Estimating DSGE Models using Multilevel Sequential Monte Carlo in Approximate Bayesian Computation

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Dynamic Stochastic General Equilibrium (DSGE) models allow for probabilistic estimations with the aim of formulating macroeconomic policies and monitoring them. In this study, we propose to apply the Sequential Monte Carlo Multilevel algorithm and Approximate Bayesian Computation (MLSMC-ABC) to increase the robustness of DSGE models built for small samples and with irregular data. Our results indicate that MLSMC-ABC improves the estimation of these models in two aspects. Firstly, the accuracy levels of the existing models are increased, and secondly, the cost of the resources used is reduced due to the need for shorter execution time.

Introduction

Recently, the importance of the Dynamic Stochastic General Equilibrium (DSGE) models has increased due to its applications in the analysis of economic cycles and in the detection of recessions[1-3]. These models are being evaluated by the result obtained in the standard deviation, the size of the sample, as well as the regularity of the data. The evaluation of the models constructed in the previous literature has shown different adjustments. For example, models constructed with non-Bayesian techniques have provided an adjustment of 0.17-0.20 standard deviation with large (more than 100 observations) and regular samples[4,5], but their adjustments are between 0.28-0.36 when constructed with small samples and irregular data[6,7]. On the other hand, models that have used Bayesian probability computational technique have obtained better results than those previously described. With large samples and with regular data these models have obtained a precision between 0.14-0.38[1,8], while in the case of small samples and irregular data the results have been 0.24-0.40[3,9]. These previous works applied computational methods such as Particle Markov Chain Monte Carlo, Sequential Monte Carlo Squared, Approximate Bayesian Computation Sequential Monte Carlo and Kalman quadratic filter. In order to solve the problems of accuracy of the existing DSGE models, this paper develops a Multilevel Sequential Monte Carlo algorithm in its variant of Approximate Bayesian Computation (MLSMC-ABC), which has already demonstrated its methodological superiority in other economic areas in carrying out accurate sampling with few observations and with irregular data distributions[10,11]. Our results show a more robust estimation in terms of accuracy, a saving in calculation costs and, a better behaviour with small and irregular samples compared to the Bayesian computational methods recently used in the previous literature[3,8,9,12].

Methods

In this work, we have built a DSGE model using Bayesian Monte Carlo ABC methods. A sample of data has been drawn from which the variables that represent the model have been extracted. The DSGE model used is the popular Smets-Wouters model, to which the MLSMC-ABC algorithm is applied for its resolution, obtaining the optimal values of the parameters of the equations that represent the model. Finally, we apply the Bayesian selection function Marginal Data Density (MDD) with which the standard deviations (STD) of the chosen computational algorithm are obtained. The methodological process applied in the present study is illustrated in Figure 1.

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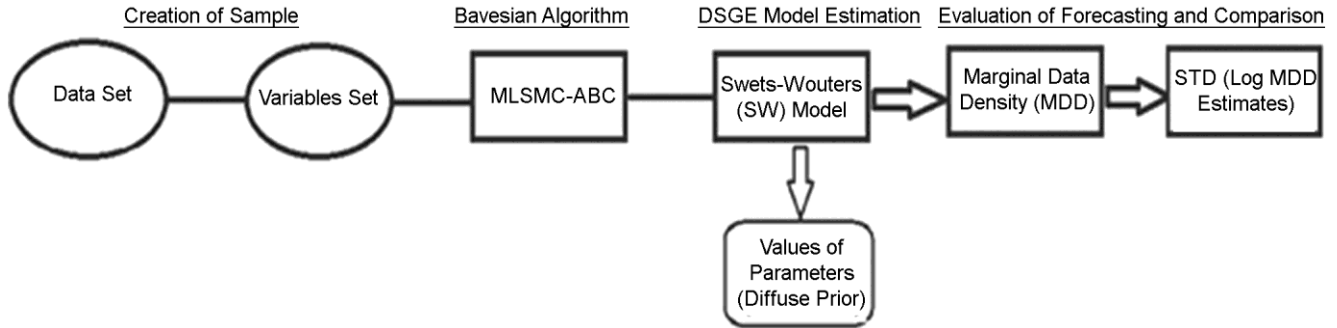E-mail: jasantos@ualg.pt

Fig. 1 — Flowchart of research

Multilevel Sequential Monte Carlo in Approximate Bayesian Computation (MLSMC-ABC)

ABC is a method without likelihood functions by which the previous distribution is updated with the information provided by the moments[11]. Multilevel Monte Carlo samplers for ABC has the property of recalculating the weights, in order to adapt to the data sample applied in an optimal way, so it is quite efficient working with a small dimensionality of data. The approach of Beskos *et al*.[13] is to avoid the problem of optimization with small samples, the method of samples of importance is applied in order to reduce the variance, as an alternative method to the sampling of acceptance-rejection commonly used in Monte Carlo simulations, forming a first distribution to which the sequential Monte Carlo samplers (SMC)[14] are applied to perform the simulation. Therefore, the function of weight distribution is defined as $G_n(x)=k_{n+1}(x)/k_n(x)$ and the discrete-time approximation function of k_n as $Z_n = \int_E K_{\in_n}(y,u) f(u|\theta) \pi(\theta) d(u, \theta)$, that help obtain the multilevel identity equation (1).

$$\eta L(\varphi) = \frac{Z_0}{Z_1}\eta_0(G_0\varphi) + \sum_{l=2}^{L} \eta_{l-1}\left(\left(\frac{Z_{l-1}}{Z_l}G_{l-1} - 1\right)\varphi\right) \quad \dots(1)$$

In order to apply the SMC sampler to obtain a set of samples that represent all the possible scenarios sequentially distributed in a way $\boldsymbol{\eta_0}, \boldsymbol{\eta_1} \dots \boldsymbol{\eta_{L-1}}$[11,15], the estimation process begins once the particle population is organized by sampling with random variables, independent and identically distributed from $\boldsymbol{\eta_0}$. Then, at each step, a Markov Chain Monte Carlo (MCMC) is resampled and applied to transform the particles. We denote it as $(\boldsymbol{X_0^{1:N_0}}, \dots, \boldsymbol{X_{L-1}^{1:N_{L-1}}})$ with $+\infty> \text{N}_0 \ge \text{N}_1 \ge \dots \text{N}_{L-1} \ge 1$, the particles, after the transformation, are re-sampled, denoting it as $\boldsymbol{X_l^{1:N_l}}$, according to the weights $\boldsymbol{G_l(X_l^i)} = (\frac{k_{l+1}}{k_l})(\boldsymbol{X_l^i})$, and

for the indices $\boldsymbol{l \in \{1, \dots, L-1\}}$. $\{\boldsymbol{M_l}\}_{1 \trianglelefteq l \trianglelefteq L-1}$ denotes a sequence of MCMC kernels, with the property $\boldsymbol{\eta_l M_l = \eta_l}$. For $\boldsymbol{\varphi: E \to \Re, l \in \{1, \dots, L\}}$, we would have the estimator of $\boldsymbol{E_{\eta_{l-1}}[\varphi(X)]}$. This mathematical development can be checked in the work of Beskos *et al*.[13]. The joint likelihood distribution for the SMC algorithm is:

$$\prod_{i=1}^{N_0} \eta_0(dx_0^i) \prod_{l=1}^{L-1} \prod_{i=1}^{N_l} \frac{\eta_{l-1}^{N_t-1}(G_{l-1}M_l(dx_l^i))}{\eta_{l-1}^{N_t-1}(G_{l-1})} \quad \dots(2)$$

Advancing after the previous procedure, you would get samples $\{X_L^i\}_{i=1}^{N_l}$, so a standard SMC estimate according to the equation (1) would be $\eta_L^{N_L}(g)$; at this point, the previous samples are discarded[11]. An approximation of SMC for equation (1) would be as it appears in equation (5).

$$\hat{Y} = \sum_{l=2}^{L}\left\{\frac{\eta_{l-1}^{N_t-1}(\varphi G_{l-1})}{\eta_{l-1}^{N_t-1}(G_{l-1})}\eta_{l-1}^{N_t-1}(\varphi)\right\}\frac{\eta_0^{N_0}(\varphi G_0)}{\eta_0^{N_0}(G_0)} \quad \dots(3)$$

Showing that the mean square error (MSE) of MLSMC method is limited by:

$$[\eta_L(\varphi) - \eta_\infty(\varphi)]^2 \frac{C}{N_0} + C\sum_{l=2}^{L}\frac{1}{N_{l-1}}\left\|\frac{Z_{l-1}}{Z_l}G_{l-1} - 1\right\|_\infty^2 +$$
$$\sum_{2 \le l \le q \le L}\left\{\left\|\frac{Z_{l-1}}{Z_l}G_{l-1} - 1\right\|_\infty \left\|\frac{Z_{q-1}}{Z_q}G_{q-1} - 1\right\|_\infty \left(\frac{k^{q-1}}{N_{l-1}} + \frac{1}{N_{l-1}^{1/2}N_{q-1}}\right)\right\}$$
$$\dots(4)$$

where $\|.\|_\infty$ shows the simulations, property and $C < +\infty, \mathbf{k} \in (0, 1)$ they are constants that do not depend on l, q.

Smets-Wouters (SW) Model

The SW model is a standard-scale New Keynesian macroeconomic model that has become a reference point in the literature of the DSGE models[16]. In our

analysis, we consider two distributions. The first distribution, which we refer to as a prior standard, includes the initial fixed parameters used by the previous literature for the SW model[3]. The second distribution is obtained after estimating the model and we call it diffuse prior (Table 2). The SW model is estimated based on seven macroeconomic time series and can be checked these initial equations in the work of Smets and Wouters[16]. As main indicators, this model includes the GDP growth, the rate of inflation, the interest rate, the growth of consumption, the growth of investment, the growth of wages, and the hours worked. In addition, given that the technological impact in the SW model is assumed to be stationary, the model variables are not transformed to introduce seasonality, and therefore, the growth rate of the technological impact does not appear in the measurement equations. Based on the aforementioned information, the SW model uses the following fixed parameters in the estimation in its previous

distributions (standard prior) are denoted as follows: $\delta = 0.025$, $g_y = 0.18$, $\lambda_w = 1.50$ and $\varepsilon_w = \varepsilon_p = 10$, where δ is the depreciation rate, g_y is the exogenous spending-GDP rate, λ_w is the steady-state mark-up in the labour market and $\varepsilon_w, \varepsilon_p$ are the parameters of the aggregators in the goods and the labour market.

In addition, the following additional parameters are used to estimate the SW model and are denoted as follows: $\varphi = 5.00$, $\sigma_c = 1.5$, $h = 0.7$, $\xi_w = 0.7$, $\sigma_l = 2$, $\xi_p = 0.7$, $\tau_w = 0.5$, $\tau_p = 0.5$, $r_\pi = 2$ and $\alpha = 0.3$, where φ is the steady-state elasticity of the capital adjustment, σ_c is the intertemporal elasticity of substitution, h is the habit parameter, ξ_w is the degree of wage stickiness, σ_l is the elasticity in labour supply, ξ_p is the degree of price stickiness, τ_w is the degree of indexation to past wage, τ_p is the degree of indexation to past inflation, r_π is the real interest and α is the share of capital in production. The log-linearized equilibrium conditions are necessary as the final step for estimating the model. These conditions are expressed in the work of Smets-Wouters[16]. These conditions mentioned are composed for fourteen endogenous variables defined as follows: output (y_t), consumption (c_t), investment (i_t), the current value of the capital stock (q_t), capital (k_t^s), the accumulation of the installed capital (k_t), the capital utilisation rate (z_t), the rental rate of capital (r_t^k), the price mark-up (μ_t^p), inflation (π_t), the wage mark-up (μ_t^ω), real wage (ω_t), labour services (l_t), nominal interest rate (r_t). The stochastic behaviour of the system of linear rational

Table 1 — SW Model: Log MDD Estimates

Algorithm (Method)	MEAN(Log MDD)	STD(Log MDD)
Standard Prior		
SMC	-906.528	0.39
ABC-SMC	-863.284	0.27
MLSMC-ABC	-827.446	0.14
Diffuse Prior		
SMC	-881.025	0.28
ABC-SMC	-858.929	0.21
MLSMC-ABC	-815.272	0.09

Table 2 — SW Model with Diffuse Prior (Posterior Comparison)

	SMC			ABC-SMC			MLSMC-ABC		
Parameter	Mean	[0.05,0.95]	STD(Mean)	Mean	[0.05,0.95]	STD(Mean)	Mean	[0.05,0.95]	STD(Mean)
σ_l	3.02	[1.43, 5.14]	0.05	3.05	[1.96, 4.17]	0.0013	3.03	[2.37, 4.09]	0.00055
l	-0.04	[-2.94, 2.96]	0.07	-0.02	[-1.92, 1.98]	0.0024	-0.05	[-1.95, 1.91]	0.00071
ι_p	0.13	[0.01, 0.28]	0.01	0.14	[0.04, 0.23]	0.0064	0.11	[0.07, 0.18]	0.00042
Φ	1.68	[1.48, 1.92]	0.01	1.69	[1.54, 1.93]	0.0040	1.70	[1.57, 1.91]	0.00084
h	0.72	[0.57, 0.77]	0.02	0.66	[0.59, 0.78]	0.0032	0.67	[0.58, 0.76]	0.00038
r_π	2.75	[2.11, 3.52]	0.03	2.78	[2.15, 3.21]	0.0035	2.79	[2.24, 3.13]	0.00027
ρ_b	0.22	[0.03, 0.46]	0.01	0.23	[0.08, 0.42]	0.0088	0.23	[0.13, 0.39]	0.00034
ϕ	8.01	[4.15, 12.51]	0.16	7.97	[5.36, 12.05]	0.017	8.05	[6.38, 10.12]	0.0014
σ_p	0.16	[0.11, 0.22]	0.01	0.12	[0.11, 0.20]	0.0045	0.13	[0.10, 0.18]	0.0006
ξ_p	0.70	[0.63, 0.81]	0.01	0.73	[0.63, 0.81]	0.0039	0.74	[0.65, 0.80]	0.00045
ι_w	0.72	[0.38, 0.95]	0.03	0.72	[0.44, 0.92]	0.0075	0.71	[0.61, 0.85]	0.00073
μ_p	0.79	[0.54, 0.97]	0.02	0.80	[0.58, 0.94]	0.0094	0.78	[0.66, 0.87]	0.00062
ρ_w	0.67	[0.19, 0.97]	0.04	0.71	[0.46, 0.99]	0.0098	0.70	[0.53, 0.92]	0.00036
μ_w	0.65	[0.09, 0.98]	0.05	0.63	[0.16, 0.97]	0.0084	0.61	[0.43, 0.91]	0.00032
ξ_w	0.95	[0.81, 0.97]	0.02	0.94	[0.82, 0.99]	0.0057	0.96	[0.85, 0.99]	0.00023

expectations equations is addressed by seven exogenous shocks: total factor productivity (ε_t^a), investment-specific technology(ε_t^i), risk premium (ε_t^b), exogenous spending (ε_t^g), price mark-up (ε_t^p), wage mark-up (ε_t^ω) andmonetary policy (ε_t^r).

Results and Conclusions

For the construction of the model, data from the US economy for the period 2008Q1-2018Q4 have been used, obtained from the Federal Reserve Economic Data (FRED) of the Federal Reserve Bank of St. Louis. The sample consists of 40 observations with the purpose of making more demanding estimates than those made in previous studies, where samples with 100 or more observations were applied[3,9]. In addition to estimating the diffuse prior distribution, it is usual to estimate the so-called Marginal Data Density (MDD)$p(Y) = \int p(Y/\theta)\ p(\theta)\ d\theta$ for DSGE models[3,11], since it provides a summary of the evidence contained in the results obtained by the model, becoming an essential indicator for the comparison of models and predictive analysis[17]. Table 1 shows the estimates of the MDD after estimating the SW model. These results demonstrate the greater stability offered by the MLSMC-ABC algorithm compared to the rest, above all, based on the deviations obtained. Also, MLSMC-ABC improves the results obtained from previous work with the SMC and ABC-SMC algorithms[3,9].

We estimate posterior moments based on our proposed MLSMC-ABC algorithm as well as SMC-ABC and a standard SMC algorithm. To assess the accuracy of the Monte Carlo methods, we run these algorithms 20 times and compute means and standard deviations of later moment estimates across runs. Table 2 shows the results of the estimates made by the different algorithms with the diffuse prior. Also included is a confidence interval of 5% where the value of each estimated parameter fluctuates. For example, the standard deviation of the estimate of the mean for μ_p (the coefficient for the salary margin) is 0.00062. The average estimate of this coefficient is 0.78, while for anyestimation, the coefficient of this variable would yield a mean value between 0.66 and 0.87 within a confidence level of 95%.Therefore, the results of the standard deviation of the mean estimated from MLSMC-ABC, are much lower than those obtained by SMC or ABC-SMC, and even, taking as reference the results obtained in previous works[3,9].

This study presents an alternative simulation technique to estimate DSGE models. We show that, when properly adapted to DSGE models, the MLSMC-ABC technique is more robust than other commonly used algorithms such as Sequential Monte Carlo and Approximate Bayesian Computation Sequential Monte Carlo. Our results show high robustness of the MLSMC-ABC algorithm for small samples with irregular data, a concern shown by the previous literature in the estimation of DSGE models. Finally, the great precision shown by this new algorithm also implies an improvement in the optimization of the calculation of macroeconomic forecasting without the need to use a large amount of available resources, nor does it have to make a broad specification of the DSGE models, being vital importance for public institutions and other interest groups in macroeconomic analysis.

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