0.75
Avg	0.69	0.72	0.72	0.8	0.84

Table 5 — DE[15] values of various contrast enhancement techniques

Methods/ Image	*HE[2]*	*BBHE[4]*	*AIEBHE[7]*	*EEBHE[8]*	*Proposed*
I1	5.5	6.55	5.81	6.72	6.82
I2	5.28	6.21	5.75	6.41	6.51
I3	5.21	6.18	5.54	6.54	6.55
I4	5.89	7.15	6.93	7.22	7.28
I5	5.58	6.81	6.37	7.01	7.01
Avg	5.49	6.5	6.08	6.75	6.83

Table 6 — PSNR[16] values of various contrast enhancement techniques

Methods/ Image	*HE[2]*	*BBHE[4]*	*AIEBHE[7]*	*EEBHE[8]*	*Proposed*
I1	13.23	17.77	16.34	21.47	27.36
I2	13.58	15.93	15.8	21.59	27.15
I3	19.23	16.66	14.12	23.31	24.86
I4	15.96	19.66	20.13	22.57	25.72
I5	16.99	21.75	18.69	18.22	22.2
Avg	15.8	18.35	17.02	23.18	25.4

Table 7 — GMSD[17] values of various contrast enhancement techniques

Methods/ Image	*HE[2]*	*BBHE[4]*	*AIEBHE[7]*	*EEBHE[8]*	*Proposed*
I1	0.24	0.15	0.17	0.07	0.05
I2	0.17	0.19	0.08	0.09	0.054
I3	0.16	0.2	0.2	0.08	0.08
I4	0.1	0.08	0.08	0.07	0.06
I5	0.17	0.13	0.11	0.08	0.07
Avg	0.17	0.15	0.13	0.07	0.06

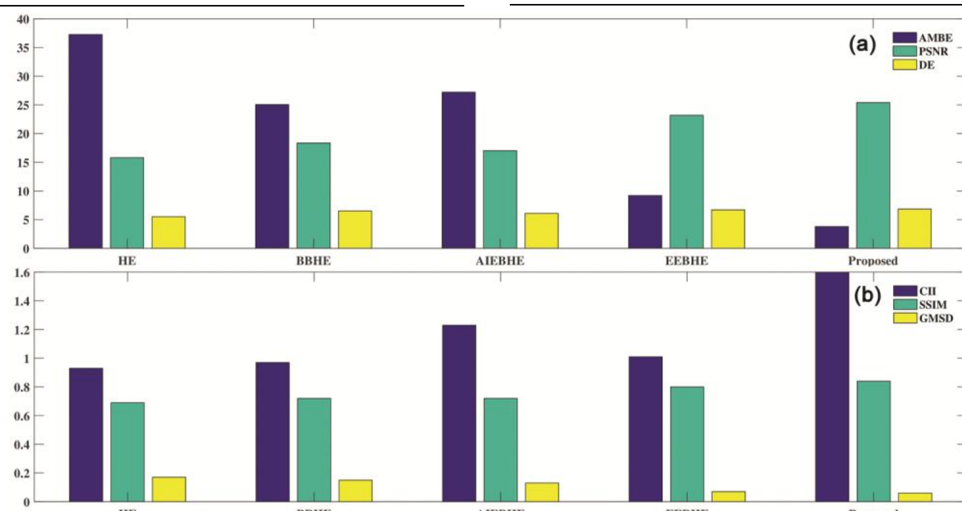

Fig. 2 — Comparison plot of performance measures for various enhancement techniques: (a) AMBE[12], PSNR[16], & DE[15], and (b) CII[13], SSIM[14], & GMSD[17].

Table 8 — Average values for various metrics for various contrast enhancement techniques[2,4,7-8]

Methods/ Metrics	*HE*[2]	*BBHE*[4]	*AIEBHE*[7]	*EEBHE*[8]	*Proposed*
AMBE	66.92	35.61	26.12	16.88	2.35
CII	0.68	0.84	1.15	0.99	1.69
SSIM	0.5	0.55	0.68	0.8	0.8
DE	4.84	5.75	5.15	6.1	6.14
PSNR	11.59	15.82	16.94	19.71	26.45
GMSD	0.2	0.17	0.13	0.09	0.05

the edge information after enhancement compared to the other techniques discussed in the literature.

The information required for the exact diagnosis in medical images was higher when compared to standard images. So entropy was considered one of the vital parameters in medical image enhancement. The entropy value for the sample images was listed in Table 5. It was observed that the proposed method results in a higher entropy value than other associated methods. It was due to the usage of Renyi entropy in the formulation of discrete function.

The PSNR values for the sample images were listed in Table 6. It was noticed that the proposed method produces a high PSNR value compared to the other techniques. This high PSNR value infers that the proposed method had good reconstruction quality post enhancement. GMSD values were listed in Table 7. The low weight of GMSD represented a less structural deviation in the processed image. From the Table 8, it was interpreted that the proposed method preserves the structural detail of the enhanced image.

From the results, it was clear that the proposed method encompasses better contrast improvement, preservation of edge details, good information preservation, and optimum mean brightness against the other techniques discussed in the literature.

4 Conclusion

In this paper, a computationally efficient contrast enhancement algorithm using Renyi entropy has been proposed. Renyi entropy has been used in this paper because of its performance towards the prominent amplitudes. The entropy has been calculated from the sub histogram obtained by dividing the histogram into two, based on its median value. A mapping function has been calculated from the distribution function acquired by normalizing the entropy function. Qualitative and quantitative experiments have shown that the proposed method can enhance and produce better results with low complexity.

References

1 Kaur H, & Rani J, *2016 International Conference on Wireless Communications, Signal Processing and Networking (IEEE, Chennai)*, ISBN:978-1-4673-9339-3, 2016, p. 770.

2 D Vijayalakshmi, Nath M K & Acharya O P, *Sens Imaging*, 21 (2020)1.

3 Dhurairajan V, Kumari T S, & Bhavani C A, *Defence S&T Technical Bulletin*, 11 (2018) 113.

4 Kim Y T, *IEEE Trans Consum Electron*, 43 (1997) 1.

5 Wang Y, Chen Q, & Zhang B, *IEEE Trans Consum Electron*, 45 (1999) 68.

6 Sim K S, Tso C P, & Tan Y Y, *Pattern Recognit Lett*, 28 (2007) 1209.

7 Tang J R, & Isa N A M, *Comput Electr*, 40 (2014) 86.

8 Mun J, Jang Y, Nam Y, & Kim J, *J Vis Commun Image Represent*, 58 (2019) 688.

9 Shanmugavadivu P, & Balasubramanian K, *Comput Electr*, 40 (2014) 757.

10 Kanmani M, & Narsimhan V, *Multimed Tools Appl*, 77 (2018) 23371.

11 MRI brain image dataset: https://www.kaggle.com/navoneel/brain-mri-images-for-brain-tumor-detection.

12 Chen S D, & Ramli A R, *IEEE Trans Consum Electron*, 49 (2003)1310.

13 Zeng P, Dong H, Chi J, Xu X, 2004 *IEEE International Conference on Robotics and Biomimetics (IEEE, Shenyang)*, ISBN:0-7803-8641-8, 2004, p. 574.

14 Wang Z, Bovik A C, Sheikh H R, & Simoncelli E P, *IEEE Trans Image Process*, 13 (2004) 600.

15 Shannon C E, *Bell Syst Tech*, 27 (1948) 379.

16 Gonzalez R C, & Woods R E, *Digital Image Processing (Pearson Education, New Delhi)*, 3rd Edn, ISBN:978-81-317-2695-2, 2009, p. 354.

17 Xue W, Zhang L, Mou X, & Bovik A C, *IEEE Trans Image Process*, 23 (2014) 684.